1. Introduction

As to the deformation of ocean surface induced by winds, the present author published his result for a zonal ocean of homogeneous water. (Hidaka, 1955) The resulting deformation was proved to be inversely proportional to the magnitude of the coefficient of horizontal eddy viscosity represented by the notation $A$. Thus the range of undulation of the surface was 64 meters for $A=7.8 \times 10^8$ c.g.s and 0.65 m for $A=7.8 \times 10^{10}$ c.g.s. for the whole meridian. This means that the range of surface undulation is of an order of magnitude corresponding to $A=10^{10}$ c.g.s., which is a value estimated too high for a usual oceanic motion. In order to reconcile this discrepancy, some modification of theory is necessary and the author came to a conclusion that a baroclinic ocean must be substituted for a homogeneous one. In addition to this, spherical coordinates will have to be employed for the motion on a rotating earth. The following discussion is the result of this modification.

2. Equations of motion and continuity

Suppose an ocean of baroclinic, inhomogeneous water completely covering a rotating globe. Let us consider that the current system in this ideal ocean is steady, maintained by a system of winds constantly blowing over the ocean surface and that this wind system is dependent only on latitude $\phi$, and not of the longitude $\lambda$. This is the so-called planetary wind system. There are of course meridional and longitudinal components of winds, but they are supposed to have no variation in the east-west direction. Then it is evident that the current system generated by this wind system would be of a similar type. Of course, there would be components of currents in meridional and longitudinal directions, but they will depend on the latitude only.

There is assumed a similar type of distribution of mass as the result of mutual adjust-ment between wind drifts and water masses generated at different parts of the ocean. Thus the density only changes in a meridional plane and its variation is independent of $\lambda$.

Take the axis of $z$ vertically downwards from an undisturbed surface closest to the ocean surface. Let the components of a current be $u$, $v$ and $w$ in east ($x$, zonal), north ($y$, meridional) and vertical directions. These are, of course, independent of longitude $\lambda$ by assumption. As the result of motion of water and the earth's rotation we shall have a deformation or an undulation $z=-\zeta$ on the sea surface. This will also be independent of longitude so that we shall have no deformation of water surface in the east-west direction. Let $z=0$ be a level surface very close to the actual sea surface $z=-\zeta$, so that $\zeta$ is not large. The vertical variation of density within the height $\zeta$ can be disregarded as small. Thus we have

$$d \rho=g \rho dz,$$

so that

$$\rho(z)=\int_{-\zeta}^{0}g \rho dz+\int_{0}^{z}g \rho(z)dz$$

$$=\rho_0 \cdot \zeta(\phi)+\int_{0}^{z}g \rho(z)dz,$$

where $\rho$ or $\rho(z)$ is density at any depth $z$ $\rho_0$ and the surface density, $g$ the acceleration of gravity, and $\int_{0}^{z}g \rho(z)dz$ the pressure exerted by the water column between $z=0$ and $z=z$. The equations of motion are given by

$$\frac{A_1}{\rho R^2 \cos \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \frac{\partial u}{\partial \phi} \right)$$

$$+\frac{A_2}{\rho} \frac{\partial^2 u}{\partial z^2} +2\omega \sin \phi \frac{\partial v}{\partial \phi} = 0,$$

$$\frac{A_1}{\rho R^2 \cos \phi} \frac{\partial}{\partial \phi} \left( \cos \phi \frac{\partial v}{\partial \phi} \right)$$
\[ + \frac{A_i}{\rho} \frac{\partial^2 v}{\partial z^2} - 2\omega \sin \phi u \]
\[ - \frac{1}{\rho} \left\{ \rho \frac{d \xi}{R \partial \phi} + \frac{\partial}{\partial \phi} \left[ \int_0^z \rho \rho(z) d \rho \right] \right\} = 0, \]  

where \( \omega \) is the angular velocity of the earth, \( R \) the mean radius of the earth, and \( A_i \) and \( A_z \) the coefficients of horizontal and vertical eddy viscosity respectively. In addition to them, the equation of continuity is

\[ \frac{1}{R \cos \phi} \frac{\partial}{\partial \phi} (v \cos \phi) + \frac{\partial w}{\partial z} = 0. \]

The conditions to be satisfied at both poles are

\[ \phi = \pm \frac{\pi}{2} : u = v = 0, \]  

while the following conditions must be satisfied on the surface of the sea:

\[ - A_i \frac{\partial u}{\partial z} \bigg|_{z=0} = \tau_E, - A_i \frac{\partial v}{\partial z} \bigg|_{z=0} = \tau_N \]

where \( \tau_s \) and \( \tau_n \) are the east and north components of the wind stress respectively. Strictly speaking, the sea surface is located at \( z = -\zeta(\phi) \). However, because \( \zeta \) is considered small, the boundary conditions at the free surface \( z = -\zeta \) are transferred to the plane \( z = 0 \). Integrate both sides of (1) with respect to \( z \) from \( z = 0 \) to a sufficiently deep level \( z = h \) at which it can be assumed either that this level is the ocean floor, or that there is no horizontal flow and the vertical shears of horizontal velocity components vanish.

Then equation (1) will lead to

\[ M_E = \int_0^h \rho \rho u dz; \quad M_N = \int_0^h \rho \rho v dz \]

If the equation of continuity (2) is integrated with respect to \( z \) from the sea surface down to the bottom or to a sufficiently deep level \( z = h \) where no horizontal flow is assumed, it follows:

\[ \frac{d}{R d \phi} (M_N \cos \phi) = -w \bigg|_{z=0} \cos \phi \]

assuming that there is no vertical flow either at the sea surface or at the depth \( h \), the right-hand member of the above equation vanishes. Thus it follows that \( M_N \cos \phi \) is equal to a constant, \( C \) or

\[ M_N = \frac{C}{\cos \phi}. \]

Now that it follows

\[ M_N = 0 \text{ for } \phi = \pm \frac{\pi}{2}, \]

from the condition (3). In order that this condition be satisfied at all latitudes, it must follow that \( C = 0 \) everywhere.

With \( M_N = 0 \), equation (5) becomes

\[ \frac{A_i}{\rho R^2 \cos \phi} \frac{d}{d \phi} \left( \cos \phi \frac{d M_E}{d \phi} \right) + \frac{\tau_E}{\rho} \]

\[ + 2\omega \sin \phi M_E = 0, \]

and

\[ \frac{\tau_E}{\rho} - 2\omega \sin \phi M_E - \frac{d}{R d \phi} \bigg|_{z=0} \rho \]

\[ \left( \int_0^z \rho(s) ds \right) dz + \frac{d \zeta}{R d \phi} = 0 \]

neglecting the variation of \( \rho \) as small.

3. Integration of the equations

In order to integrate (7), assume

\[ \tau_E = X_1 \cos \phi + X_2 \sin 2\phi + X_3 \cos 3\phi \]

\[ + \ldots \ldots + \text{ad inf} \]

and

\[ M_E = M_1 \cos \phi + M_2 \sin 2\phi + M_3 \cos 3\phi \]

\[ + \ldots \ldots + \text{ad inf}. \]

This expansion make \( M_E = 0 \) at both poles because of the boundary conditions (3). Integration of (7) from \( \phi = -\pi/2 \) to \( \phi = \phi \) gives
\[
\frac{A}{\rho R^2} \cos \phi \frac{dM}{d\phi} + \int_{-\pi/2}^{\phi} \tau_\beta(\phi) \cos \phi d\phi = 0. \tag{11}
\]

Now that it follows
\[
\int_{-\pi/2}^{\phi} \tau_\beta(\phi) \cos \phi d\phi = \frac{X_1}{2} (\phi + \frac{\pi}{2})
\]
\[\quad - \frac{1}{2} X_2 \cos \phi + \frac{1}{2 \cdot 2} (X_1 + X_2) \sin 2\phi
\]
\[\quad - \frac{1}{2 \cdot 3} (X_2 + X_3) \sin 2\phi + \frac{1}{2 \cdot 4} (X_3 + X_4) \sin 4\phi
\]
\[\quad - + + \ldots \quad \tag{12}
\]
The expansion of \(\phi + \pi/2\) in a trigonometric series is
\[\phi + \frac{\pi}{2} = \frac{2}{1} \cos \phi + \frac{2}{3} \sin 2\phi - \frac{2}{3} \cos 3\phi
\]
\[\quad + \frac{2}{4} \sin 4\phi + \frac{2}{5} \cos 5\phi + \frac{2}{6} \sin 6\phi
\]
\[\quad - + + \ldots \quad \tag{13}
\]
Multiplying (13) by \(X_1/2\) and substituting it into (12),
\[
\int_{-\pi/2}^{\phi} \tau_\beta(\phi) \cos \phi d\phi =
\]
\[\frac{1}{1} X_1 - \frac{1}{2} X_2 \cos \phi,
\]
\[+ \left( \frac{1}{2} X_1 + \frac{1}{4} X_1 + \frac{1}{4} X_2 \right) \sin 2\phi
\]
\[+ \left( - \frac{1}{3} X_1 - \frac{1}{6} X_1 - \frac{1}{6} X_4 \right) \cos 3\phi
\]
\[+ \left( - \frac{1}{4} X_1 + \frac{1}{8} X_3 + \frac{1}{8} X_5 \right) \sin 4\phi
\]
\[+ \left( + \frac{1}{5} X_1 - \frac{1}{10} X_4 + \frac{1}{10} X_6 \right) \cos 5\phi
\]
\[+ \left( + \frac{1}{6} X_1 + \frac{1}{12} X_5 + \frac{1}{12} X_7 \right) \sin 6\phi
\]
\[+ \left( - \frac{1}{7} X_1 - \frac{1}{14} X_4 - \frac{1}{14} X_8 \right) \cos 7\phi
\]
\[+ \ldots \quad \text{ad inf.} \quad \tag{14}
\]
On the other hand
\[
\frac{A}{\rho R^2} \cos \phi \frac{dM}{d\phi} = \frac{2}{2} M_2 \cos \phi
\]
\[+ \left( \frac{M_1}{2} - \frac{3M_3}{2} \right) \sin 2\phi
\]
\[+ \left( \frac{2M_2}{2} + \frac{4M_4}{2} \right) \cos 3\phi
\]
\[+ \left( - \frac{3M_3}{2} - \frac{5M_5}{2} \right) \sin 4\phi
\]
\[+ \left( \frac{4M_4}{2} + \frac{6M_6}{2} \right) \cos 5\phi
\]
\[+ \left( - \frac{5M_5}{2} - \frac{7M_7}{2} \right) \sin 6\phi
\]
\[+ \left( \frac{6M_6}{2} + \frac{8M_8}{2} \right) \cos 7\phi
\]
\[+ \left( - \frac{7M_7}{2} - \frac{9M_9}{7} \right) \sin 8\phi + \ldots \quad \text{ad inf.} \quad \tag{15}
\]
Substituting (14) and (15) in (11) and equating to zero the coefficients of \(\cos\phi\), \(\sin 2\phi\), \(\cos 3\phi\), \ldots, it follows
\[
2M_2 + \left( \frac{1}{1} X_1 - \frac{1}{2} X_2 \right) \cdot \frac{2\rho R^2}{A_i} = 0,
\]
\[
2M_2 + 4M_4 + \left( - \frac{1}{3} X_1 - \frac{1}{6} X_2 - \frac{1}{6} X_4 \right) \cdot \frac{2\rho R^2}{A_i} = 0,
\]
\[
4M_4 + 6M_6 + \left( + \frac{1}{5} X_1 - \frac{1}{10} X_4 - \frac{1}{10} X_6 \right) \cdot \frac{2\rho R^2}{A_i} = 0,
\]
\[
6M_6 + 8M_8 + \left( - \frac{1}{7} X_1 - \frac{1}{14} X_4 - \frac{1}{14} X_8 \right) \cdot \frac{2\rho R^2}{A_i} = 0, \ldots
\]
\[
34M_{34} + 36M_{38} + \left( - \frac{1}{35} X_1 - \frac{1}{70} X_{34} \right) \cdot \frac{2\rho R^2}{A_i} = 0;
\]
\[
- \frac{1}{1} X_1 - \frac{1}{4} X_1 + \frac{1}{4} X_3
\]
\[
- \frac{M_1}{2} - 3M_3 + \left( - \frac{1}{4} X_1 + \frac{1}{8} X_3 + \frac{1}{8} X_5 \right) \cdot \frac{2\rho R^2}{A_i} = 0,
\]
\[
- 3M_3 - 5M_5 + \left( - \frac{1}{4} X_1 + \frac{1}{8} X_3 + \frac{1}{8} X_5 \right) \cdot \frac{2\rho R^2}{A_i} = 0.
\]
where the distance between both poles is divided into 36 equal parts or every 5° of meridional arc.

If the coefficients \( X_1, X_2, X_3, \ldots \) are known, \( M_1, M_2, M_3, \ldots \) can be determined. Further it follows that

\[
\begin{align*}
-2w \sin \phi M_1 + \frac{\tau_N}{\rho} = \frac{g h}{R} \frac{d \zeta}{d \phi},

\tau_N = \int_0^1 \rho(s) ds
\end{align*}
\]

From (12), it follows that

\[
-2w \sin \phi M_1 = \frac{-2w R^2}{A_1} (M_1 \cos \phi + M_2 \sin \phi + \ldots) \sin \phi
\]

\[
= \frac{2w R^2}{A_1} \left( \frac{1}{2} M_2 \cos \phi + \frac{1}{2} (M_1 - M_3) \sin 2\phi + \ldots \right)
\]

\[
+ \frac{1}{2} (M_3 - M_5) \sin 4\phi
\]

\[
+ \left( -\frac{1}{2} M_1 + \frac{1}{2} M_4 \right) \cos 3\phi + \ldots
\]

(17)

where the coefficients \( M_3, M_5, M_7, \ldots \) can be computed from (16). If further \( \tau_N(\phi) \) can be given by the expression:

\[
\tau_N(\phi) = Y_1 \cos \phi + Y_2 \sin 2\phi + Y_3 \cos 3\phi + \ldots Y_{36} \sin 36\phi,
\]

(19)

it follows that

\[
\frac{d \zeta}{R d \phi} + \frac{1}{g \rho h R} \frac{d}{d \phi} \left\{ \int_0^1 \rho(s) ds d\zeta \right\}
\]

\[
= \frac{1}{g \rho h} \left( -2w \sin \phi M_1(\phi) + \frac{\tau_N(\phi)}{\rho} \right)
\]

\[
= \frac{1}{g \rho h} \left( -2w R^2 \frac{M_1 - M_3}{A_1} + \frac{Y_1}{\rho} \right) \cos \phi
\]

\[
+ \left( \frac{-2w R^2}{A_1} \frac{M_1}{2} - Y_2 \right) \sin 2\phi
\]

\[
+ \left( \frac{-2w R^2}{A_1} \frac{M_1 - M_3}{2} - Y_3 \right) \cos 3\phi + \ldots
\]

(20)

This series can be evaluated because \( M_1, M_3, M_5, \ldots \), are known. Thus the surface deformation \( \zeta(\phi) \) is given by

\[
\zeta(\phi) + \frac{1}{g \rho h} \int_0^1 \rho(s) ds d\zeta
\]

\[
= \frac{1}{g \rho h} \left( -2w R^2 \frac{M_1 - M_3}{A_1} + \frac{Y_1}{\rho} \right) \cos \phi
\]

\[
- \frac{1}{2} \left( -2w R^3 \frac{M_1 - M_3}{A_1} + \frac{Y_2}{\rho} \right) \cos 2\phi
\]

\[
+ \frac{1}{3} \left( -2w R^3 \frac{M_1 - M_3}{A_1} + \frac{Y_3}{\rho} \right) \sin 3\phi
\]

\[
- \frac{1}{4} \left( -2w R^3 \frac{M_1 - M_3}{A_1} + \frac{Y_4}{\rho} \right) \cos 4\phi + \ldots
\]

(21)

where \( C \) is a constant.

Let \( \zeta(\phi) \) represent

\[
\zeta(\phi) = -\frac{1}{g \rho h} \int_0^1 \rho(s) ds d\zeta
\]

(22)

and the right-hand side of (21) be denoted by \( \zeta_w(\phi) \), it follows

\[
\zeta(\phi) = \zeta_w(\phi) + \zeta_s(\phi).
\]

(23)

It is now stressed that the quantities \( \zeta_w(\phi) \) and \( \zeta_s(\phi) \) can be easily calculated from data obtained through practical observations.

If the quantity \( \zeta_s(\phi) \) is defined as the steric water height and the right-hand member of (21) as the wind-produced water height \( \zeta_w(\phi) \) corresponding to wind action, then it follows that the elevation \( \zeta(\phi) \) of the surface of a zonal ocean, either barotropic or baroclinic, is given by (23).

The elevation \( \zeta(\phi) \) of the surface of a baroclinic zonal ocean thus can be known because
all the terms in the right-hand sides of (20) and (21) can be computed exclusively from observed data.

It may be easily understood that \( \zeta_{p}(\phi) \) only exists when the stratification is baroclinic and therefore this quantity may be disregarded in case of a homogeneous ocean. Thus the deformation in a zonal ocean will be given by the right-hand member of (21). The constant \( C \) can be included in \( \zeta(\phi) \), for example, so as to make \( \zeta(\phi)=0 \) at the equator.

4. Computation of the shape of sea surface induced by wind in an ocean of uniform water

In a uniform ocean for which \( \rho \) is a constant, the expression (22) or

\[
\zeta_{w}(\phi) = -\frac{1}{g\rho h} \int_{0}^{z} \left( \int_{0}^{s} \rho(s) ds \right) dz
\]

is a constant, so that this quantity has no influence on the variation of sea surface in a meridional direction.

The wind-produced deformation \( \zeta_{w}(\phi) \) was computed by using \( X_{m}, Y_{m}, R, A_{i} \) and the expressions (16) and (18). Thus the relation of \( \zeta_{w}(\phi) \) versus the latitudes are given for \( A_{i}=10^{8} \) and \( A_{i}=10^{10}, \) and compiled in Table 1.

For smaller values of \( A_{i} \), the deformation is inversely proportional to \( A_{i} \) to a very close approximation, so that it can be obtained by multiplying the values of \( \zeta_{w}(\phi) \) for \( A_{i}<10^{8} \) c.g.s by multiplying \( 10^{8}/A_{i} \).

Table 1. Deformation of Water Surface \( \zeta_{w}(\phi) \) due to the Existing Mass Distribution in the Central Pacific Ocean

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \zeta_{w}(\phi) )</th>
<th>( \phi )</th>
<th>( \zeta_{w}(\phi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>70° S</td>
<td>197 cm</td>
<td>3° N</td>
<td>20 cm</td>
</tr>
<tr>
<td>68° S</td>
<td>198</td>
<td>6° N</td>
<td>-8</td>
</tr>
<tr>
<td>66° S</td>
<td>197</td>
<td>9° N</td>
<td>43</td>
</tr>
<tr>
<td>64° S</td>
<td>183</td>
<td>12° N</td>
<td>18</td>
</tr>
<tr>
<td>62° S</td>
<td>170</td>
<td>15° N</td>
<td>-40</td>
</tr>
<tr>
<td>60° S</td>
<td>147</td>
<td>18° N</td>
<td>-19</td>
</tr>
<tr>
<td>58° S</td>
<td>138</td>
<td>21° N</td>
<td>3</td>
</tr>
<tr>
<td>56° S</td>
<td>119</td>
<td>24° N</td>
<td>-1</td>
</tr>
<tr>
<td>54° S</td>
<td>112</td>
<td>27° N</td>
<td>-67</td>
</tr>
<tr>
<td>52° S</td>
<td>102</td>
<td>30° N</td>
<td>-7</td>
</tr>
<tr>
<td>50° S</td>
<td>76</td>
<td>35° N</td>
<td>-4</td>
</tr>
<tr>
<td>48° S</td>
<td>64</td>
<td>40° N</td>
<td>21</td>
</tr>
<tr>
<td>46° S</td>
<td>66</td>
<td>45° N</td>
<td>42</td>
</tr>
<tr>
<td>44° S</td>
<td>74</td>
<td>50° N</td>
<td>50</td>
</tr>
<tr>
<td>42° S</td>
<td>74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40° S</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38° S</td>
<td>76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36° S</td>
<td>60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34° S</td>
<td>62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>32° S</td>
<td>52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30° S</td>
<td>19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>28° S</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>26° S</td>
<td>17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24° S</td>
<td>-13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22° S</td>
<td>-3</td>
<td></td>
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<td>20° S</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18° S</td>
<td>-16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>16° S</td>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14° S</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The diagrams in Figs. 1 and 2 show the meridional variation of sea surface with respect to a level surface close to the former. For both values of the horizontal coefficient of eddy viscosity, or \( A_{i}=10^{8} \) and \( 10^{10} \) c.g.s., the sea surface is low in the southern ocean, nearly flat between 30° S and 30° N, but it goes up northward at high latitudes of the northern hemisphere. This means that more water occupies the northern hemisphere than the southern. This tendency is also seen in a former paper by the present author, although somewhat deformed. (Hidaka, 1954)

5. Calculation of \( \zeta_{p}(\phi) \)

Elevation of the sea surface corresponding
to the meridional distribution of density \textit{in situ} was computed from several meridional sections prepared from data of the stations occupied by Japanese research vessels. These are the section prepared from data of the stations occupied by the “Ryofu Maru” under the leadership of Dr. Jotaro Masuzawa, the Japan Meteorological Agency, the sections of KH-68, KH-69 and KH-70 prepared from data of the stations occupied in 1968, and 1969 and 1970 on board the Hakuho Maru under the leadership of Professor Sumio Horibe, the Ocean Research Institute, University of Tokyo. All of these four sections are along different meridians but not very far apart in the Pacific Ocean.

A comparison of these four sections reveals that the quantity

\[ \zeta_w(\phi) = \frac{1}{g \rho_0 h_0} \int_0^h \rho \left( \int_0^r \rho(s) ds \right) dz \]

distributed approximately symmetrically on both sides of the equator. It is higher at or close to the equator and goes down toward the high latitudes of both hemispheres. This tendency is not necessarily distinct in the Ryofu Maru section of 1967, but can be recognized in all the other four sections obtained by the “Hakuho Maru”. The difference in water height over a level surface is less than two meters.

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**Table 2. Deformation of Water Surface \( \zeta_w(\phi) \) induced by Winds on an Ocean of Uniform Density**

<table>
<thead>
<tr>
<th>( \sin \phi )</th>
<th>( \phi )</th>
<th>( \zeta_w(\phi) )</th>
<th>( \sin \phi )</th>
<th>( \phi )</th>
<th>( \zeta_w(\phi) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.0</td>
<td>90.0° S</td>
<td>-613 cm</td>
<td>0.1</td>
<td>5.7° N</td>
<td>-2 cm</td>
</tr>
<tr>
<td>-0.9</td>
<td>64.2° S</td>
<td>-458 cm</td>
<td>0.2</td>
<td>11.5° N</td>
<td>4</td>
</tr>
<tr>
<td>-0.8</td>
<td>53.2° S</td>
<td>-339 cm</td>
<td>0.3</td>
<td>17.5° N</td>
<td>5</td>
</tr>
<tr>
<td>-0.7</td>
<td>44.4° S</td>
<td>-257 cm</td>
<td>0.4</td>
<td>23.6° N</td>
<td>6</td>
</tr>
<tr>
<td>-0.6</td>
<td>36.9° S</td>
<td>-180 cm</td>
<td>0.5</td>
<td>30.0° N</td>
<td>42</td>
</tr>
<tr>
<td>-0.5</td>
<td>30.0° S</td>
<td>-131 cm</td>
<td>0.6</td>
<td>36.9° N</td>
<td>66</td>
</tr>
<tr>
<td>-0.4</td>
<td>23.6° S</td>
<td>-69 cm</td>
<td>0.7</td>
<td>44.4° N</td>
<td>128</td>
</tr>
<tr>
<td>-0.3</td>
<td>17.5° S</td>
<td>-28 cm</td>
<td>0.8</td>
<td>53.2° N</td>
<td>203</td>
</tr>
<tr>
<td>-0.2</td>
<td>11.5° S</td>
<td>-14 cm</td>
<td>0.9</td>
<td>64.2° N</td>
<td>296</td>
</tr>
<tr>
<td>-0.1</td>
<td>5.7° S</td>
<td>-3 cm</td>
<td>1.0</td>
<td>90.0° N</td>
<td>361</td>
</tr>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>0 cm</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

---

**Fig. 3. Distribution of Dynamic Steric Water Heights \( \zeta_s(\phi) \) in a Meridian.**

6. Actual sea surface

An actual sea surface \( \zeta(\phi) \) is given, as already stated, by the sum of \( \zeta_w(\phi) \), the deformation of the sea of uniform water, and \( \zeta_s(\phi) \) which can be computed from the distribution of density \textit{in situ}. Although the dynamic steric deformation represented by \( \zeta_s(\phi) \) is independent of \( A_1 \), the barotropic deformation represented by \( \zeta_w(\phi) \) is inversely proportional to \( A_1 \). Thus the superposition of these two deformation of different sources gives a very complicated surface form of the actual ocean.

However, it can still be concluded that the northern hemisphere has more water than the southern, because \( \zeta_s(\phi) \) is distributed in nearly a symmetric form, while \( \zeta_w(\phi) \) has an asymmetric distribution as shown by the
right-hand side of (21).

7. Summary

The above will be summarized as follows:

(1) The dynamical solution of the sea surface deformed by the rotation of the earth and wind stress exerted on the surface of a baroclinic zonal ocean was obtained by solving the dynamical equations, use being made of the actual wind-stress distribution along a number of meridians approximately at the centre of the Pacific, and the steric structure of the ocean along the meridians. The sea level due to wind action is found to be higher in the northern than in the southern hemisphere.

(2) The total deformation $\xi(\phi)$ consists of the uniform water height $\xi_w(\phi)$ which comes from the wind action on the ocean of uniform water only and steric water height $\xi_s(\phi)$ which arises from the mass distribution along the same meridian.

(3) The uniform water height $\xi_w(\phi)$ is inversely proportional to $A_I$, the coefficient of horizontal eddy viscosity, while the steric water height depends upon neither $A_I$ nor the wind stress, but only on the steric structure of sea water.

(4) Distribution of $\xi_s(\phi)$ is approximately symmetric with respect to the equator, so that more water is kept in the northern than in the southern hemisphere.

8. Discussion

(1) The fact that the coefficient of horizontal eddy viscosity must be more than 100 times as great as usually assumed in oceanic phenomena of smaller scales may be explained in much the same way as in the case of the Antarctic Circumpolar Current, in which $A_I$ has to be $10^9 - 10^{10}$ times larger in order to explain the speed of this ocean current. (Hidaka, 1953) The author has been looking for a reasonable explanation.

(2) If the coefficient of lateral eddy viscosity increases indefinitely, it will follow that $\xi_w(\phi) = 0$, and the expression for $\xi(\phi)$ will become $\xi(\phi) = \xi_s(\phi)$ and the surface deformation is entirely due to mass distribution, although (22) gives the least possible deformation that is expected.

This is in an approximate conformity with A. Defant’s estimation that the difference in water heights between 10° N and 20° N in the Atlantic is 26.8 cm. (Defant, 1941) From Table 1, it can be perceived that nearly the same difference exists between these latitudes.

(3) Henry Stommel (Stommel, 1964) and Eugene Lisitzen (Lisitzen, 1965) published the dynamical topography of the world oceans, and both concluded that the maximum difference in sea level is of the same order of magnitude. R. B. Montgomery (Montgomery, 1969) also accepted this value from various evidence.

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