A Numerical Study of Two-Dimensional Airflow over an Isolated Mountain

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Abstract

In order to simulate numerically one of the most remarkable phenomena of airflow over a mountain, i.e., the hydraulic jump on the lee side of the mountain, several numerical experiments are conducted with the shallow fluid flow system. Since the jump is a kind of shock phenomenon in the fluid, we need to use a highly accurate finite difference scheme. In the present study the one-step Lax-Wendroff method is adopted.

The following four cases of uniform flows are given as the initial conditions over the whole domain, i.e., two cases of subcritical flows and remaining two cases of supercritical flows. The results are in good agreement with those of the one-dimensional flow obtained by Houghton and Kasahara (1968). In the present experiment the jump is set up on the lee side of an isolated bell-shaped obstacle in one out of two subcritical flows and in one out of two supercritical flows. In another case of the subcritical flow the wave of wave length which is close to the width of the obstacle predominates. The flow pattern similar to that of the one-dimensional flow is observed in the remaining case of the supercritical flow. The zonal flow crossing over the obstacle is more predominant than that passing around the obstacle.

1. Introduction

The atmospheric hydraulic jump is sometimes observed at the lee side of the mountain such as Mt. Fuji (Magata, 1968). In many cases when a strong wind blows over the lee side of mountain, the inversion layer appears over the slope (Arakawa, 1968). The flow under the inversion layer is similar in its character to the shallow fluid flow. The one-dimensional airflow over an isolated mountain at the bottom of the atmosphere has therefore been investigated as the shallow fluid flow by Jih-Ping, Kung-Kuan and Ming (1964), Larsen (1966), Houghton and Kasahara (1968), Arakawa (1968) and Arakawa and Oobayashi (1968). Furthermore the three-dimensional airflow passing over an isolated mountain was first studied by Magata (1968), using a two-layer model for a nonhydrostatic atmosphere.

On the other hand the laboratory experiment on the jump phenomenon was carried out first by Long (1954). In this experiment an obstacle was fixed at the bottom of a channel filled with water and occurrence of hydraulic jump in the fluid was demonstrated for a certain range of velocities over the obstacle.

Few works have so far been done, however, on the motion of the two-dimensional horizontal air flow in hydrostatic equilibrium over the isolated mountain. In order to advance our understanding of the hydraulic jump phenomena observed in the real atmosphere, the present author considers that extension of the computational model to the two-dimensional horizontal flow is a necessary and effective step to approach eventually to the three-dimensional treatment of the continuous stratified fluid.

The purpose of the present study is to investigate numerically the dynamic mechanism of the pressure jump in this direction. We use the two-dimensional time-dependent shallow water equations as the governing equations for the motion of an incompressible, homogeneous, inviscid, and hydrostatic fluid crossing over a bell-shaped obstacle. The model may well represent qualitatively the nature of the atmosphere for the simulation of a special phenomenon like the pressure jump (Tepper, 1955).

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In order to obtain numerically an accurate discontinuous solution for the jump, we have to use the higher order finite difference scheme. In the present calculation, we take up the one-step Lax-Wendroff method.

2. Basic equations

For the rather limited objective of this study, we will use the incompressible, homogeneous, hydrostatic and inviscid atmosphere. We ignore the pressure gradient at the top of the inversion layer and surface friction. The Coriolis force is also neglected because of the smallness of the scale. The motion of the fluid may be described by the following well-known shallow water equations;

\[
\frac{\partial U}{\partial T} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + g' \frac{\partial}{\partial X} (h + m) = 0
\]

(2.1)

\[
\frac{\partial V}{\partial T} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + g' \frac{\partial}{\partial Y} (h + m) = 0
\]

(2.2)

\[
\frac{\partial h}{\partial T} + \frac{\partial}{\partial X} (hU) + \frac{\partial}{\partial Y} (hV) = 0
\]

(2.3)

where

\[g' = \frac{\rho' - \rho}{\rho} - g\]

The two space variables \(X\) and \(Y\) show Cartesian coordinates. In (2.1)-(2.3) we define \(U\) and \(V\) as the \(X\) and \(Y\) components of the fluid velocity, respectively; \(h\) the height of the fluid layer under the inversion; \(m\) the height of an obstacle which is a function of both \(X\) and \(Y\); and \(T\) time. \(g\) denotes the acceleration due to gravity, \(\rho'\) and \(\rho\) are the air densities above and under the inversion, respectively (see Fig. 1).

Fig. 1(a) shows the domain of the present model, which is a fixed rectangular region with sides parallel to the coordinate axes. The lateral boundary is set to be free everywhere so that a disturbance can flow smoothly outward or inward through the boundary.

A bell-shaped obstacle is placed on the bottom at the center of the domain. As a cross-sectional view of the flow in Fig. 1(b) shows, the bottom is set flat outside the obstacle region.

We now introduce the following nondimensional variables:

\[
\begin{align*}
&m = \frac{n}{C_0}^* \\
&u = \frac{U}{C_0^*}, \quad v = \frac{V}{C_0^*}, \quad c = \frac{c}{C_0^*}, \\
&x = \frac{X}{L}, \quad y = \frac{Y}{L}, \quad t = \frac{C_0^*}{L} T
\end{align*}
\]

(2.4)

where

\[C_0^* = g' H_0, \quad c^* = g'h, \quad n^* = g'm \tag{2.5}\]

\(C_0^*\) is related to the fluid height over the undisturbed regions; \(L\) denotes an arbitrary standard length, and here we take \(L\) a half of the width of the obstacle (see Fig. 1).

In terms of the non-dimensional variables the basic equations (2.1)-(2.3) may be transformed as follows;

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial}{\partial x} (c^2 + n^2) = 0 \tag{2.6}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial}{\partial y} (c^2 + n^2) = 0 \tag{2.7}
\]

\[
\frac{\partial c^2}{\partial t} + \frac{\partial}{\partial x} (c^2 u) + \frac{\partial}{\partial y} (c^2 v) = 0 \tag{2.8}
\]

Hereafter, we shall employ equations (2.6)-(2.8) for the numerical study.
3. Initial condition
The following pre-initial and initial states are assumed. For \( t < 0 \) and \( -\infty < (x, y) < +\infty \), the fluid has a constant horizontal velocity denoted by the vector \( \mathbf{v}_0 \) and there is no obstacle, and then at \( t = 0 \) the bell-shaped obstacle is suddenly and inviscidly inserted into the fluid. At \( t = 0 \) we impose as

\[
\begin{align*}
  u &= u_0, \\
  v &= 0, \\
  c^2 &= c_0^2 = 1.0,
\end{align*}
\]

over the whole domain. (3.1)

Since the initial value of \( c^2, c_0^2 \), is the nondimensional fluid height of the magnitude of unity, \( u_0 \) represents the Froude number of the initial state. Since the initial condition assumes the uniformity of the flow and constant thickness of the fluid over the whole domain, the velocity and mass fields are specified independently at the starting point and therefore wave motions for the mutual adjustment of both fields are expected to arise near the obstacle.

In the present numerical experiment the following four cases are taken up for the initial velocity of the uniform flow, i.e.,

Case (1) \( u_0 = 0.2 \),
Case (2) \( u_0 = 0.4 \),
Case (3) \( u_0 = 1.2 \),
Case (4) \( u_0 = 2.0 \).

The maximum nondimensional height of the obstacle \( n_c^2 \) is set as

\[ n_c^2 = 0.5 \]

Fig. 2 shows the \( F_i, \gamma_c \)-diagram for the one-dimensional flow, which is reproduced from Arakawa and Oobayashi (1968), where \( F_i = |v_i|/c_i \) is the Froude number of approaching flow, \( v_i \) is equivalent to \( \mathbf{v}_0, c_i \) is the same as \( C_0^* \), and \( \gamma_c = n_c^2/c_c^2 \) corresponds to the height of the obstacle. According to the study of the one-dimensional flow, the regions \( a \) and \( d \) show regimes for steady state flow and the regions \( b \) and \( c \) unsteady flow.

The values for the four cases are plotted on the \( F_i, \gamma_c \)-diagram. Cases (1) and (4) are in the steady state region, and Cases (2) and (3) in the unsteady state region, respectively (see Fig. 2). Since the regimes of motion in Fig. 2 were determined for the one-dimensional flow, it is not insured that the results are applicable also for the two-dimen-

4. Finite difference equations
For the purpose of numerical integration with respect to time, it is convenient to rewrite equations (2.6)-(2.8) in the form of conservation law, i.e.,

\[
\frac{\partial W}{\partial t} + \frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + K = 0
\]

where

\[
W = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}, \quad A = \begin{pmatrix} \frac{\alpha^2 + \beta^2}{\gamma} \\ \frac{\alpha \beta}{\gamma} \\ \alpha \end{pmatrix}, \quad B = \begin{pmatrix} \frac{\alpha \beta}{\gamma} \\ \frac{\gamma + \gamma^2}{2} \\ \frac{\beta^2}{\gamma} \end{pmatrix}, \quad K = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\]

\[
\alpha = \gamma u, \quad \beta = \gamma v, \quad \gamma = c^2
\]
\(\alpha\) and \(\beta\) denote the \(x\) and \(y\) components of the momentum with respect to a unit column, and \(\gamma\) is the thickness of the fluid. The finite difference scheme developed by Lax and Wendroff (1960) is applied to equation (4.1), because one of our main purposes is to obtain the discontinuous solutions of equation (4.1) accurately and the one-step Lax-Wendroff method is considered to be one of the most suitable finite difference scheme to deal with such a shock phenomenon. The scheme may best be applied to the system of equations which conserves momentum and mass. (Lax and Wendroff (1960), Richtmyer (1963) and Burstein (1967)). The one-step Lax-Wendroff method may be expressed in the following form:

\[
W(x,y,t+\Delta t) = W(x,y,t) - \Delta t \left( \frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + K \right) + \frac{\Delta t^2}{2} \left\{ \frac{\partial}{\partial x} \left[ P \left( \frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + K \right) \right] + \frac{\partial}{\partial y} \left[ Q \left( \frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} + K \right) \right] - \frac{\partial K}{\partial t} \right\}
\]

where

\[
P = \begin{pmatrix}
\frac{2\alpha}{\gamma} & 0 & -\frac{\alpha^2}{\gamma^2} \\
\frac{\beta}{\gamma} & \alpha & -\frac{\alpha\beta}{\gamma^2} \\
1 & 0 & 0
\end{pmatrix}, \quad Q = \begin{pmatrix}
\frac{\beta}{\gamma} & \frac{\alpha}{\gamma} & -\frac{\alpha\beta}{\gamma^2} \\
0 & \frac{2\beta}{\gamma} & \frac{\beta^2}{\gamma^2} \\
0 & 1 & 0
\end{pmatrix}
\]

\[
\frac{\partial K}{\partial t} = - \left[ \begin{pmatrix}
\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} & \frac{\partial n^2}{\partial x} \\
\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} & \frac{\partial n^2}{\partial y} \\
\frac{\partial A}{\partial x} + \frac{\partial B}{\partial y} & 0
\end{pmatrix} \right].
\]

The space derivative is replaced by the centered difference formula to secure the second order accuracy.

The computational stability of the scheme is guaranteed if \(\lambda\) satisfies the condition

\[
\lambda (|v|_{max} + c_{max}) < \frac{1}{\sqrt{2}}
\]

(4.4)

where \(\lambda = \Delta t/\Delta s\), \(\Delta x = \Delta y = \Delta s\), \(|v|_{max}\) is the magnitude of the largest possible flow velocity and \(c_{max}\) the maximum value of the fluid height.

Next, the boundary condition for the present calculation is described. At the lateral walls of the domain free boundaries are assumed (see Fig. 1). The disturbance is permitted to flow smoothly outward or inward through the walls. In the present experiment the Neumann boundary condition is taken up, i.e.,

\[
\frac{\partial M}{\partial B_N} = 0
\]

(4.5)

where \(B_N\) denotes a unit of the normal component at the boundary, \(M\) is a dependent variable, for example, \(\alpha, \beta, \) or \(\gamma\) in (4.2). In the finite difference form, the condition (4.5) yields

\[
M_N = M_{N-1}
\]

(4.6)

where \(M_N\) shows the variable at a boundary point, \(N\), and \(M_{N-1}\) that at the next neighboring point to the boundary.

5. Values of computational parameters

The square domain is covered by a net of the square lattice with interval \(\Delta x = \Delta y = \Delta s\). The size of the domain is \(6L \times 6L\). The origin of the spatial coordinates is taken at the center of the domain (see Fig. 1).

The obstacle used here is bell-shaped and is expressed as follows;

\[
n^2 = \frac{nc^2}{2} \left( 1 + \cos R \right), \text{ for } |R| \leq 1
\]

\[
n^2 = 0, \text{ for } |R| > 1
\]

(4.7)

where \(R^2 = x^2 + y^2\), and \(n_c^2\) corresponds to the maximum height of the obstacle. Nondimensional values for the parameters are shown below.

\[
L = 5\Delta s, \quad \Delta s = 0.2, \quad C_0 = 1.0
\]

(4.8)

The time step \(\Delta t\) is chosen to satisfy the stability condition (4.4), and \(\lambda = 0.1\) is adopted in the cases (1) and (2), \(\lambda = 0.05\) in the cases (3) and (4). All the four cases are integrated up to 250 time steps.

6. Results of the numerical experiment

(i) Case (1): \(u_0 = 0.2\). The contour patterns of height deviation from the initial height field, \(C_0 = 1.0\), are shown in Fig. 3 with an interval of 0.05 in a series of four figures taken at \(t = 2, 3, 4\), and 5. The obstacle is depicted with a circle in these figures. The bottom of Fig. 3 illustrates the cross-sectional view of the height distribution crossing the center of the obstacle at \(t = 5.0\).
Fig. 3

Fig. 4
Fig. 3. Evolution of the height deviation field of the case (1). The contours are drawn with an interval of 0.05. Nondimensional time is indicated in the upper right corner of each chart. The obstacle is shown by the circle. The bottom figure shows the cross-sectional view of the flow relative to the obstacle at $t=5.0$.

Fig. 4. Evolution of the height deviation field of the case (2). The contours are drawn with an interval of 0.1. The bottom figure shows the cross-sectional view relative to the obstacle at $t=5.0$.

The dashed contours in upper figures indicate the negative deviation of height and the solid contours the positive deviation. The heavy solid contours denote zero lines. At the beginning stage the height of inversion surface increases on the windward side of the obstacle and decreases on the lee side as seen in the top of Fig. 3. This initial phase of the height distortion appears in common in all the four cases. The subsequent development of the flow patterns, however, becomes quite different according to the intensity of the initial flow.

A disturbance is generated by the obstacle and propagates outward across the boundaries. The train waves are generated afterwards as seen in patterns at $t=5.0$. From the horizontal profile of the flow, the predominant wave length is estimated to be close to the width of the obstacle. The train waves move downstream. From the behavior of numerical solution it may be said that there is no steady state in this case.

(ii) Case (2): $u_0=0.4$. At the beginning the flow pattern is similar to that in the case (1). (Fig. 4) The height of the fluid surface increases on the windward side of the obstacle and the phase of rise propagates upstream. On the contrary, the height decreases on the lee side and the phase of fall propagates downstream. As time advances, the quasi-steady state pattern is observed to form in Fig. 4. Throughout whole computation time there is a minimum height of the fluid near the crest of the obstacle, and a jump exists on the lee side. The situation is similar to the steady state jump for subcritical flow ($F<1$) in the one-dimensional case. Waves of the wave length $2\Delta s$ are observed to appear almost regularly especially in the lee side of the jump. It may be speculated that these noises are computational modes in the numerical solution or some sort of unstable waves like Helmholtz wave of computational origin.

In Fig. 5, $y$-component of the horizontal velocity and the composite wind vector of the stream at $t=4.0$ for the case (2) are shown. The solid contour indicates positive deviation of the velocity $v$ from $v_0=0$ and the dashed contour negative deviation. The heavy solid contour denotes zero line. The arrow indicates the horizontal wind vector. Total configuration of the wind distribution does not depend on the initial wind (see also Fig. 7). The $y$-component of velocity shows a symmetrical pattern about the center of the obstacle, but the intensity is very weak. The arrows show the predominant zonal flow with the maximum near the crest of the obstacle.

(iii) Case (3): $u_0=1.2$. Fig. 6 shows the computational results for the supercritical case with contours of the interval of 0.1 or 0.2. Quite different patterns were obtained in this case. The positive height deviation on the windward side of the obstacle does not propagate upstream, because the flow is supercritical ($F>1$). Instead the fluid surface on the windward side rises as time goes on. The fluid tends to accumulate at a fixed position and ultimately the jump is expected to take place there. On the other hand, the fluid surface on the lee side falls down as
time proceeds and the jump occurs at a particular point on the lee side. At \( t=2.0 \) the jump becomes remarkably sharp. The computation was still continued after this time. The feature of the jump was then destroyed probably due to the computational dispersion. (The computational dispersion was studied by Wurteleles (1961) and Matsuno (1966), and they showed that this kind of dispersion arose when an impulsive force was imposed). The breakdown of the jump is depicted with a dashed line in the bottom of Fig. 6. Afterwards the modified jump is supposed to propagate downstream (Houghton and Kasahara, 1968; Arakawa and Oobayashi, 1968). In order to simulate more accurately the jump phenomenon, the governing equations should include the internal and surface frictions, because according to the turbulent theory there is much kinetic energy dissipation at the area of the jump region.

Fig. 7 shows the \( y \)-component of the fluid velocity, \( v \), and the composite horizontal wind vector just like Fig. 5 for the case (2). Again we obtain the weak flow around the obstacle in comparison with the dominant zonal current. Total wind speed decreases on the windward side and increases on the lee side. The maximum wind velocity in this case appears on the foot of
Fig. 8. Evolution of the height deviation field of the case (4). The contours are drawn with an interval of 0.2. The bottom figure shows the cross-sectional view relative to the obstacle at $t=2.5$.

The results are in good agreement with the fall wind or the lee side jump.

(iv) Case (4): $u_0=2.0$. The computed patterns of the flow are shown in Fig. 8. In this case, the flow pattern is different from three other cases. The height of the fluid surface on the windward side increases with respect to time and tends to be a quasi-steady state above the crest of the obstacle. In the case of the one-dimensional flow, the maximum height of the fluid surface also appears above the crest for supercritical flow ($F>1$). The disturbance on the fluid surface occurring on the lee side shifts downstream. This is illustrated in the bottom of Fig. 8.

Now, in order to examine the equilibrium state of the numerical solutions, the steady state condition of the two-dimensional flow considered. In the steady state, equations (2.6) and (2.7) yield

$$(\nabla \cdot \mathbf{u}) \mathbf{u} + \mathbf{F}(\mathbf{c}^2 + \mathbf{n}^2) = 0$$

where $\mathbf{v}=(u,v)$ is the horizontal wind vector;

$$\mathbf{F} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)$$

the horizontal nabla operator. From equation (6.1), we get

$$\mathbf{F} \left( \frac{1}{2} \mathbf{v}^2 + \mathbf{c}^2 + \mathbf{n}^2 \right) = \mathbf{v} \times \mathbf{rot} \mathbf{v}$$

Along each stream line, equation (6.2) may be replaced by the well-known Bernoulli equation;

$$\frac{1}{2} \mathbf{v}^2 + \mathbf{c}^2 + \mathbf{n} = E_0$$

where $E_0$ denotes constant energy when no disturbance exists. Froude number $F$ is defined as

$$F^2 = \frac{\mathbf{v}^2}{\mathbf{c}^2}$$

Substituting (6.4) into (6.3), we obtain the equation;

$$F^2 = 2 \left( \frac{\varepsilon}{c^2} - 1 \right)$$
where

\[ e = E_0 - n^2 \]  

(6.6)

Equation (6.5) implies the relation;

\[ c^2 \gtrsim \frac{2}{3} e \] 

supercritical flow

\[ c^2 \approx \frac{2}{3} e \] 

critical flow

\[ F \lesssim 1 \] 

subcritical flow

Numerical solution for the different value of \( F \) should approach asymptotically to a steady state in the region near and above the obstacle. Thus in order to examine the present results equation (6.5) may be adopted as the steady state condition. In Fig. 9, the solid line shows the relation (6.5). \( F^2 \) and \( c^2/e \) are computed on each case at the last time step, i.e., at \( t=5.0 \) for the cases (1) and (2), and at \( t=2.5 \) for the cases (3) and (4). Since gridpoints in the steady state are different from case to case, the number of points plotted in Fig. 9 are limited and do not coincide with total number of gridpoints in the domain. Agreement of plotted points with the steady state line is good, especially in the case (4). This means that the case (4) has reached nearly steady state. The numerical solution approaches monotonically to the quasi-steady state.

The grid interval used in the present study is so coarse that the sharp jump is not resolved correctly and the computed jump is smoothed out. In the case (2) the supercritical flow is observed over the lee side of the obstacle. This suggests the existence of the sharp jump on the lee side, but numerical solution does not exhibit clearly.

7. Conclusions

A numerical study was conducted to clarify the dynamical effect of mountain upon the small-scale motion of the atmosphere. The computational model used for the present experiment is based upon the shallow water equations in Eulerian form.

The results of of the numerical simulation may be summarized as follows:

(1) The results are in good agreement with those of the one-dimensional flow for the cases (2), (3) and (4). In the case (2) the minimum height of the fluid surface is obtained near the crest of the obstacle and the jump arises on the lee side. In the case (3) the jump occurs on the lee side and strong wind blows on the foot of the obstacle. The jump shifts downstream as time advances. In the case (4) the maximum height of the fluid surface is obtained above the crest. The jump in this case is not remarkable as in the case (3).

(2) The flow is predominantly zonal and blows slightly around the obstacle.

(3) In the case (1) the wave-shaped pattern is obtained. The predominant wave length is close to the width of the obstacle. The train waves generated by the obstacle propagate downstream. The case (1) is considered to be in the critical point, though in the one-dimensional case it falls in the steady state as is shown in Fig. 2.

The results mentioned above qualitatively
simulate the jump at the lee side of the mountain like Mt. Fuji or the fall wind. Although in the present study the fluid above the inversion layer is ignored, the motion of the upper layer generally has the important effect on the inversion layer as is often observed in the nature. It seems to the present author interesting to take account of the dynamical effects of the upper layer. Furthermore, the turbulent transfer of the momentum in the boundary layer, friction on the earth’s surface and vertical coupling of the stratified fluid also be incorporated when the small-scale motions are treated.

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孤峰を越す2次元流の数値実験

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従来1次元の場合について行われてきた孤峰を越す流れの研究を2次元流に拡張し、障害物によって生じるhydraulic jump の力学的機構を調べるのが本論文の目的である。強風が山の風下側に起こり、そこでhydraulic jump がみられる場合には、上層に逆流層が存在することが多い。これに着目すると、逆流層下の運動は浅水方程式で表現できる。所で浅水方程式を用いて数値実験を行うためには、jump が shock 現象の一種であるから高い近似の差分表示が必要である。本研究では、one-step Lax-Wendroff method を用いた。孤峰に相当する障害物としてはベル状の山をおいた。

初期条件として、全領域が subcritical flow の場合を2例、supercritical flow の場合を2例それぞれ与えた。数値積分の結果、subcritical flow の1例と supercritical flow の2例については1次元の実験例とよく一致した。すな
わち、subcritical flow の 1 例と supercritical flow の 1 例について、障害物の風下側の頂上付近と山ろく付近に jump を得た。また、supercritical flow の他の例については、頂上付近で逆転面の盛り上がりが最大になった。subcritical の進い流れについては、逆転面に波動が起こり、その最も卓越する波長は障害物の幅の大きさであった。

ここで用いた数値実験モデルの結果では、流れの殆どが障害物を越え、障害物のまわりをまわる効果は小さいことがわかった。