NOTES AND CORRESPONDENCE

Interrelationship between Leaf Density and Mixing Length within Plant Canopies

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(Manuscript received 5 January 1972)

Recently Kondo (1971) suggested that in tall and dense plant communities the nondimensional group

$$A \equiv \gamma(h-d) \quad (1)$$

is practically the same for a whole line of various crops. Kondo's argument was based directly on the meaning of γ and $(h-d)$, and the particular numerical value of A ($A=1$) was presumably based on the few results which he quoted (actual range of A was about 0.5 to 1.5).

The present note offers a somewhat different point of view and indicates the conditions under which A could be expected to be constant among, as well as within, canopies.

Adopting Inone's (1963) model

$$\gamma = \left(\frac{ac}{4l^2} \right)^{1/3} \quad (2)$$

and

$$h-d = \frac{l}{k} \quad (3)$$

so that

$$A \equiv \gamma(h-d) = \frac{1}{k} (lac/4)^{1/3} \simeq 1.57(lac)^{1/3} \quad (4)$$

If $A = \text{const}$ as Kondo suggested, then

$$lac = \text{const} \quad (5)$$

and in particular, if $A \simeq 1$, then

$$lac \simeq 0.26 \quad (6)$$

Equation (5), if substantiated by additional experimental data, may serve as an important practical tool to determine the mixing length from known a and c values. Such a relationship may be particularly useful within canopies whose

elements (leaves) are similar in shape, while the leaf density varies markedly with height.

Let us consider now the cases for which the relationship (5) is expected to hold. First consider a high Reynolds Number flow within rigid canopy models. If two such models are geometrically similar, then la , being a non-dimensional combination of geometrical parameters, must be independent of scale. The drag coefficient of similar configurations at high Re is also independent of scale, so that the group $A' = lac$ is invariant with geometric scale. Inasmuch as grasses may be considered geometrically similar, they may have the same value of A' if c does not vary too much with windspeed. This may also be true among similar broad leaf species.

Consider now a more specific geometrical arrangement, consisting of slender horizontal cylinders directed perpendicular to the flow and randomly spaced at a density of n per unit area.

From geometrical considerations

$$l \propto 1/\sqrt{n} \quad (7)$$

while

$$a \propto bn \quad (8)$$

If the cylinders are far enough from each other to eliminate any interaction among them (e.g. $1/(b\sqrt{n}) = 10$, Thom, 1971), then c is independent of n and

$$A' = lac \propto b\sqrt{n} \quad (9)$$

If b remains constant with height in the 'canopy' but n varies (as is the case in many real canopies), A' cannot be expected to remain constant, but rather increase with density.

On the other hand, if the 'canopy' elements are close enough to each other so that interaction is present, the drag coefficient decreases with element

Fig. 1. Relationships between the density of elements and the drag coefficient. Marshall's (1971) data are for short vertical cylinders whose height was only twice their diameter. He used as reference velocity the velocity at the height of the elements. Kays and London's (1955) data are for long tubes at $Re=4000$. Thom's (1971) result is implied from his paper. Empty circles denote staggered (or diamond) arrangements and full circles denote in-line arrangements.

density as can be seen in Fig. 1. If in this range the relation between c and n and b may be roughly approximated by

$$c \propto 1/(b\sqrt{n}) \quad (10)$$

as the figure (especially Marshall's data) suggests, then

$$A' \simeq lac \propto \frac{1}{\sqrt{n}} \cdot bn \cdot \frac{1}{\sqrt{n}} = \text{const} \quad (11)$$

The preceding general argument should also hold for three dimensional elements (say, randomly oriented and spaced discs). In that case one finds for non-interacting elements

$$A' \propto (BN^{1/3})^2 \quad (12)$$

requiring

$$c \propto 1/(BN^{1/3})^2 \quad (13)$$

in the interaction region, to compensate for the increase of la with density.

Thom (1971) found that in a beans crop whose data he analysed, the shelter (interaction) factor was about 4, indicating that the actual average drag coefficient was only about a quarter of that of the individual elements. This finding may be considered as supporting the view that actual canopy elements do interact, thus exhibiting a reduced drag coefficient.

Nomenclature

a - leaf area density	(cm^2/cm^3)
A - defined by Equation (1)	(—)
A' - lac	(—)
b - cylinder diameter	(cm)
B - characteristic length of three dimensional canopy elements	(cm)
c - drag coefficient	(—)
d - displacement height	(cm)
h - crop height	(cm)
k - von Kármán's constant (~ 0.4)	(—)
l - mixing length in canopy	(cm)
n - density of two-dimensional canopy elements	(cm^{-2})
N - density of three dimensional canopy elements	(cm^{-3})
Re - Reynolds Number	(—)
γ - decay coefficient of canopy wind profile	(cm^{-1})

References

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植物群落内部の葉面密度と混合距離の相互関係

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