NOTES AND CORRESPONDENCE

On the Determination of the Magnitude of the Mean Horizontal Wind Vector

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The magnitude of the mean horizontal wind vector, $\bar{u}$, is an extensively used parameter in the theory of micrometeorology. The definitions of different intensities of turbulence include this parameter. Businger *et al.* (1971) suggest that it would be more appropriate to treat $\bar{u}$ as a parameter in micrometeorological studies than the mean horizontal wind speed, $\bar{s}$.

Nevertheless, s is measured in every meteorological and climatological station by means of the inexpensive cup or propeller anemometer, whereas measurement of u is limited to special stations, equipped with a sonic or a hot wire anemometer, or with some kind of a digitizing-computing machine.

Because measurements of s are more readily available than u , $\bar{s}$ is usually used instead of $\bar{u}$ with the assumption that the error will be small (Businger *et al.*, 1971).

Frenkiel (1951) showed that in a first approximation

$$\bar{s} \approx \bar{u} \left(1 + \frac{\overline{v'^2}}{2\bar{u}^2} \right) = \bar{u} \left(1 + \frac{G_y^2}{2} \right), \quad (1)$$

where G_y is the intensity of turbulence.

Let us examine first if this approximation can be used to derive $\bar{u}$ from measurement of $\bar{s}$.

Consider a horizontal plane with wind components u and v along and perpendicular to the mean wind direction. Then $\bar{v}=0$ and the horizontal wind speed is given by

$$s^2 = u^2 + v^2, \quad (2)$$

and

$$\bar{s}^2 = \bar{s}^2 + \overline{s'^2} = \bar{u}^2 + \overline{u'^2} + \overline{v'^2}, \quad (3)$$

therefore

$$\bar{s}^2 = \bar{u}^2 \left(1 + \frac{\overline{v'^2}}{\bar{u}^2} \right) + \overline{u'^2} - \overline{s'^2}. \quad (4)$$

Combining Eqs. (4) and (1) gives the interesting result

$$\overline{s'^2} = \overline{u'^2} - \left(\frac{\overline{v'^2}}{2\bar{u}} \right)^2, \quad (5)$$

which is equivalent to Eq. (1) and accurate to the same degree.

One can immediately see that two difficulties arise that prevent the use of Eq. (1) in order to deduce $\bar{u}$ from measurement of $\bar{s}$:

1. Eq. (1) incorporates G_y which is equally difficult to measure as $\bar{u}$ and therefore it is measured only in specially equipped stations.

2. Assuming $\overline{v'^2} \sim \overline{u'^2}$ (Swanson and Cramer, 1965) and using measurements of G_y [Swanson and Cramer (1965), Yokoyama (1969), measurements by the present author using a set up described in Skibin (1972)], we see that the ratio between the second and first right hand side terms in Eq. (5) can be as high as 0.1. The result that $\overline{u'^2}$ could be larger than $\overline{s'^2}$ (by as much as 10%) rules out the use of Eq. (1).

It might be of interest to devise a simple method of obtaining $\bar{u}$ from $\bar{s}$ using standard meteorological equipment, *i.e.* wind vane and cup anemometer.

Consider the width of the trace of a windvane. It has maximum angular width, $2\bar{\theta}$, which can be determined directly from the trace. A rule of thumb (Markee, 1963) relates $2\bar{\theta}$ to the standard deviation of wind azimuth, σ_θ .

Let T be the averaging time interval, then

$$\bar{u} = \frac{1}{T} \int_0^T s \cos \theta \, dt, \quad (6)$$

where θ is the instantaneous wind direction measured from the mean. From Eq. (6) it follows that

$$\begin{aligned}\bar{s} \cos \bar{\theta} &= \cos \bar{\theta} \frac{1}{T} \int_0^T s \, dt = \frac{1}{T} \int_0^T s \cos \bar{\theta} \, dt \\ &\leq \frac{1}{T} \int_0^T s \cos \theta \, dt \leq \frac{1}{T} \int_0^T s \, dt = \bar{s},\end{aligned}\quad (7)$$

that is

$$\bar{s} \cos \bar{\theta} \leq \bar{u} \leq \bar{s}. \quad (8)$$

Let us select

$$\bar{u} = \bar{s} \frac{1 + \cos \bar{\theta}}{2}, \quad (9)$$

then the maximum error in determining $\bar{u}$ will not exceed

$$\Delta \bar{u} \leq \bar{s} \frac{1 - \cos \bar{\theta}}{2}, \quad (10)$$

and the relative error will be

$$\frac{\Delta \bar{u}}{\bar{u}} \leq \frac{1 - \cos \bar{\theta}}{1 + \cos \bar{\theta}}. \quad (11)$$

As an example, take $\theta = 20^\circ$ (neutral conditions), then

$$\frac{\Delta \bar{u}}{\bar{u}} \leq 0.0311, \quad (12)$$

that is, using Eq. (9) under stable or neutral conditions, $\bar{u}$ will be determined with an error that will not exceed 3%. The error will probably be less since for $G_y > 0$, $\bar{u}$ is certainly less than $\bar{s}$ and it is probably always greater than $s \cos \bar{\theta}$ so that the sign in Eq. (8) should read $<$ instead of $\leq$. For $\bar{\theta} = 30^\circ$ (width of trace of 60°) the error will not exceed 7%.

Equation (9) is particularly useful because it is a definition with which we can routinely determine $\bar{u}$. The maximum error that could then result would be given by Eq. (11). It is important that in the derivation of Eq. (9) no a-priori assumptions were made concerning the structure or intensity of turbulence which could limit the use of Eq. (9). Its use is limited only by the individually-determined desired accuracy in obtaining $\bar{u}$.

Comparing Eq. (1) to Eq. (9) yields the relation

$$G_y^2 = 2 \frac{1 - \cos \bar{\theta}}{1 + \cos \bar{\theta}}. \quad (13)$$

This is correct with the same degree of accuracy as Eqs. (1) and (9) together, *i.e.*, it is correct under stable and near neutral conditions with low turbulence level and relatively high wind speed so that

$$u', v' \ll \bar{u}. \quad (14)$$

These conditions imply also that

$$\bar{\theta} \ll 1, \quad (15)$$

so that $\cos \theta$ can be developed into a series, and to a first approximation

$$\cos \bar{\theta} = 1 - \frac{\bar{\theta}^2}{2}. \quad (16)$$

Using Eqs. (15) and (16), Eq. (13) becomes simply

$$G_y^2 = \frac{\bar{\theta}^2}{2}. \quad (17)$$

Under the conditions of Eq. (14), Eq. (17) can easily be used to determine the lateral intensity of turbulence from a wind vane trace.

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水平風速ベクトルの大きさの算定

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