On the Role of Convective Momentum Exchange in the Mesoscale Gravity Wave

By S. Matsumoto and K. Ninomiya

Meteorological Research Institute, Tokyo, Japan

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Abstract

The mesoscale disturbances are characterized by the gravity wave which accompanies the field of convergence of the order of 10^{-4}sec^{-1} . This amount of convergence strongly controls the convective activity and has a significant influence upon the wind field through the momentum exchange process.

Numerical experiments are made by means of the simplified two layer version of sheared flow incorporated with convective momentum transfer which is parameterized empirically. It is shown that the downward momentum transfer in the lower-level convergence area has an effect of increasing the lower-level wind velocity and decreasing the upper-level wind velocity and thus results in the self-intensification of lower-level convergence and increasing the velocity of propagation downstream. The redistribution in the wind profile, however, contributes to the strengthening of frictional dissipation, which is partly compensated by the energy conversion from convective kinetic energy.

1. Introduction

There have been so many analytical case studies concerning mesoscale disturbances since they are closely related to the local violent phenomena such as thunderstorm (Fujita, 1959), squall line (Tepper, 1950), heavy snowfall (Matsumoto, Ninomiya and Akiyama, 1967 a) and so on. However, little is known about the mechanism or the dynamics of mesoscale phenomena. According to Tepper's (1950) inference, a prefrontal squall line is considered as a kind of gravity wave. Matsumoto and Ninomiya (1965) pointed out that the mesoscale disturbance observed in a dome-shaped cold airmass has the characteristics of a gravity wave in many respects, such as phase relationship, phase velocity, pressure anomaly and so on. Sasaki (1959) made a numerical experiment basing on the primitive equation to explain the development of a prefrontal squall line. However, the gravity wave embedded in the density stratification is, generally speaking, a stable phenomenon and does not develop by itself unless some other external conditions come into play.

Matsumoto, Ninomiya and Akiyama (1967 a) obtained some important information concerning the influence of convective activity upon mesoscale

disturbances by making numerical analyses of dynamical balance. They suggested that the mesoscale convergence of the order of 10^{-4}sec^{-1} provides a favorable condition for cumulus development and that the cumulus activity contributes to redistribution of physical properties such as vapor, heat, momentum and so on. The momentum exchange tends to increase the wind velocity in the lower levels and decrease the wind velocity in the upper levels of the convective layer, and, consequently, the low-level convergence and the associated upper-level divergence are accentuated. It may be inferred that this circumstance helps the disturbances to propagate forward down the mean flow, otherwise the progressive and the retrogressive wave are equally probable (Matsumoto, Ninomiya and Akiyama, 1967 b).

The importance of convective activity lies in producing precipitation on one hand, and in transporting energy and momentum on the other. The latter function of cumulus convection has recently been recognized extensively in atmospheric processes. The convective transfer under a winter monsoon situation was studied analytically by Matsumoto (1967) and in more detail by Ninomiya (1968) who showed clearly the necessity of cumulus activity in the field of convergence of the

order of 10^{-4}sec^{-1} . Many authors are now making numerical experiments to find out the role of cumulus convections under various conditions in the general field and it seems to have been verified that a convergence more than 10^{-4}sec^{-1} is required for maintaining cumulus activity. Now this amount of convergence, as is well known, is not expected in a synoptic scale atmospheric motion but is often observed in association with mesoscale disturbances as mentioned above.

As a first step to understanding the relationship between convective motion and larger scale motion, the role of convective momentum exchange in the mesoscale gravity wave will be discussed by introducing an empirical formula for the convective momentum transfer.

2. Two-layer presentation of the sheared flow subject to momentum exchange process

It is well known that a remarkably stable layer develops just above the cloud top level of highly convective regions such as the tropics (Malkus, 1958) or the Japan Sea under winter monsoon prevalence (Matsumoto and Ninomiya, 1967). Let us consider a convective layer bounded by an inversion and divide this layer into two layers in order to take into consideration the effect of convective momentum transfer and the effect of baroclinic general flow on the gravity waves. Denoting the surface frictional layer by the subscript 0, the middle level, where the convective transfer is to be defined, by the subscript 2 and the top boundary layer, where the gravity wave develops, by the subscript 4 as is shown in Fig. 1, we have the momentum equations on the lower layer represented by the level 1 and the upper layer represented by the level 3 respectively as follows:

Fig. 1. Two-layer presentation of sheared flow and notations.

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + w_1 \frac{u_2 - u_0}{\Delta Z} + \frac{\overline{u'w'} - F_0}{\Delta Z} = -g' \frac{\partial H}{\partial x}, \quad (1)$$

$$\frac{\partial u_3}{\partial t} + u_3 \frac{\partial u_3}{\partial x} + w_3 \frac{u_4 - u_2}{\Delta Z} + \frac{F_4 - \overline{u'w'}}{\Delta Z} = -g' \frac{\partial H}{\partial x}, \quad (2)$$

where H and ΔZ are the undulation of the inversion and the depth of the divided sublayer respectively, and g' is the acceleration of gravity reduced by the factor of percentage density difference causing the buoyancy. F_0 and F_4 are the momentum flux on the surface frictional layer and that on the inversion layer respectively and $\overline{u'w'}$ gives symbolically the convective momentum flux across the mid level. The effect of the earth's rotation is not considered here since we are concerned with the two-dimensional gravity wave propagating along x direction alone as a first approximation. We also have the equation of continuity written as

$$w_4 = \frac{\partial H}{\partial t} + u_4 \frac{\partial H}{\partial x} = -\Delta Z \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial x} \right), \quad (3)$$

where the fractional mass divergence due to the elevation of the boundary surface is neglected for the sake of simplicity.

Assuming a linear distribution for the wind profile, the wind velocity at the bottom and the top of the convective layer is expressed in terms of u_1 and u_3 , i.e.

$$u_0 = \frac{3u_1 - u_3}{2}, \quad (4)$$

$$u_4 = \frac{3u_3 - u_1}{2}. \quad (5)$$

The vertical velocity w_1 and w_3 are obtained by linear approximation

$$w_1 = \frac{w_2}{2} = -\frac{\Delta Z}{2} \frac{\partial u_1}{\partial x}, \quad (6)$$

$$w_3 = \frac{w_4 + w_2}{2} = -\Delta Z \frac{\partial u_1}{\partial x} - \frac{\Delta Z}{2} \frac{\partial u_3}{\partial x}. \quad (7)$$

The set of non-linear equations (1)~(3) together with the relations of linear interpolation (4)~(7) give us the conservation of total energy $\Delta Z/2 \cdot (u_1^2 + u_3^2) + g' H^2/2$, provided that we exclude the frictional dissipation at the top and bottom boundaries, the kinetic energy conversion from or to the convective motion and the triple correlation showing the interaction at the top surface.

3. Formulation of convective transfer and boundary transfer of momentum

According to the numerical analysis made on the mesoscale disturbance, Matsumoto, Ninomiya and Akiyama (1967 a and b) found that a large amount of residue comes out from the budget of divergence equation applied on the lowermost layer of the atmosphere and it is distributed exclusively over the convergence area. Since this residue is to be regarded as a contribution from the convective motion involved, we obtain empirically the following approximated formula:

$$\frac{\overline{w'D'}}{\Delta Z} \propto D \quad (\text{for } D < 0) \quad (8)$$

where D is the divergence and primes denote the convective field as mentioned before. The coefficient of proportionality was then about 10^{-3}sec^{-1} because $-\overline{w'D'}/\Delta Z = O(10^{-7}\text{sec}^{-2})$ and $-D = O(10^{-4}\text{sec}^{-1})$.

From an empirical knowledge combined with results of numerical experiments on cumulus convection, we can express $\overline{w'D'}$ approximately as

$$\overline{w'D'} \simeq \sigma w_c \delta D, \quad (9)$$

where σ , w_c and δD are respectively the area ratio of the updraft column, the upward velocity in a cumulus core and the excess divergence (the difference of divergence inside of a cumulus from outside divergence). Taking into account the fact that the compensating downdraft w_f may practically be ignored, eq. (9) is again approximated as

$$\overline{w'D'} \simeq w \delta D, \quad (9')$$

since we have $w = \sigma w_c + (1 - \sigma)w_f$ where w is the mesoscale vertical velocity and may be replaced by $-D\Delta Z$. Thus we can understand that the proportionality coefficient mentioned above has a reasonable value because $-\delta D$ may be considered to be the convergence characterizing a cumulus tower, that is, of the order of 10^{-3}sec^{-1} .

Now let us try to estimate the correlation $\overline{u'w'}$ indirectly from the value of $\overline{w'D'}$ simply by multiplying a scale factor L_c which specifies the size of an element convective cell, and write

$$\frac{\overline{u'w'}}{\Delta Z} = \frac{L_c \overline{u'D'}}{\Delta Z} = L_c \delta D \cdot \frac{w}{\Delta Z} = A \frac{\partial u_1}{\partial x},$$

$$\begin{cases} A=0 & \text{for } \frac{\partial u_1}{\partial x} \geq 0, \\ A=A_0 & \text{for } \frac{\partial u_1}{\partial x} < 0. \end{cases} \quad (10)$$

The proportionality coefficient A , then, has a positive value A_0 of the order of 10m sec^{-1} within the area of lower-layer convergence and vanishes elsewhere, that is in areas of nonconvective activity.

The physical interpretation of A_0 will also be given in a similar manner as mentioned above. From the derivation

$$\overline{u'w'} \simeq \sigma w_c \delta u = w \delta u \cdot \sigma w_c / w = -D \Delta Z \cdot \delta u \cdot \sigma w_c / w$$

we find that A_0 corresponds to $-\delta u \cdot \sigma w_c / w$, where $-\delta u$ is the momentum deficit in an updraft core as compared to the surrounding air.

Next let us formulate the turbulent eddy momentum flux through the lower and upper boundaries. At the surface friction layer, the momentum flux is proportional to the wind shear. Therefore we may write simply

$$\frac{F_0}{\Delta Z} = -B u_0, \quad (11)$$

where the reasonable value of coefficient B will be of the order of 10^{-4}sec^{-1} judging from the turbulent theory. Similarly we assume the momentum flux at the upper boundary layer in the following manner:

$$\frac{F_4}{\Delta Z} = -B(U - u_4), \quad (12)$$

where U is the undisturbed wind velocity above the inversion layer and the same value of proportionality coefficient is adopted for the sake of simplicity. We often observe nearly the same amount of wind shear develop in the lower and upper boundaries of the convective layer (see for example Matsumoto, Ninomiya and Akiyama, 1967 b).

4. Results of numerical integrations

Basing on the two-layer version of the sheared flow represented by eqs. (1)~(3), the interaction between the non-linear gravity wave and the convective activity is investigated by numerical experiments for various situations. Since our interest lies in the mesoscale phenomena in which a large amount of convergence is expected, integrations are performed in domains whose dimensions range from $50\text{km} \sim 400\text{km}$ with cyclic lateral boundary conditions. The values of A_0 and B are changed within a reasonable range.

Our primary purpose is to simulate the mesoscale disturbances observed over the Japan Sea coastal area under the influence of a predominant cold

air outburst. Therefore, the depth of the convective layer is assumed to be 5000m ($\Delta Z=2500\text{m}$) and we take the value of effective gravity $g'=10\text{cm sec}^{-2}$ about 1/100 of the acceleration of the gravity judging from the density difference between inside and outside of the cold airmass. The initial mean wind field at level 1 ($Z=1250\text{m}$), at level 3 ($Z=3750\text{m}$) and undisturbed wind field are assumed as

$$u_1=10\text{m sec}^{-1},$$

$$u_3=20\text{m sec}^{-1},$$

$$U=30\text{m sec}^{-1},$$

throughout all experiments.

As to the initial conditions, simple harmonic, solitary and random disturbances are superposed on the general wind field, but no disturbance is given for the pressure field H . The time step ΔT and the grid size Δx applied for integration are chosen in such a way that the relation

$$\Delta x/\Delta T \approx 90\text{m sec}^{-1} \quad (\Delta T=2\text{min for } \Delta x=10\text{km})$$

Fig. 2. Time change of maximum and minimum velocity deviation at the lower level (thinner line), at the higher level (solid line) and that of the elevation of inversion layer for various values of A_0 . Figures on the left are for an initial simple harmonic wave disturbance of 200km wave length and the figures on the right for that of 100km wave length.

holds, so that the criterion for computational stability is satisfied because the velocity of the gravity wave and the mean wind velocity in the convective layer are about 24m sec^{-1} and 15m sec^{-1} respectively. The number of grid points is either 20 or 40. The centered difference scheme is applied for both time and space except for the first-step extrapolation.

a) simple harmonic initial disturbances

The velocity disturbance superposed on levels 1 and 3 is assumed to have the same amplitude of 2.5m sec^{-1} with opposite signs, so that the lower-level convergence is accompanied by the same amount of upper-level divergence and the total divergence $\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial x}$ vanishes. Therefore initially, there is no tendency to change in the height of the inversion. In due time, however, the total divergence comes to have a certain value because of the difference in advective velocity, and the gravity wave starts to develop.

Some examples of the results are presented in Fig. 2. The value of A_0 is changed as indicated in each figure, but B has a fixed value of $0.25 \times 10^{-4}\text{sec}^{-1}$. The figures on the left-hand side are for the wave length of 200km and those on the right-hand side for the wave length of 100km. In each figure are given the maximum and minimum velocity deviations at the lower level (thinner line) and at the upper level (solid line) and the elevation of the inversion layer (heavy line) as a function of time, respectively. The hatched area gives an approximate measure of the maximum convergence (or divergence) at the lower level. It is seen that the disturbance damps out within the time prescribed by the value of B if A_0 assumes a smaller value. When the value of A_0 exceeds a certain value, the disturbance begins to develop and the convergence takes a larger value than the initial value, showing that the disturbance kinetic energy is supplied from the convective kinetic

Fig. 3 Time sections of convergence for various values of A_0 . The initial disturbance is a simple harmonic wave with a wave length of 200km. Contours are labeled with the value of convergence in every $-0.25 \times 10^{-4}\text{sec}^{-1}$.

energy*. It is also observed that the average wind velocity increases at the lower level while it decreases at the upper level as is expected from the downward momentum transfer. The undulation appearing on the top surface takes a quite reasonable value of several hundred meters as was suggested by the authors (1965).

There seems to exist a critical value of A_0 at about 10 m sec^{-1} , beyond which the energy conversion overcomes the frictional dissipation. This appears more clearly on the smaller-scale disturbance.

In Fig. 3 are presented the time sections of the divergence field for various A_0 values. The wave length is 200 km and $B = 0.5 \times 10^{-4} \text{ sec}^{-1}$. The shaded area shows convergence at the lower level and contours are drawn at intervals of $-0.25 \times 10^{-4} \text{ sec}^{-1}$. The strongly convergent area is double hatched. Here again we can observe clearly the fact mentioned above. The phase shift appearing on each figure shows the propagation of disturbance downstream with a velocity approximately coinciding with the average wind velocity. The phase velocity becomes larger for the larger value of A_0 . Another noticeable fact is that the convergence area concentrates in a narrower zone with increasing intensity and, at the same time, smaller wave disturbances develop in due course of time. This tendency is quite remarkable if the value of A_0 is large. Fig. 4 shows this non-linear process in more detail. Successive curves are the wind velocity at level 1 at 5-minute intervals. The wave length of initial disturbance is 100 km and $A_0 = 15 \text{ m sec}^{-1}$. The asymmetrical distribution is gradually exaggerated and a secondary wave develops.

Fig. 4. Time evolution of the lower-level wind field labeled with integration time. The initial disturbance is a simple harmonic wave with the wave length of 100 km . The value of A_0 is 15 m sec^{-1} .

* The total kinetic energy decreases but the disturbance kinetic energy increases because of the energy conversion in the form of $\partial u / \partial z \cdot \overline{u'w'}$.

b) solitary wave

Instead of placing a simple harmonic wave, a solitary disturbance is given initially under otherwise similar conditions to the above. The results are given in Fig. 5. This figure is the same as Fig. 3 except that contours of convergence are given at intervals of $0.5 \times 10^{-4} \text{ sec}^{-1}$. We obtain conclusions qualitatively similar to the above. One thing to be added is the retrogressive wave which appears in cases of smaller value. Therefore it may be inferred that the convective momentum transfer tends to propagate gravity waves towards the lee side of the general flow. In the actual atmosphere, almost all of the disturbances are obviously progressive.

Fig. 6 gives the evolution of disturbance starting from a solitary wave. The lower-level wind field is presented at intervals of 100 minutes. The remarkable feature in this is the development of a concentrated convergence field within several hours and, thereafter, the shifting of the prevailing wave number due to the non-linear interaction. It must also be pointed out that the average wind velocity on this level increases by about 4 m sec^{-1} in 9 hrs. because of convective momentum transfer.

c) random disturbance

On each of the 40 grid points with 10 km intervals, random velocity disturbance less than $\pm 1 \text{ m sec}^{-1}$ is given at levels 1 and 3 independently. With increasing integration time, the frictional dissipation drops down first, and then the interaction between general flow A_0 and convective motion works to increase mesoscale kinetic energy. In Fig. 7 are summarized the results obtained by numerical experiments under various values of interaction coefficient A_0 and dissipation coefficient B . Full lines in the diagram indicate the time in which the frictional dissipation decreases to half of the initial value, and broken lines indicate the time when the interaction between convective motion and mean motion becomes minimum and starts increasing. The half reduction period given by full lines has a reasonable value of a few hours corresponding to the value of B , and this period is prolonged when A_0 has a large value and thus a large amount of momentum is transferred downward. On the other hand, the time required for the interaction to become effective is shortened, as indicated by broken lines in Fig. 7, when A_0 has a large value. It is also noticed that this time is shortened when a large value of B is adopted. The circumstance may be explained as follows.

Fig. 5. Time section of convergence for various values of A_0 . The initial disturbance is a cyclic solitary wave with a wave length of 200km. Contours are labeled with the value of convergence in every $-0.5 \times 10^{-4} \text{ sec}^{-1}$.

Fig. 6. Lower-level wind field in every 100min. The value of A_0 is 15 m sec^{-1} .

Under a large frictional dissipation coefficient, the initial disorganized field is damped out very quickly and, after that, a systematic wind field is built up in the vertical distribution, which favors the energy conversion and reflects on the redevelopment of frictional dissipation.

Fig. 7. Diagram showing the half reduction period of frictional dissipation (full lines) and the time required for mesoscale disturbance-convection interaction to become minimum (broken lines) as a function of interaction coefficient A_0 and dissipation coefficient B .

An example of wind distribution is given in Fig. 8. This experiment is made under the condition $A_0 = 20 \text{ m sec}^{-1}$ and $B = 0.25 \times 10^{-4} \text{ sec}^{-1}$. The integrated time is 300min, which corresponds

Fig. 8. Lower-level and higher-level wind field after 300min starting from random disturbance. Integration domain is 400km and the values of A_0 and B are 20 m sec^{-1} and $0.25 \times 10^{-4} \text{ sec}^{-1}$ respectively.

to the redevelopment stage. Therefore the wind field is organized in such a way that the distribution at the lower level and that at the upper level are reverse to each other, the lower-level convergence being accompanied by the upper-level convergence and vice versa. An interesting aspect is the prevailing wave length, which is 200km at the time shown in Fig. 8. We can observe different stages of development, i.e., the prevailing wave length is 100km at one time and is 50km or smaller at another just as in the spectral change of the general circulation. This is obviously caused by the non-linear interaction.

5. Remarks

In this paper is studied the influence of convective momentum transfer on the mesoscale gravity wave which develops on the boundary surface at the top of a convective layer. By means of numerical integration experiments we can obtain some information very suggestive in many respects, i.e. the concentration of low-level convergence, the progressive propagation with reasonable phase velocity, the vertical distribution of wind velocity, the phase relationship between pressure field and velocity field and their magnitude, the development of mesoscale disturbances starting from random distributions under the effect of non-linear interaction, and so on.

However, the discussions have been made on a dry model in which only the momentum equation is applied for the two-level version of one-dimensional disturbances. Our model is essentially represented by the external gravity wave. The thermodynamic relationship and moisture continu-

ity as well as the condensation process must make primary contributions, especially in the smaller-scale phenomena. The life time, the preferred scale and other important aspects of mesoscale disturbances have undoubtedly much to do with those subjects. Therefore we will proceed in future to the internal gravity wave with moist processes.

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Appendix

A remark on instability due to interaction between vertical wind shear and convective momentum exchange

In order to linearize eqs. (1)~(3), we first approximate them by

$$\begin{aligned} \frac{\partial u_1}{\partial t} + \left(U_1 - \frac{AU}{2} \right) \frac{\partial u_1}{\partial x} \\ + A \left(\frac{\partial u_1}{\partial x} - D_{max} \right) - B \frac{u_3 - 3u_1}{2} = -g' \frac{\partial H}{\partial x}, \end{aligned} \quad (1')$$

$$\begin{aligned} \frac{\partial u_3}{\partial t} + \left(U_3 - \frac{AU}{2} \right) \frac{\partial u_3}{\partial x} - AU \frac{\partial u_1}{\partial x} - A \left(\frac{\partial u_1}{\partial x} \right. \\ \left. - D_{max} \right) - B \left(U - \frac{3u_3 - u_1}{2} \right) = -g' \frac{\partial H}{\partial x}, \end{aligned} \quad (2')$$

$$\frac{\partial H}{\partial t} + U_4 \frac{\partial H}{\partial x} = -AZ \left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial x} \right), \quad (3')$$

where D_{max} is the maximum value of positive divergence. Here A is regarded to have a constant value $A_0/2$ instead of the nonlinear value given by eq. (10). Let us next impose the following requirements:

$$\left. \begin{aligned} U_3 &= -\frac{AD_{max}}{2B} + \frac{3}{4}U, \\ U_1 &= \frac{AD_{max}}{2B} + \frac{1}{4}U. \end{aligned} \right\} \quad (13)$$

Then we have the linear perturbation equations,

$$\frac{\partial u_1'}{\partial t} + \left(U_1 - \frac{\Delta U}{2} \right) \frac{\partial u_1'}{\partial x} + A \frac{\partial u_1'}{\partial x} - B \frac{u_3' - 3u_1'}{2} = -g' \frac{\partial H}{\partial x} \quad (1)''$$

$$\frac{\partial u_3'}{\partial t} + \left(U_3 - \frac{\Delta U}{2} \right) \frac{\partial u_3'}{\partial x} - \Delta U \frac{\partial u_1'}{\partial x} - A \frac{\partial u_1'}{\partial x} + B \frac{3u_3' - u_1'}{2} = -g' \frac{\partial H}{\partial x} \quad (2)''$$

$$\frac{\partial H}{\partial t} + U_4 \frac{\partial H}{\partial t} = -\Delta Z \left(\frac{\partial u_1'}{\partial x} + \frac{\partial u_3'}{\partial x} \right). \quad (3)''$$

The same perturbation equations will be obtained by simply linearizing eqs. (1)~(3) under the assumption that the momentum exchange coefficient has a constant value and upward momentum transfer is allowed in the positive divergence area.

The frequency equation is obtained for a simple harmonic wave disturbance in a form of $\exp i\alpha\{x - (c - U_2)t\}$ as follows:

$$\begin{vmatrix} -\Delta U + A - i\frac{3}{2}\frac{B}{\alpha} - c & \frac{i}{2}\frac{B}{\alpha} & c_g \\ \Delta U - A + \frac{i}{2}\frac{B}{\alpha} & -i\frac{3}{2}\frac{B}{\alpha} - c & c_g \\ \frac{c_g}{2} & \frac{c_g}{2} & \Delta U - c \end{vmatrix} = 0, \quad (14)$$

where $\alpha = 2\pi/L$ (L is the wave length) and $c_g = \sqrt{g' \cdot 2\Delta Z}$. Eq. (14) is obviously a third degree algebraic equation of c with complex coefficients, and the solutions are generally complex and written as

$$c = c_r + i c_i. \quad (15)$$

The growth rate is given by

$$G = \alpha c_i, \quad (16)$$

showing that a positive imaginary part of c corresponds to an amplifying wave component.

Let us, for the sake of simplicity, consider the inviscid atmosphere ($B=0$). The solutions of eq. (14) are then summarized in Fig. 9. The phase velocity c_r is given in the upper figure and the growth rate αc_i for a disturbance with a wave length of 100km in the lower figure and both are presented as functions of $\Delta U/c_g$ (ordinate) and A/c_g (abscissa). The shaded area indicates the instability caused by the interaction between the wind shear and the convective momentum transfer. The isolines of the growth rate are

Fig. 9. Phase velocity (upper figure) as functions of $\Delta U/c_g$ and A/c_g . The wave length of disturbance is assumed to be 100km. Only the phase velocity of progressive waves is given in the upper figure for the sake of simplicity. The shaded area and the stippled area are respectively the instability region for inviscid atmosphere and the additional instability region under the effect of surface friction.

labeled with the time G^{-1} within which the amplitude of disturbance ($L=100\text{km}$) grows to e times of the initial amplitude. It is seen that a disturbance with the wave length of 100km can develop within a few hours if sufficiently large convective exchange acts in the atmosphere having a vertical wind shear within a proper range. The instability is found under the condition given by $A > \Delta U > 0$.

The mechanism of instability would be understood by examining the phase velocity diagram as follows. There are three solutions of eq. (14), one moving approximately with the general flow (hereafter called synoptic mode) and the others progressive and retrogressive gravity waves. These are modified under the effect of vertical wind shear and convective exchange. The

phase velocity of the synoptic mode is shown by broken lines and that of the progressive gravity wave mode by full lines in Fig. 9. The synoptic mode is accelerated by the convective exchange process because the wind velocity in the lower layer convergence is increased and thus an additive convergence is created on the lee side of the convergence. The gravity wave mode is much influenced by the vertical wind shear. The synoptic mode and the gravity wave mode become confluent with each other at a certain combination of ΔU and A values as is seen in Fig. 9. Under this situation, the divergence field related to the undulation of the boundary surface and thus inherent in the gravity wave resonates with the divergence field generated by the convective momentum transfer. It is seen in Fig. 9 that both modes have the same phase velocity in the instability region as shown by dotted lines.

The surface friction B behaves in a somewhat

Fig. 11. Growth rate as a function of wave length in the additional instability region for viscous atmosphere indicated by stipples in Fig. 9. $\Delta U/c_g=0.65$, $B=0.6 \times 10^{-4} \text{ sec}^{-1}$.

Fig. 10. Change of growth rate due to surface friction. Full line, broken line and dotted line are for $B/c_g=0.0$, 0.02 and 0.04 respectively. Upper figure is for $\Delta U/c_g=0.85$ and lower figure for $\Delta U/c_g=0.45$.

strange manner. The solution of the frequency equation (14) with complex coefficients shows that the instability region is expanded to the stippled region in Fig. 9. The change of growth rate is presented in Fig. 10 for $\Delta U/c_g=0.85$ (upper figure) and for $\Delta U/c_g=0.45$ (lower figure). We find that the instability comes out for smaller A values while the growth rate decreases for larger values when the surface friction is taken into account.

Fig. 11 shows how the growth rates depend on wave length for disturbances in a viscous atmosphere under particular situations indicated by the stippled region. We find that the increase of the growth rate is very much reduced in the regime of smaller wave lengths. It is inferred that a preferred scale would possibly be found within a mesoscale range if an appropriate amount of lateral mixing be introduced into our model.

Inspecting the characteristic field of the eigen function in the instability region, we find that the potential energy is transported downward through mid level and causes an increase of the kinetic energy in the lower. It is also shown that the upper layer serves as a source region of the potential energy and that the kinetic energy of the upper layer is supplied by the effect of convective momentum transfer.

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中規模重力波に及ぼす運動量の対流輸送の役割

松本 誠一・二宮 洸三

気象研究所

中規模擾乱は 10^{-4}sec^{-1} のオーダーの収束場を伴う重力波の性格をもっている。この収束量は対流活動を強く支配し、運動量交換過程を通して風の場合に重要な影響を持つものである。

垂直シアーのある流れを簡単に2層モデルで表現し、対流運動量輸送を経験的にパラメタライズして数値実験を行った。下層収束域における運動量の下向き輸送は下層の風速を強め上層の風速を弱める効果を持ち、かくして下層の収束は自ら強まり下流への伝播速度は増大することになる。然しながら風速が再分布されることによりまさつ消耗が強化されるが、これは対流運動エネルギーからのエネルギー交換により一部補償される。

線型モデルにより、シアーのある基本流において対流運動量輸送が不安定効果をもつことが示される。