NOTES AND CORRESPONDENCE

Numerical Solutions of Geopotential with Different Forms of Balance Relationships in the Tropics

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1. Introduction

In the past, the well known divergence equation obtained from the equation of motion has formed the starting equation for deriving various forms of wind-pressure balance relationships. In the present paper, the following two forms of the divergence equation will receive attention:

$$\frac{\partial D}{\partial t} = \zeta \eta + \vec{k} \cdot \nabla \eta \times \vec{V} - \nabla^2 \frac{\vec{V} \cdot \vec{V}}{2} - \nabla^2 \phi \quad (1)$$

$$\begin{aligned} -\frac{\partial D}{\partial t} = & -(D^2 + \vec{V} \cdot \nabla D) + 2J(u, v) \\ & - u\beta + f\zeta - \nabla^2 \phi \end{aligned} \quad (2)$$

where u, v are the components of wind velocity vector $\vec{V}$ along axes x, y pointing to east and north respectively, ζ is the vertical component of the relative vorticity vector $\nabla \times \vec{V}$, f is the coriolis parameter, η is the absolute vorticity, D is the velocity divergence $\nabla \cdot \vec{V}$, ϕ is the geopotential of an isobaric surface, β is the latitudinal variation of the coriolis parameter, t is the time, $\vec{k}$ is the vertical unit vector, and the vector operators have their usual meanings.

Equation (2) with the assumption of non-divergence of wind in which $\vec{V} = \vec{k} \times \nabla \psi$, where ψ is a stream function, has been widely applied as a balance relation for large-scale flow patterns to compute stream function from observed values of geopotentials. The reverse process of computing geopotential from this equation using wind information has also been tried. Miyakoda (1960) was the first to demonstrate the feasibility of such a scheme in respect of the tropics.

In the present note, balance relations as derived from equations (1) and (2) have been tried to compute geopotential fields from observed wind

data over the Indian Ocean region at four standard isobaric levels in two specific monsoon situations, using different boundary conditions. The main interest of the paper lies in investigating to what extent observed synoptic systems in tropical and equatorial latitudes are preserved by different approximated forms of the balance relations. A discussion of results obtained with different relations together with an analysis of errors in each case is presented.

2. Balance relations

The balance relations tested are the following:

$$\nabla^2 \phi_v = \eta \zeta + \vec{k} \cdot \nabla \eta \times \vec{V} - \nabla^2 \frac{\vec{V} \cdot \vec{V}}{2} \quad (3)$$

$$\nabla^2 \phi_b = 2J\left(\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}\right) + \nabla \cdot (f \nabla \psi) \quad (4)$$

$$\nabla^2 \phi_l = \nabla \cdot (f \nabla \psi) \quad (5)$$

$$\nabla^2 \phi_g = f \nabla^2 \psi \quad (6)$$

where $\phi_v, \phi_b, \phi_l, \phi_g$ denote balanced geopotentials computed from equations (3), (4), (5) and (6) respectively.

Equation (3) may be called the vorticity form of the steady-state divergence equation (1), since it involves vorticity explicitly on the right-hand side. Equation (4) is the $(\psi - \phi)$ balance equation obtained from the divergence equation (2) by omitting all terms involving divergence explicitly. Equation (5) is an approximated form of equation (4) with the non-linear Jacobian term omitted. For this reason, equation (5) has been called the linear form of the balance equation.

Equation (6) is a further approximation of equation (5), from which the term containing the meridional variation of the coriolis parameter has been omitted. Equation (6) is the well-known

quasi-geostrophic approximation of the balance equation and the relation, $\phi = f\psi$, has been widely used in middle and high latitudes as a fairly satisfactory balance relation.

Equation (3)-(6) represent four different forms of balance relations in which the right-hand side depends on wind information only and may, therefore, be solved numerically as Poisson equations with suitable boundary conditions, using the method of relaxation. Ellsaesser (1968) made a comparative study of different forms of wind-geopotential relationship to compute balanced wind fields from observed geopotential fields and use the reconverted geopotential fields in numerical weather prediction models. His finding is that the linear form of the balance relation as given by equation (5) is, from all considerations, the most satisfactory form of the balance equations to be used for initialization for NWP models tested by him. The balanced geopotential computed in the present paper in different cases has not yet been tested extensively in NWP models dealing with equatorial atmosphere. However, an assessment of the deviation of the computed geopotential field from observed values made in different cases reflects the relative capability of each to depict important synoptic patterns particularly equatorial systems. How each of them will fare in an initialization scheme for a dynamical forecasting model is yet to be seen.

3. Boundary conditions, grid and data

Forcing functions in equation (3)-(6) involve computation of vorticity and stream function. Vorticity is computed by using the relation, $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, and the non-divergent stream function is computed from the relation, $\nabla^2\psi = \zeta$, using suitable boundary conditions. Two types of boundary conditions were used for computing ψ , the Dirichlet as well as the Neumann boundary conditions. The Jacobian was computed by using the Arakawa form of finite-differences (Arakawa, 1963). Equation (3)-(6) were solved by the method of relaxation using two types of boundary values for ϕ . In one type, the values of ϕ at the boundary were taken from analysis of observed ϕ -field, and in the other the boundary values were computed using geostrophic relationship. In the latter case, two observed values of ϕ were taken one at the starting corner-point and the other at

the diametrically-opposite corner point of the computational field. The values of ϕ at successive grid-points along the x -axis were computed using the finite-difference geostrophic relation

$$\Delta\phi = fvd \quad (7)$$

where $\Delta\phi$ denotes change in ϕ over grid length d along the x -direction. With known boundary values at all the corner points and taking into account the variation of f in the y -direction, the values of ϕ at grid-points along the y -direction were then computed using the one dimensional second-order linear differential equation

$$\frac{\partial^2\phi}{\partial y^2} = -u\beta - f\frac{\partial u}{\partial y} \quad (8)$$

An over-relaxation coefficient of 0.72 and a convergence criterion of 0.1×10^{-12} were used throughout.

The computations are carried out over an area which extends from 40°E to 120°E and from 30°S to 30°N on a mercator projection. The grid points used are 2.5° latitude-longitude intersections. The initial data consists of observed wind for two distinct synoptic situations, one a northern winter situation at 00Z on 1 January 1964, and the other a northern summer monsoon situation at 00Z on 7 July 1963, at four standard pressure levels, viz. 850, 700, 500 and 300 mb. The situations were selected because of their satisfactory coverage of data during the International Indian Ocean expedition period (1963-64). However, data position in the southern hemisphere was not as satisfactory as in the northern hemisphere. For this reason, a variation of the area of computation with the same longitudinal coverage but with the latitudinal belt extending from 10°S to 30°N was also tried in the summer monsoon situation case. Fig. 1 shows the analyses of observed wind and geopotential fields at 700 mb for the summer situation for the computational area which extends latitudinally from 30°S to 30°N . For economy of space, one level only is presented. As already mentioned, stream function was computed using both Dirichlet and Neumann boundary conditions. However, as there was no perceptible improvement by employing Neumann boundary condition, the results using Dirichlet boundary condition only are presented. Computations were carried out with the CDC-3600

Fig. 1. Analyzed maps of observed wind and geopotential at 700 mb at 00Z 7 July 1963:
(a) Observed wind field, C-Cyclonic, A-Anticyclonic.
(b) Observed geopotential field, H-High, L-Low.

computer at the Tata Institute of Fundamental Research, Bombay.

4. Results and discussion

The geopotential fields computed with the different forms of the balance relation as given by equations (3)-(6) and observed boundary heights are presented in Fig. 2 (a)-(b) for the summer situation. Results of computation with geostrophically derived boundary heights are not presented, for lack of space. A comparison of the computed geopotential height patterns with the observed height patterns appears to show that except in the case of quasi-geostrophic relation, synoptic height patterns are depicted reasonably well in all other cases with minimum deviation in the case of the vorticity form of the balance

relation (Equation 3). Table 1 shows the root-mean-square (r.m.s.) distribution of errors of the computed geopotential field at different pressure levels obtained by using both the observed and geostrophically-derived boundary heights in the winter situation. For computing the r.m.s. errors, the observed height pattern was obtained by manual analysis of charts. Corresponding error statistics in respect of the summer situation are presented in Table 2. As already mentioned, a variation in computational area using a reduced area bounded by longitudes 40°E and 120°E and latitudes 10°S and 30°N was tried in the summer situation case and the error table in respect of this reduced area is given in Table 3.

Tables 1, 2, and 3 would appear to reveal the following broad features:

Fig. 2. Computed geopotential at 700 mb at 00Z 7 July 1963, with different forms of balance relation (boundary heights observed):
(a) Vorticity form, (b) Balance,
(c) Linear balance, (d) Quasi-geostrophic.

Table 1. Root-mean-square (r.m.s.) error distribution (meters)-1 January 1964.

Boundary heights	Levels (mb)	Vorticity form	Balance	Linear balance	Quasi-geostrophic
Observed	850	5.5	5.6	4.9	14.0
	700	8.3	8.1	7.1	12.0
	500	20.0	16.0	15.0	19.0
	300	48.0	58.0	56.0	56.0
Geostrophically derived	850	34.0	28.0	29.0	42.0
	700	30.0	29.0	27.0	23.0
	500	31.0	29.0	28.0	29.0
	300	81.0	74.0	73.0	89.0

Table 2. Root-mean-square (r.m.s.) error distribution (meters)-7 July 1963.

Boundary heights	Levels (mb)	Vorticity form	Balance	Linear balance	Quasi-geostrophic
Observed	850	10.8	12.4	12.0	22.7
	700	10.5	17.4	16.7	20.9
	500	12.8	16.5	15.3	19.7
	300	25.9	28.5	24.5	24.8
Geostrophically derived	850	59.7	62.5	62.6	73.6
	700	42.0	50.1	50.1	54.5
	500	24.2	21.8	22.1	21.9
	300	42.9	41.6	42.4	52.5

Table 3. Root-mean-square (r.m.s.) error distribution (meters)-7 July 1963.

Boundary heights	Levels (mb)	Vorticity form	Balance	Linear balance	Quasi-geostrophic
Observed	850	6.4	6.6	6.4	18.2
	700	7.9	8.5	8.0	19.2
	500	5.0	5.1	4.9	10.1
	300	11.5	13.9	13.2	25.5
Geostrophically derived	850	9.5	10.5	10.5	21.8
	700	12.8	13.6	13.1	14.9
	500	7.9	7.1	7.2	13.7
	300	25.4	27.3	26.7	37.0

Fig. 3. Distribution of r.m.s. errors of computed geopotential (meters) at 700 mb with latitude for different forms of balance relations:
(a) 00Z 1 January 1964. (b) 00Z 7 July 1963.

- (i) Relatively large errors are produced with boundary heights derived geostrophically;
- (ii) Amongst the different forms of balance relation tried, the quasi-geostrophic relation appears to produce the maximum errors;
- (iii) Root-mean-square error produced by the vorticity form of the steady-state divergence equation (Equation 3) appears to be comparable with those produced by the balance as well as the linear-balance relations (Equations 4 and 5).
- (iv) In the two situations studied, the error in the summer case appears to be larger than that in the winter case in the lower troposphere. Reverse appears to be the

case in the upper troposphere.

- (v) A comparison of errors presented in Tables 2 and 3 would seem to suggest that r.m.s. errors increase with enlargement of the computational area.

Figure 3 shows the latitudinal distribution of the r.m.s. errors at 700 mb obtained with observed boundary heights in the cases of the different balance relations. It may be seen that the largest errors occur in the case of the quasi-geostrophic relation with maximum a few degrees south of the equator during winter and north of the equator during summer. In general, the vorticity form of the wind-pressure relationship appears to produce minimum r.m.s. error, although in the winter case

(Fig. 3(a)), r.m.s. errors due to balance and linear balance relations are comparable with that due to the vorticity form.

References

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熱帯地方において種々のバランスの関係を仮定した場合の ジオポテンシャルの数値解

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