Instability of planetary waves in a zonal current

By M. Aihara and H. Imai

Meteorological Research Institute, Tokyo
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Abstract

Stability properties of planetary-scale motions in a baroclinic zonal current are studied numerically by means of a simple primitive equation system. Mathematical technique used is a form of the method of the initial value which is employed after Brown, Jr. (1969). Some special devices and extensions are made so as to make it applicable to the primitive equation model.

It is concluded that all waves are unstable and move always eastward. Results are compared with those of the multi-layer geostrophic model.

1. Introduction

Instability problems of planetary scale motions, that is, cyclone waves and ultralong waves, have been investigated by many authors in recent decades.

Most of these works are based upon the linearized perturbation equations. Eigenvalues and eigenfunctions are sought by assuming a wave type solution. Either analytical or numerical method is employed to solve them. In evaluation of eigenvalue, the recent trend is to use the matrix method which is a simple and common mathematical technique in utilizing the high-speed computer in the stability problem.

However, there are other techniques to solve linearized equations. The so-called initial value method is one of them. One of the authors has adopted this method, or which is more appropriately called the energy method, and discussed the stability properties of the disturbance in a barotropic atmosphere under the guidance of the late Prof. S. Syono. This paper treats the stability characteristics of planetary waves in a zonal current. Although the subject taken up is a rather classical one, the authors wish to examine the problem more closely based on the primitive equation system. The numerical technique in solving the primitive equation is due to the method of the initial value, but different from the aforementioned one. The method employed here is the same as that used by Wiin-Nielsen (1962), and especially by Brown, Jr., (1969), in the geostrophic stability theory. Development and modifications of the method are specially made so as to make it applicable to the primitive equation system. Stability properties of the planetary-scale motions of the atmosphere are studied numerically by means of this method.

2. Basic equations

In order to study the behaviours of planetary-scale motion in the atmosphere, we take up a set of primitive equations rather than the customarily used geostrophic model. This is mainly due to the fact that the latter model fails to describe the balance relation between the pressure and the wind field for ultralong waves as far as the meridional scale of the motion is ignored.

Perturbation equations used are as follows:

\[
\frac{\partial u}{\partial t} = -U \frac{\partial u}{\partial x} - \frac{dU}{dp} \omega + f v - \frac{\partial \phi}{\partial x} + A_m \frac{\partial^2 u}{\partial x^2} + \frac{\partial \tau_x}{\partial p} \tag{1}
\]

\[
\frac{\partial v}{\partial t} = -U \frac{\partial v}{\partial x} - f u + A_m \frac{\partial^2 v}{\partial x^2} + \frac{\partial \tau_y}{\partial p} \tag{2}
\]

\[
\frac{\partial \theta}{\partial t} = -U \frac{\partial \theta}{\partial x} - f \frac{dU}{d\theta} \frac{\partial \theta}{\partial p} + \frac{\partial \phi}{\partial x} - \frac{\partial \tau_w}{\partial p} \tag{3}
\]

\[
0 = \frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial p} \tag{4}
\]
\[ 0 = \frac{\partial \phi}{\partial p} + \frac{dH}{dp} \theta \quad (5) \]

where, \( x \) and \( p \) are taken positively eastward, and downward respectively, and \( t \) is the time.

Other quantities are those customarily used, i.e., small letters express perturbation quantities and capital letters are of mean quantities. \( H \) is the so-called Exner function defined by \( C_p(P/P_0) \) with \( \kappa = R/C_p \). In order to introduce \( \beta \)-effect, we use vorticity and divergence equations, instead of equations of motion. They are written,

\[ \frac{\partial}{\partial t} \frac{\partial u}{\partial x} = -U \frac{\partial^2 u}{\partial x^2} - \frac{\partial U}{\partial p} \frac{\partial \omega}{\partial x} + f \frac{\partial v}{\partial x} - \beta u - \frac{\partial \phi}{\partial x^2} + A_T \frac{\partial^3 u}{\partial x^3} + \frac{\partial}{\partial x} (\frac{\partial \tau_s}{\partial p}) \quad (6) \]

\[ \frac{\partial}{\partial t} \frac{\partial v}{\partial x} = -U \frac{\partial^2 v}{\partial x^2} + f \frac{\partial \omega}{\partial p} - \beta v + A_T \frac{\partial^3 v}{\partial x^3} + \frac{\partial}{\partial x} (\frac{\partial \tau_v}{\partial p}) \quad (7) \]

\( A_T \) and \( A_T \) are coefficients of eddy momentum and thermal diffusion, where \( P_r = A_T/A_T \) is put equal to unity for brevity’s sake. \( \tau_s \) and \( \tau_v \) means shearing stress and \( h \), the vertical transport of heat by eddies.

3. Method of solution

We shall explain the method of solution used in this paper. This is of the type of the initial value problem which was successfully applied to the study of hydrodynamic stability problem by Brown, Jr., (1969). This method is especially fits for computer calculation. For the outline of this method, one should refer to the paper cited above. One of the authors extended this method furthermore to make it applicable to the hydrostatic primitive equation system which contains, in general, three different types of physical waves.

Consider a simple harmonic wave solution, \( q = \hat{q} e^{ik(x-ct)} \) where \( q \) is any meteorological element in a complex representation. \( \hat{q} \) is the amplitude, \( k \) the wavenumber \( 2\pi/L \), \( C \) the complex wave speeds \( C = C_r + iC_i \).

Then, a convergent solution is said to be obtained if the wave speeds defined below,

\[ C_r = \frac{1}{k} \text{Im}(q \frac{\partial q^*}{\partial t}/q^*q) \]

\[ C_i = \frac{1}{k} \text{Re}(q \frac{\partial q^*}{\partial t}/q^*q) \quad (8) \]

coincide with each other at each grid point within a certain allowable limit \( \varepsilon \). \( q \) represents either \( u \), \( v \) or \( \theta \) in this case, and \( \text{Im} \) and \( \text{Re} \) mean Imaginary and Real part, respectively. \( q^* \) means the complex conjugate of \( q \).

That is, a convergency criterion is written

\[ \max |\Delta C_r|, |\Delta C_i| \leq \varepsilon \]

where \( \Delta C_r \) and \( \Delta C_i \) denote differences of phase speed calculated at each grid point. \( \varepsilon \) is taken to be 0.08 m sec\(^{-1}\).

The primitive equation system, however, contains three different types of waves, one of which is significant to describe planetary-scale motions, the other two being high-frequency, fast-moving waves, which are irrelevant to our purpose. So, the method of computation consists of two procedures, namely;

(I) Elimination of unwanted high-frequency wave or noise.

(II) Short time interval to estimate accurate values of phase speed by means of formula, (8).

For the elimination of noise wave, i.e., the first step mentioned above, we take up a low-pass filter.

It has been proved that, for example, the so-called modified Euler-backward scheme (cf. Kurihara (1965)) is highly effective to this end. Accordingly, in numerical integration of perturbation equations, the time interval \( \Delta t \) was chosen to be \( kC_n \Delta t = 1 \), where \( C_n \) means the noise speed which is close to the inertia-internal gravity wave modified by the mean flow. The time interval thus selected is known to be most efficient to damp out the noise.

In order to estimate the time tendency \( \partial q/\partial t \) in (8) as accurately as possible, an iterative scheme, similar to Runge-Kutta iteration, is used. That is, \( \partial q/\partial t = F \) is replaced by the following iterative process;

\[ q^{(n+1)}(t+\Delta t) = q(t) + \frac{1}{\Delta t} \left[ q^{(n)}(t+\Delta t_1) + F(t) \right] \]

where, \( F^{(n)}(t+\Delta t_1) = F(t) \) and \( n \) is the number of iteration. At the stage of con-
vergence, inequality is satisfied,
\[ |q^{(n+1)} - q^{(n)}| \leq \delta \]
where \( \delta \) is taken to be 0.05 m sec\(^{-1}\) or \( ^\circ \)C, and \( \Delta t = 1 \) min.

In actual computation, we divide the atmosphere into \( N \) layers of equal thickness \( \Delta p \). Each level is numbered from 0 to \( 2N \) successively downward. Prognostic values \( u, v \) and \( \theta \), and pressure \( \phi \) are assigned at odd-number

\[
\begin{align*}
0 & \quad \omega_0 = 0 \\
1 & \quad \omega_1 \\
2 & \quad \Delta p \\
3 & \quad \omega_3 \\
\vdots & \quad \omega_j \\
\text{odd, } j & \quad (\Psi, \theta, \phi)_j \\
4 & \quad \omega_4 \\
\vdots & \quad \omega_j \\
2N - 2 & \quad \omega_{2N-2} \\
2N - 1 & \quad \omega_{2N} = 0
\end{align*}
\]

Fig. 1 N-layer atmosphere. Assignment of the meteorological elements is shown in the figure.

levels. Vertical \( p \)-velocity \( \omega \) is at an even level (Fig. 1). Terms relating vertical differences are carefully determined by noticing the consistency of energy transformation in a layered atmosphere.

Basic equations in a layer model are written as follows,
\[
\begin{align*}
\frac{\partial}{\partial t} u_j &= -U_j \frac{\partial u_j}{\partial x} - f v_j + A_m \frac{\partial^2 v_j}{\partial x^2} + \frac{1}{\Delta p} (\tau_{j+1} - \tau_{j-1})_x \\
\frac{\partial}{\partial t} v_j &= -U_j \frac{\partial v_j}{\partial x} - f \left( \frac{\partial u_j}{\partial x} \right) v_j - \frac{1}{2 \Delta p} \left[ (D \theta j)_j + (D \theta j)_{j-1} \right] + A_x \frac{\partial^2 \theta_j}{\partial x^2} + \frac{1}{\Delta p} (h_{j+1} - h_{j-1}) \\
\frac{\partial}{\partial t} \theta_j &= \frac{\partial \omega_j}{\partial x} + \frac{1}{\Delta p} (\omega_{j+1} - \omega_{j-1}) \\
0 &= \phi_j - \phi_{j+2} - \Delta II_{j,j+2} \theta_{j+1}
\end{align*}
\]

where, \( DU_{j+1} = U_{j+2} - U_j \), \( D \theta j = \theta_{j+1} - \theta_j \), \( \Delta II_{j,j+2} = II_{j+2} - II_j \), respectively.

As an initial condition of the perturbation quantity, a simple velocity distribution is assumed,
\[
\begin{align*}
u(x, p) &= v_0(p) \sin kx \\
v(x, p) &= \gamma v(x, p) \\
\omega(x, p) &= 0
\end{align*}
\]

where \( \gamma \) is a small quantity. Then, pressure \( \phi \) is obtained from the divergence equation by assuming a balance condition, \( \frac{\partial}{\partial t} \frac{\partial u_j}{\partial x} = 0 \),
potential temperature $\theta$ is determined by hydrostatic relation providing $\theta_{2N-1} = 0$.

Mean state of the atmosphere is presented in Fig. 2. The profile of zonal flow $U_y$ is assumed to be linear with $p$, but the intensity of the vertical shear $\Lambda$ is varied widely in numerical computation. The mean winter situation of $U_y$ is shown in the same figure for comparison. A 10-layer atmosphere is assumed throughout the examples presented in this paper.

4. Result of computation without eddy dissipations

This and the following section are devoted to the description of the results of numerical computation made with and without eddy dissipations. In this section, we concern ourselves with the result of computation without eddy dissipation, an inviscid case. Stability characteristics of planetary-scale motions are shown in Fig. 3. This diagram was obtained by changing vertical shear $\Lambda$ and wavenumber $k$ as parameters. Wavenumber $k$ at $45^\circ$N was altered from 1 to 10 sequentially, and, then 14 and 28 are added supplementarily. Cases treated are twelve in all for a fixed value of zonal vertical shear. Fig. 3 shows all waves are unstable. In particular, the existence of two major instabilities, the stronger one in cyclone scale and the weaker one in ultralong wave scale, is to be seen.

The main features are quite similar to the results obtained with the multilayer geostrophic model by, for example, Green (1960), Hirota (1968), and recently, Garcia et al. (1970). However, the instability that appeared in the ultralong wave region in this model is rather emphasized than that of the geostrophic model. The reason why this is so will be considered in a later part of this paper.

Energy transformation functions and phase speed for the linear shear case are shown in Fig. 4. In the upper half of the figure, energy transformation functions $(P \cdot K')$, $(K \cdot K')$ for...
a particular shear $\Lambda=3 \text{ m sec}^{-1}100^{-1} \text{ mb}$ are shown.

$$\left\{ \vec{P} \cdot K' \right\} = -\frac{dU}{dp} \vec{\theta} \cdot \vec{\omega}$$

$$\left\{ \vec{K} \cdot K' \right\} = -\frac{dU}{dp} \vec{u} \cdot \vec{\omega}$$

where the square bracket means vertical integration. The former means the transformation of mean potential into disturbance kinetic energy due to the ascent of warm air and the sinking of cold air. The latter means the transformation of mean kinetic into eddy kinetic energy due to the eddy transport of momentum in the vertical under the existence of zonal vertical shear. The former plays a predominant role, while the latter process only a minor one in the growth of the disturbance. A large amount of energy transformation $\{\vec{P} \cdot K'\}$ appears around wave number $k=9$, the most unstable wave, and the second maximum is seen at $k=3$. The amount of $\{\vec{K} \cdot K'\}$ gradually decreases from $k=14$ to $k=4$, and then attains a weak maximum at $k=2$. It is easily known that the vertical transport of momentum, $\vec{u} \cdot \vec{\omega}$, is predominantly downward, especially in the troposphere.

The phase speed of the unstable waves is given in the lower half of the figure. It is readily recognized that the unstable wave always moves eastward with a velocity greater than the minimum zonal speed, $U_{\text{min}}$. As a consequence, there exists a steering level in the lower portion of the atmosphere, say, near the 700 mb level. In his theoretical considerations on the large-scale geostrophic disturbance, Miles (1964, 1965) has derived the possible range of phase speed of the unstable disturbance as a necessary condition and summarized in the so-called semi-circle theorem. That the existence of the steering level is a necessary condition of the unstable wave, is partly reproduced in our simple primitive equation system, too. As to the eastward movement of unstable waves, particularly ultralong waves, some comments will be given in connection with their vertical structures. In order to study the characteristic features of the growing disturbance more precisely, it is most appropriate to show the vertical distribution of physical quantities and their transformations.

Figures 5(a) and (b) express the dependence of meteorological elements on altitude for two selected cases, the most unstable wave, $k=9$ and the ultralong wave, $k=2$. The main features of the wave, $k=9$, a cyclone wave, are those commonly accepted and coincide fairly well with those of theoretical works and synoptic analyses hitherto published.

Energy transformation from mean potential to eddy kinetic $\{\vec{P} \cdot K'\}$ takes place in the lower part of the atmosphere centered about the 700 mb-level. However, eddy kinetic energy thus created is transferred further downward and upward by eddy transportation, $\vec{u} \cdot \vec{\omega}$. As a result of such mechanism, the maximum of eddy meridional kinetic energy appears very near to the earth's surface. The cyclone wave is thus said to be a surface phenomenon.

On the other hand, energy transformation $\{\vec{K} \cdot K'\}$ of $k=2$ is positive throughout the troposphere and negative in the stratosphere, though it is small in magnitude. Transforma-
tion is most active in the mid-troposphere, but disturbance kinetic energy thus generated is carried away from there and accumulated in the stratosphere. The amplitude of ultralong waves is predominantly large in the upper atmosphere, in that case, the vertical transportation of energy $\theta'\omega'$ is an important process. In a cyclone wave, both eddy meridional kinetic energy and northward transport of sensible heat concentrate in the surface layers of the atmosphere. On the contrary, they are emphasized in the upper atmosphere for ultralong waves. Zonal cross section of unstable planetary waves is given in Fig. 6(a) and (b). The characteristic features of the cyclone wave, i.e., vertical motion, temperature and divergence field relative to pressure trough, are satisfactorily reproduced in the figure by numerical calculation. Fig. 6(b) shows those of the ultralong wave. It is one of the remarkable features of the unstable wave that the pressure trough tilts westward with increasing height, but the tilting angle of the ultralong wave is much more emphasized than the cyclone wave. The correlation between temperature and vertical motion is rather high, even for the ultralong wave, while it is absolutely zero for the geostrophic disturbance with infinite meridional extent, (cf. Hirota (1968)), due to the lack of the term $\beta u$ in (10), which will be shown later.

The outstanding feature of the ultralong wave is to be observed in the divergence field in Fig. 6(b), that is, the coincidence of the pressure trough and the line of nondivergence. It is nothing but the establishment of the so-called Burger's relation in vorticity equation (7), and a balance relation between pressure and wind field in eq. (6) for ultralong waves.

Namely,
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\[ \omega = \frac{-3v}{a} \quad \text{and} \quad \omega a = \frac{-u}{a} \]

\( f \frac{\partial \omega}{\partial p} = \beta v \quad \text{(9)} \)

\( f \frac{\partial v}{\partial x} - \beta u = \frac{\partial^2 \phi}{\partial x^2} \quad \text{(10)} \)

The wind is westerly throughout the low pressure area and easterly in the whole domain of high pressure, while a southerly wind blows in front of the trough and a northerly wind at the rear of the trough. Therefore, the phase relation between pressure \( \phi \) and wind \( u, v \) satisfies both balance relations above as seen from Fig. 7. It turns out that both divergence and vorticity equations become di-

Fig. 7 Zonal cross-section of the disturbance zonal and meridional wind, for ultralong wave, \( k=2 \).
agnostic for ultralong waves. We can notice that the residual terms of the divergence and vorticity equation in (6) and (7) contain time derivative and are one order smaller in magnitude than eqs. (9) and (10). The residual terms tell that the perturbed winds \( u \) and \( v \) are advected by the mean zonal wind \( U \) toward the east.

5. Results of computation with eddy dissipations

In this section, we shall briefly discuss the effect of eddy dissipations upon the development of planetary waves. Horizontal diffusion and vertical transport of momentum and heat due to eddy are introduced in calculation. The numerical technique is the same as in the inviscid case. The modified Euler backward scheme is applied in numerical integration to eliminate noise waves and the time interval, \( \Delta t = 1 \) min. is taken in accurate evaluation of the phase speed \( C_r \) and \( C_i \) at each level. Numerical constants and dependency of the coefficient on height are assumed as follows,

\[
A_M = A_T = 10^6 \text{m}^2 \text{sec}^{-1}
\]

\[
h = \left( K \frac{\partial \theta}{\partial p} \right)
\]

\[
(\tau_x, \tau_y) = \left( A_T \frac{\partial}{\partial p} (u, v) \right)
\]

and \( K = A_T = \alpha p^3 \), where \( \alpha = 2.754 \times 10^{-7} \text{ sec}^{-1} \). At the earth's surface eddy stress is assumed to be

\[
(\tau_x, \tau_y) = -g \rho_s C_a V_s (u, v)
\]

and the input of sensible heat is given by the bulk formula,

\[
h_s = \gamma_s (T_\theta - \theta_{eN})
\]

where, \( T_\theta \) is the surface temperature of the earth acting as an external parameter. Coefficients \( C_a \) and \( \gamma_s \) are as follows;

\[
C_a = 0.001
\]

\[
\gamma_s = 5.0 \times 10^{-6} \text{ sec}^{-1}
\]

The phase speed and growth rate are shown in Fig. 8 for a particular value of vertical shear \( \Lambda = 3 \text{ m sec}^{-1} \text{100 mb}^{-1} \). Curves with \( T - S \) mean those with eddy dissipation, and those of the inviscid case are also shown for the sake of comparison. In general, the growth of the wave is suppressed by the introduction of dissipative terms. Especially, the disturbance of wave number \( k = 4 \text{ msec}^{-1} \text{100 mb}^{-1} \) \( \phi = 45^\circ \text{N}, \text{N}=10 \text{ \text{INVISCID}} \)

\[
\begin{align*}
\Lambda &= 3 \text{ m sec}^{-1} \text{100 mb} \\
\phi &= 45^\circ \text{N}, \text{N}=10
\end{align*}
\]

of dissipative terms. Especially, the disturbance of wave number \( k = 4 \text{ msec}^{-1} \text{100 mb}^{-1} \) is considerably modified and becomes a stationary and neutral wave. The phase speed of the unstable wave is little influenced by dissipation effect as might be expected. Vertical distributions of thermal and mechanical energy dissipations due to eddy are plotted in Figs. 9 and 10, respectively. Both types of energy dissipations are large in the lower portion of the atmosphere for the cyclone wave as seen from Fig. 9, while the primary maximum of energy dissipations is observed at the tropopause level and the secondary maximum near the surface boundary layer for ultralong waves. This is the result of the predominance of the amplitude of ultralong waves in the upper at-
mosphere, since the coefficient of vertical eddy process is assumed to decay with altitude proportional to the square of pressure.

In his numerical analysis, Kung (1967) has noticed that, there is a dissipation layer near the tropopause level or a jet stream level according to his presentation, based upon synoptic evidence. His analysis is based on 5-year data and the area treated is rather restricted, i.e., over the whole area of North America.

6. Some theoretical considerations on the instability of ultralong waves

In this section, we will briefly comment on the unstable character of ultralong waves.

Basic equations are simplified as follows based upon the results of numerical computations in the foregoing two sections.

\[
\begin{align*}
\frac{\partial \theta}{\partial t} = & -U \frac{\partial \theta}{\partial x} - f \frac{dU}{dx} \frac{\partial \theta}{\partial p} - \frac{\partial \omega}{\partial p} \\
\beta \psi = & \frac{\partial \omega}{\partial p} \\
f \frac{\partial \psi}{\partial x} - \beta u = & \frac{\partial \phi}{\partial x} \\
\frac{\partial \phi}{\partial p} = & -\frac{dU}{dp} \frac{\partial \theta}{\partial p} \\
\frac{\partial u}{\partial x} = & -\frac{\partial \omega}{\partial p}
\end{align*}
\]
After elimination of dependent variables $u$, $v$ and $\phi$, we have a set of equations concerning $\theta$ and $\omega$,

\[
\begin{align*}
0 &= \frac{\partial}{\partial t} + U \frac{\partial \theta}{\partial x} + \frac{f^2}{\beta} \frac{dU}{dp} \frac{\partial \omega}{\partial p} + \frac{\partial \theta}{\partial p} \omega \\
0 &= \frac{\partial^2}{\partial p^2} \left( \frac{f^2}{\beta} \frac{\partial \omega}{\partial x^2} + \beta \omega \right) + \frac{d\Pi}{dp} \frac{\partial \theta}{\partial x^2}
\end{align*}
\]

The differential equation of the amplitude of vertical motion $\Omega$ can be derived by assuming a wave-form perturbation for $\theta$ and $\omega$. Thus,

\[
(U-C) \frac{d^2 \Omega}{dp^2} + \mu \left( \frac{f^2}{\beta} \frac{dU}{dp} \frac{d\Omega}{dp} - \sigma \Omega \right) = 0
\]

where,

\[
\begin{align*}
\mu &= \frac{k^2 \beta}{\beta^2 - f^2 k^2}, \quad \text{negative for all waves in higher latitudes} \\
\sigma &= -\frac{d\Pi}{dp} \frac{d\theta}{dp}, \quad \text{static stability which is positive}
\end{align*}
\]

The boundary condition is simply,

$\Omega(0) = \Omega(p_0) = 0$.

Through the transformation

$\Omega = XY$

and $X = (U-C)^{-1}$

It is written,

\[
\frac{d^2 Y}{dp^2} + Q(p, C) Y = 0
\]  

(11)

where,

\[
\begin{align*}
\lambda &= \frac{f^2}{2\beta} \mu \\
Q(p, C) &= \frac{\lambda(1-\lambda)}{(U-C)^2} \left( \frac{dU}{dp} \right)^2 - \frac{\lambda}{U-C} \left( \frac{d^2 U}{dp^2} \right) \\
&- \frac{\mu\sigma}{U-C}
\end{align*}
\]

and $Y(0) = Y(p_0) = 0$.

By multiplication of $Y^*$, complex conjugate of $Y$, in eq. (11), and integrating vertically from the top to the bottom of the atmosphere, we have the following integral relation;

\[
C \int_0^p \left[ \frac{dY}{dp} \right]^2 - Q(p, C) |Y|^2 dp = 0
\]

The imaginary part gives a necessary condition of instability, that is to say,

\[
\text{Im}(Q_c) = \frac{\mu}{(U-C)^2} \left[ \frac{f^2 \Delta^2 (1-\lambda)(U-C)}{U-C} \right] - \sigma
\]

since we confine our discussion to the linear shear case. Therefore, in order to have an unstable wave $C_i > 0$, $\text{Im}(Q_c)$ must change signs within the interval $(0, p_0)$. Since the static stability $\sigma$ and the factor $(1-\lambda)$ are positive throughout the integral domain, it follows that, at least partly, $(U-C)$ must be positive, namely, there must be a steering level. This conclusion is consistent with the one already derived by Miles for cyclone scale disturbances by means of the geostrophic model.

7. Concluding remarks

In this paper, emphasis is laid on the differences between the results obtained by the primitive equation system and those by the geostrophic model, especially for ultralong waves.

Based upon the results of computation, we conclude that, in the growth of ultralong waves, the energy source is mainly the conversion of mean potential into disturbance kinetic energy in contrast to the results of the geostrophic model. In the latter model, the energy source of the unstable ultralong wave must be sought in other places.

Unstable planetary waves move eastward, and their structures and principal features are quite similar to those given by the geostrophic model. An eastward movement of unstable ultralong waves is rather a rare occurrence, but it is reported that such movement is actually observed, for instance, in a pre-sudden warming period in the stratosphere.

The method of solution adopted in this paper is a type of the so-called initial-value method, and has a wide applicability to other geophysical phenomena. However, some modiﬁcatory devices are sometimes required, as in its application to large-scale motions in the equatorial region or smaller-scale phenomena in which frequencies of waves are very close to each other and difﬁcult to separate. A sharp band-pass filter may be a possible way to
In the primitive equation model, the three-dimensionality of the motion must be taken into consideration, as has been pointed out by Phillips (1964). The method of solution taken up in this paper has one advantage over the matrix method commonly used, since the rank of matrix will be enormous in such a case. Case studies are now going on.

Numerical computations are performed by HITAC 5020 F installed at JMA, and programs are coded by FORTRAN double precision complex arithmetics.

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