Some Problems in Reproducing Planetary Waves by Numerical Models of the Atmosphere

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Abstract

The effects of vertical resolution and the upper boundary on the structure of the planetary waves in numerical models are investigated with a simplified one dimensional model, using the quasi-geostrophic, mid-latitude β-plane approximation. The results obtained by models with different vertical resolution and different height of the upper boundary are compared with the control solution which is obtained by using a small grid increment of 0.25 km and by including large dissipative effects near the upper boundary located at 90 km.

The influence of lowering the upper boundary is not fatal to the structure of the waves in the troposphere if the upper boundary is placed at the middle stratosphere and a model has several layers in the stratosphere, because Newtonian cooling or other damping effects of realistic magnitudes well attenuate the wave reflected at the top.

Stationary planetary waves obtained by low resolution models (5 or 6 layers) show apparent similarity to the true (control) solution in a qualitative sense. But to obtain quantitatively correct solutions, vertical grid increments should be taken as small as Δz = 1~2 km in the troposphere and Δz = 2~3 km in the stratosphere and the top should be placed in the middle stratosphere. When the time integration is performed with the use of a low resolution model, using the true data (control solution) as the initial, the waves can no longer be stationary but tend to transform into the model’s own stationary state. For resolving this problem we must use higher resolution models as mentioned in the stationary case.

Analogous results are obtained for the simulation of the waves in the stratosphere.

1. Introduction

It is well known that stationary planetary waves are generated by the earth’s surface topography and diabatic heating or cooling in the lower troposphere and propagate upward into the stratosphere and sometimes into the mesosphere. Within the frame work of linear theory, dynamics of the waves have been discussed by many authors, e.g. Charney and Drazin (1961), Eliassen and Palm (1961), Matsuno (1970) and now the natures of the waves as mentioned above are well established.

In numerical models of the general circulation, the stationary planetary waves are reproduced at least qualitatively (Manabe and Terpstra, 1974; Kasahara et al. 1973). Nevertheless, the stationary planetary waves are not adequately simulated in atmospheric models currently used in the numerical weather prediction, probably because they have relatively low vertical grid resolution and sometimes do not include the stratospheric levels owing to the limitation of computer capacity. The scarcity of the global observational data may be another cause of the difficulty. All these factors are fatal to simulate and forecast the planetary waves. (For the baroclinic unstable waves, it is not so fatal because they are confined in the troposphere and their scale is smaller.) For example, Ito and Isono (1971) showed the westward progression of the very long wave of zonal wavenumber one in their hemispheric prediction model which were stationary in the real atmosphere. The model used by them has only 4 layers and its top was placed at 100mb so that it doesn’t contain the stratosphere.

The false retrogression of the ultra-long waves with Rossby wave speed has been known since the time when hemispheric numerical prediction started and it seems that the problem has not yet
been fully solved. As mentioned previously, poor vertical resolution is considered to be a cause of the error. By adopting a model with finite number of layers, we are forced to introduce “the upper boundary” of the atmosphere which has no physical reality. This is also a part of the present problem and will be discussed in detail.

There have not been so many discussions about vertical resolution of numerical models compared with those about horizontal resolution. Manabe and Hunt (1968) compared the 18-level GCM model with the 9-level model of Smagorinsky et al. (1965) and showed that the increase of the model’s vertical resolution brings about remarkable improvements in the model’s similarity to the real atmosphere. Especially, the pattern in the lower stratosphere was notably improved. However, we cannot deduce the general information from their study about the effect of vertical resolution on the stationary planetary waves including the upper boundary condition, because they do not incorporate the effects of inhomogeneity of the earth’s surface in their model and hence the forced stationary planetary disturbances are not produced. Moreover, the general circulation models involve many physical processes and simulate not only planetary waves of global scale but also baroclinic waves and other phenomena and we see the coupled behaviour of all of them, so that the general circulation model is too complicated to get a simple concept about the sole effect of vertical resolution on the stationary planetary waves. Therefore, it is worthy to examine the effects of vertical resolution as well as the forced stationary planetary disturbances separately from the problem of vertical resolution. However, at present the method is not yet fully established and we will not be concerned with this method in the present paper.

We must be careful about the effect of the upper boundary in a finite difference model. In the real atmosphere the planetary waves are produced in the troposphere and propagate upward. They are sometimes reflected by strong westerly winds in the stratosphere or dissipated in the upper atmosphere by the damping effects. They can never reach the infinite height ($z = \infty$) within a finite time so that the condition $\omega = 0$ at $p = 0\text{mb}$ ($z = \infty$) has no influence on the waves. However, in a finite difference model the waves can reach the upper boundary and may be reflected downward by the artificially placed top of the model. The waves in the lower atmosphere are thus affected by this unnaturally reflected waves. This is true even if the top is placed at 0mb, because the waves can reach the uppermost layer within a finite time. Thus the formally correct condition may bring about incorrect results, so far as we use finite difference models with poor vertical resolution. In this way, the problem of the upper boundary is inseparably tied with the problem of vertical resolution.

Recently it has been proposed to introduce the so-called sponge layer into the atmospheric models (A. Arakawa and T. Tokioka, 1974, personal communication). The layer is placed in the uppermost part of the model and it is designed to absorb incident waves but not to reflect or emit waves. By adopting such a device we may be able to treat the problem of upper boundary separately from the problem of vertical resolution. However, at present the method is not yet fully established and we will not be concerned with this method in the present paper.

In the paper we first obtain the stationary state solutions of planetary waves by use of the models with different resolutions and make intercomparison among them.

Next, we discuss the transient states of planetary waves in low resolution models by giving initial conditions, so as to get an idea about the errors in numerical weather forecasting.

The two problems mentioned above are somewhat different. As mentioned before, the stationary planetary waves do not remain stationary but move westward with time because the vertical resolution is not sufficient (Itoo and Isono, 1971). It is pointed out that the rapid retrogression of the planetary waves does not occur and the waves move slowly in either direction or the amplitudes of the waves change gradually in models with increasing vertical resolution (Fawcett, 1969; Miyakoda et al. 1972). From these facts, we may guess that even if the simulation of the steady state in a coarse vertical resolution model is successful in a qualitative sense, the numerical forecast where the initial data of the real atmosphere is used will result in failure, because in the model the waves may behave to approach the model’s own stationary state which is different from that in the real atmosphere.

2. Basic equations

Since we are concerned with planetary waves in the middle latitudes, the quasi-geostrophic, $\beta$-plane approximations are applicable. The linearized vorticity equation and the thermodynamic equation for perturbations superimposed on a zonal flow may be expressed as
\[
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \rho^2 \phi + \beta \frac{\partial \phi}{\partial x} - f^2 \frac{\partial \omega}{\partial \rho} = K_{mh} \frac{\partial^2 \phi}{\partial \rho^2} + g^2 \frac{\partial}{\partial \rho} \left\{ K_{mx} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \phi}{\partial \rho} \right) \right\} + \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \frac{\partial \phi}{\partial \rho} - \frac{\partial U}{\partial \rho} - \frac{\partial \phi}{\partial x} + S \phi + Q \right)
\]

Where

\[
\rho = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad S = -\frac{1}{\rho} \frac{\partial l_n \dot{v}}{\partial \rho}, \quad \theta = T(p_n/p)^k.
\]

The notations used here are as follows:

- \( x \) eastward directed horizontal coordinate
- \( y \) northward directed horizontal coordinate
- \( p \) pressure
- \( g \) acceleration of gravity
- \( f \) Coriolis parameter
- \( \beta \) variation of Coriolis parameter with latitude
- \( U \) mean zonal wind
- \( \phi \) deviation of isobaric surface from the basic state due to perturbation
- \( \bar{p} \) density of the basic state
- \( \rho \) density of the basic state
- \( S \) static stability
- \( Q \) perturbation diabatic heating rate
- \( K_{mh} \) horizontal eddy viscosity
- \( K_{mx} \) vertical eddy viscosity
- \( K_{th} \) horizontal kinematic thermal diffusivity
- \( \tau \) time constant of Newtonian cooling

Assuming that the eastward wave length is \( L(=2\pi/k) \) and the northward width is \( D \), we separate variables as \( \phi(x, y, p, t) = \phi(p, t) \sin((\pi/D)y) \), \( e^{ikx} \), and we obtain,

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \frac{\partial^2 \phi}{\partial x^2} + \beta \frac{\partial \phi}{\partial x} - f^2 \frac{\partial \omega}{\partial \rho} &= K_{mh} \frac{\partial^2 \phi}{\partial \rho^2} + g^2 \frac{\partial}{\partial \rho} \left\{ K_{mx} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \phi}{\partial \rho} \right) \right\} + \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \frac{\partial \phi}{\partial \rho} - \frac{\partial U}{\partial \rho} - \frac{\partial \phi}{\partial x} + S \phi + Q \\
&= \left( k^2 + \pi^2 \right)^2 K_{mh} \phi - \left( k^2 + \pi^2 \right) g^2 \rho^2 \frac{\partial \phi}{\partial \rho} \\
&\quad \cdot \left( K_{mx} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \phi}{\partial \rho} \right) \right) + \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \frac{\partial \phi}{\partial \rho} - \frac{\partial U}{\partial \rho} \frac{\partial \phi}{\partial x} + S \phi + Q \\
&= \left( \frac{k^2 + \pi^2}{D^2} \right) K_{th} \frac{\partial \phi}{\partial \rho} - \frac{1}{\tau} \frac{\partial \phi}{\partial \rho} \\
&= \frac{\partial}{\partial t} \left\{ \frac{\partial}{\partial \rho} \left( \frac{1}{S} \frac{\partial \phi}{\partial \rho} \right) \right\} - \frac{\partial^2 \phi}{\partial \rho^2} - \frac{\partial \omega}{\partial \rho} \frac{\partial \phi}{\partial \rho} + g^2 \frac{\partial}{\partial \rho} \left\{ K_{mx} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \phi}{\partial \rho} \right) \right\} + f^2 \frac{\partial}{\partial \rho} \left\{ K_{mx} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial \phi}{\partial \rho} \right) \right\} + \frac{1}{\tau} \frac{\partial \phi}{\partial \rho} + S \phi + Q
\end{align*}
\]

Equation (5) is the basic equation of our problem, which corresponds to the potential vorticity equation.

We assume, as the upper boundary condition, that the vertical \( p \)-velocity vanishes at the top of a model and free slip condition, i.e.,

\[
\omega = 0, \quad \frac{\partial \phi}{\partial \rho} = 0 \quad \text{at} \quad p = p_t \ (or \ z = z_t)
\]

where \( p_t (z_t) \) is the pressure (height) of the top of a model. If a model has enough number of layers and extends to \( p = 0 \) mb \((z = \infty)\), above condition should be a good approximation to physical boundary condition that vertical \( p \)-velocity vanishes at the real top of the atmosphere, i.e.,

\[
\omega = 0 \quad \text{at} \quad p = 0 \ \text{mb} \ (or \ Z = \infty)
\]

As the lower boundary condition, we assume that the vertical \( p \)-velocity is caused by surface topography and by convergence in the Ekman boundary layer. According to Charney and Eliassen (1949) the expression for them is given as follows:

\[
\omega = \omega_m + \frac{HF}{f^2} \phi \left( k^2 + \frac{\pi^2}{D^2} \right) \phi, \quad \frac{\partial \phi}{\partial \rho} = 0
\]

at \( p = p_b \ (or \ z = z_b) \)

where \( p_b (z_b) \) is the pressure (height) of the top of the Ekman layer assumed to be 900 mb \((1 \ km)\) in this study. \( \omega_m \) is the vertical \( p \)-velocity caused by surface topography. The second term on the right-hand side of the first equation expresses the effect of the Ekman layer. \( H \) is the scale height, assumed to be 8 km, and \( F \) is expressed as

\[
F = \sqrt{\frac{f}{2} \sin 2\alpha H}
\]

where \( \alpha \) is the angle between the geostrophic and the surface wind and \( K \) is the eddy viscosity. In our model \( F \) is fixed to \( 2.0 \times 10^{-8} \ \text{sec}^{-1} \).
3. The finite difference scheme

Since the effect of vertical resolution may depend on a finite difference scheme, we must be careful to construct a scheme. In this paper, we use a scheme which seems to be the best one for the geostrophic equations and is generally used in numerical prediction. We divide the atmosphere from \( p=p_l \) to \( p=p_b \) into \( N \) layers, with variable grid intervals \( 2\Delta p_k \) \((k=1, \ldots, N)\), as shown in Fig. 1. The variable \( \phi \) and parameters \( U \) and \( K_{mh} \) are defined at the middle of a layer and \( \omega, S, K_{mz}, \) and \( \tau \) at the interface of two adjacent layers.

![Vertical distribution of variables](image1)

Fig. 1 Vertical distribution of variables

We derive the finite difference form of (3) at the \( \phi \)-level and that of (4) at the \( \omega \)-level, using the approximation such as,

\[
\left( \frac{\partial \phi}{\partial p} \right)_k \approx \frac{\phi_k - \phi_{k-1}}{\Delta p_k + \Delta p_{k-1}}, \quad \left( \frac{\partial \omega}{\partial p} \right)_k \approx \frac{\omega_{k+1} - \omega_k}{2\Delta p_k} \quad (8)
\]

This scheme satisfies integral constraint such as

\[
\int_{p_l}^{p_b} \frac{\partial \omega}{\partial p} dp = \omega(p_b) - \omega(p_l)
\]

From these equations and the finite difference form of the lower boundary condition (7), we eliminate \( \omega_k \) \((k=1, \ldots, N+1)\) and obtain \( N \) linear algebraic equations corresponding to the eq. (5), which contain the variables \( \phi_k \) \((k=1, \ldots, N)\). In the case of the steady state, we neglect the term \( \left( \frac{\partial \phi}{\partial t} \right)_k \) and obtain \( \phi_k \) directly solving them. When we perform time integration, the leap-frog scheme is used. By changing \( \Delta p_k, N \) and \( p_l \), we make various models from low resolution one to high resolution one.

4. The basic state

Since we are interested in the performance of general circulation models and numerical prediction models which simulate or predict the actual planetary waves, it is desirable to choose the basic state and various parameter values in our models as close as possible to those of the real atmosphere. The integration region extend from 1 km up to 90 km high. Static stability is calculated with the mean temperature and density obtained from the U.S. Standard Atmosphere 1962 (See Fig. 2). For damping coefficients such as \( K_{mh}, K_{mz}, K_{th} \) and \( \tau \), correct values are not fully known. So we shall give one of plausible values, referring to some literatures, as shown in Fig. 3. In general, the upward propagating waves are reflected back at the top of a model. Large values of \( K_{mh} \) and \( K_{th} \) are incorporated above 40 km in order to eliminate such unrealistic effects of the upper boundary condition (See Fig. 3). The basic zonal winds are shown in Fig. 4. Three typical cases are considered, the strong
Fig. 3 Vertical profiles of $K_mh$, $K_mz$ and $\tau$ (assuming $K_{th}=K_{mz}$)

Fig. 4 Mean zonal wind for cases of UI, UII, and UIII.

polar-night jet in the winter, weak westerly in the equinoctical seasons and easterly wind in the stratosphere in the summer. We denote each profile as UI, UII, and UIII. As a forcing function for planetary waves, we assume only vertical $p$-velocity caused by surface topography and assume its amplitude to be $10^4$ mb sec$^{-1}$ and $Q$ is 0. Other parameters used here are: $f=10^{-4}$ sec, $\beta=1.6\times10^{11}$ m$^{-1}$ sec$^{-1}$.

5. The control solution

At first we shall consider what characters the solutions are to have according to the theory by Charney and Drazin (1961). Assuming $\phi(p,t)=\sqrt{S\Psi(p)e^{i\omega t}}$, and neglecting the dissipation and heating terms, we have from eq. (5)

$$\frac{\partial^2 \Psi}{\partial p^2} + n^2 \Psi = 0$$

where

$$n^2 = \frac{S}{f^2} \left( \beta - f^2 \frac{\partial}{\partial p} \left( \frac{1}{S} \frac{\partial U}{\partial p} \right) \right) \left( k^2 + \pi^2 \frac{D^2}{S^2} \right)$$

If $n^2$ is positive, waves propagate vertically, i.e., sinusoidal solutions are possible while if $n^2$ is negative, waves are evanescent, i.e., only exponentially damping solutions result. Then the condition of vertical propagation is given as follows:

$$0 < U - C < U_r = \frac{\beta - f^2 \frac{\partial}{\partial p} \left( \frac{1}{S} \frac{\partial U}{\partial p} \right)}{k^2 + \pi^2 \frac{D^2}{S^2} + f^2 \frac{\partial^2}{\partial p^2} \left( \frac{1}{S} \frac{1}{\sqrt{S}} \right)}$$

At the level $z=z_r$ where $U - C = U_r$, waves are reflected while at another critical level $z=z_c$ where $U - C = 0$, waves are absorbed. Another feature of vertical propagation is the tilt of the wave axis. The wave axis is tilted westward with increasing height when the wave is upward propagating and eastward for downward propagation. When the upward propagating waves are reflected back toward the lower, the upward and downward propagating waves interfere with each other and sometimes a node is formed where the wave phase shifts $180^\circ$. As can be seen from eqs. (9) and (10), these properties are determined by a
mean zonal wind $U$, wavelength $L$, wave width $D$ and phase velocity $C$.

Since we are interested in the behaviours of stationary planetary waves in low resolution models, we need a control solution which is the most precise approximation to analytical solution. We obtain such solutions with a finite-difference model with 356 layers in the vertical and $\Delta z$ is 250 m. Its top is placed at 90 km and by using artificially large horizontal eddy viscosity above 40 km, upward propagating waves are dissipated there and therefore unnatural reflection at the top should be eliminated (See Fig. 3 and Fig. 5).

Here we shall examine the structure of the waves thus obtained as shown in Figs. 6 and 7.

For the case UI-2007 ($L=20,000$ km, $D=7,000$ km, the strong westerly basic flow), the wave is reflected at about 45 km where $U$ equals to $U_r$ and makes a node at about 15 km. Such a feature is also found from the analysis of stationary planetary waves in the winter (See Sato, 1974, Fig. 1).

For the case UIII-2007 ($L=20,000$ km, $D=7,000$ km, the weak westerly basic flow), the amplitude of the wave increases exponentially with increasing height. This means that at all heights the mean zonal wind is lower than $U_r$ and hence the wave can propagate upward without reflection. The increase of the amplitude is due to the decrease of the mean density with increasing height. Above 60 km, the wave decays owing to the effect of large eddy viscosity. Unnatural reflection at the top looks to be so small to have no influence in the lower layers (See Fig. 15 (b)).

For the case UIII-2007 ($L=20,000$ km, $D=7,000$ km, the easterly basic flow), the wave is absorbed at the critical level about 20 km where $U$ equals to 0 and no disturbances penetrate above the level.

For the case UI-1007 ($L=20,000$ km, $D=7,000$ km), the wave is reflected at the lower level, about 27 km and the amplitude of the wave becomes very small. For the case UI-2005 ($L=20,000$ km, $D=5,000$ km), the wave is reflected at about 32 km. In both cases, the node as appeared at 15 km in UI-2007 vanishes and the wave is confined to the lower layer. From these results, it is supposed that the control solutions may be very good approximations to the corresponding exact solutions. We take these solutions as the control in the following discussion. Typical situations of wave propagation are re-
presented by these five cases, which are listed on Table 1.

<table>
<thead>
<tr>
<th>Case</th>
<th>( U_r ) (m/s)</th>
<th>Reflection</th>
<th>Absorption</th>
</tr>
</thead>
<tbody>
<tr>
<td>UI-2007</td>
<td>54</td>
<td>45 km</td>
<td>no</td>
</tr>
<tr>
<td>UII-2007</td>
<td>54</td>
<td>no</td>
<td>weak westerly</td>
</tr>
<tr>
<td>UIII-2007</td>
<td>54</td>
<td>no</td>
<td>20 km</td>
</tr>
<tr>
<td>UI-2005</td>
<td>32</td>
<td>30 km</td>
<td>no</td>
</tr>
<tr>
<td>UI-1007</td>
<td>28</td>
<td>27 km</td>
<td>no</td>
</tr>
</tbody>
</table>

6. The steady state solutions by low resolution models

Next we investigate how many layers are needed to represent the vertical structure of the stationary planetary waves, such as a node, tilt of the axis and absorption. We also discuss how high the top should be placed in order to avoid its unnatural influence on the lower atmosphere. In the following we treat models listed in Fig. 5, and each model is referred to by symbols shown in the bottom.

The models T3, T4 and T18 have constant grid intervals with respect to pressure and their upper boundaries are assumed to be 0 mb, so that they express the stratosphere and the mesosphere with only one or two layers. They are essentially tropospheric models. In the models G7 and G17 the top is placed at 5 mb (35 km). They are designed to model the troposphere and the lower stratosphere, but G7 has only 3 layers in the stratosphere. The lowering of the upper boundary may cause the unnatural reflection. In the models S18, S23 and S33 the top is placed at 90 km and they have a large dissipating layer above 40 km, which is the same as the control solution.

First we shall examine the steady state solutions obtained by imposing topographical forcing as described before. The profiles of disturbance geopotential in the troposphere in various models are shown in Fig. 8 through Fig. 14.

For the case UI-2007, there must be a node at 100 mb (Figs. 8(a) and 8(b)). However, nodal structure at this height is not found in the results of T3 and T4. Since they express the upper atmosphere above about 200 mb with only one layer and the mean wind is smaller than \( U_r \), there, the reflection at the level where \( U \) equals to \( U_r \) does not occur. In the lower troposphere, the wave axis is tilted westward and the wave phase reverses in the upper troposphere in T3 and T4. Namely a node is formed and the result looks similar to the control solution, except that the node is located at the lower troposphere, which is not the right place. This apparent similarity is considered to be due to the artificial reflection at the top (\( p=0 \) mb). We need to be cautious about this point.

In T18 the wave structure is fairly well reproduced including the node, since it has a few layers at about 100 mb. As seen Fig. 8(b), a significant improvement in the wave structure in the troposphere is achieved when the stratosphere is included. This is because, in this particular situation (UII), the wave reflection by a strong westerly occurs in the stratosphere and has an appreciable influence on the troposphere. In G7 the amplitude is twice as large as that of the control solution. The number of layers of G7 is too small to express a complex structure such as a node. This short-coming can be seen in all cases, though G7 expresses the wave pattern in a qualitative sense.

For the case UII-2007, the amplitudes of the waves in the troposphere in the models T3, T4 and T18 differ greatly from that of the control solution (Fig. 9(a)). This is probably caused by the unnatural reflection at the upper boundary.
of the models, because they cannot resolve the damping layer above 40 km which exists in the control solution. The wave axis are tilted westward in T3 and T4 as well as the control solution. But this does not mean that the waves propagate freely upward but that the amplitude of the reflected waves are small compared with that of the upward propagating wave. In G7 and G17, the amplitude of the waves agree well with the control solution below 200 mb (Fig. 9(b)). It seems that, although their tops are located at 35 km, the influence of the unnatural reflection is small in the troposphere. This is because, if a model has several layers in the stratosphere, damping effects such as Newtonian cooling dissipate downward waves which are reflected at the upper boundary. In a model which has only one or two layers in the stratosphere such as T18, the damping does not work effectively.

It should be noted that if there were no such damping effects or they were very much smaller, the upper boundary would have an influence on the troposphere even in G7 and G17. In order to examine this problem several calculations were performed by changing values of various damping parameters. The result are shown in Figs. 10 and
Fig. 9(a) Same as Fig. 8(a), but for the case *UII-2007*.

Fig. 9(b) Same as Fig. 8(b), but for the case *UII-2007*.

Fig. 10 The damping effect for the case *UII-2007*. Amplitudes (right) and positions of ridge (left) of geopotential heights of the steady state in the troposphere, in models, the control solution (solid lines), G17 (dashed lines), G17 with $\tau$ only (dotted lines), and G17 without $\tau$ (chained lines).
Fig. 11 Same as Fig. 12, but in models, control solution (solid lines), $G_{17}$ (dashed lines) and $G_{17}$ with $\tau=40$ day in the stratosphere and $\tau=80$ day in the troposphere (dotted lines).

Fig. 12(a) Same as Fig. 8(a), but for the case $UIII-1007$.

Fig. 12(b) Same as Fig. 8(b), but for the case $UIII-2007$. 
11. From Fig. 10 we can see that among many damping effects Newtonian cooling works most effectively to the planetary scale waves. When we use a damping which is four times as slow as the one chosen in the previous calculations, 40 day in the stratosphere and 80 day in the troposphere, a nodal structure appears clearly in the troposphere on the patterns of both phase and amplitudes even in G17 (See Fig. 11). Thus, the favorable results about G7 and G17 obtained previously would not be obtained if $\tau$ differs by a factor 3 or 4.

In Figs. 12(a) and (b) are shown the results of the case UIII-2007. In T3 and T4, the patterns differ greatly from the control solution as in the former cases. Since the models cannot express the easterly wind in the stratosphere, the waves do not feel the critical level and the unnatural reflection at the top ($p=0$ mb) occurs. In T18 a fairly good result is obtained except that a depression of the amplitude appeared at 100 mb and that the phase changes by about 180 degree there. This erroneous result may be due to the partial reflection of waves, the origin of which may be attributed to sparcity of layers near the critical level at 20 km. Since that level is a singular point as can be seen in eq. (9), grid intervals should be sufficiently small near the critical level. In the models G7, G17 and S23, which express the critical level properly, good approximations are obtained except near 100 mb.

In Fig. 13(a) and (b), the results of the case UI-2005 are shown. In the models T3, T4 and T18 the amplitude is much larger than that of the control solution and the phase shifts as much as 60°. A node at about 400 mb in the control solution shifts upward and becomes unrecognizable in the amplitude distribution. Since the wave

![Fig. 13(a)](image1)

Fig. 13(a) Same as Fig. 8(a), but for the case UI-2005.

![Fig. 13(b)](image2)

Fig. 13(b) Same as Fig. 8(b), but for the case UI-2005.
are reflected at the lower level, the downward propagating wave can reach the lower troposphere more deeply than in the case UI-2007. Therefore, unnatural reflection at the upper level has a much more influence on the lower troposphere. The cause of erroneous enhancement of the waves may be interpreted as follows. In the control solution the wave energy is large in the stratosphere and small in the troposphere. However because the low resolution models have only few layers in the stratosphere, the waves are not properly expressed in the stratosphere. Hence the wave energy is confined in the troposphere and become large there. The same effect can be seen in G7, which has only 3 layers in the stratosphere. When models have enough layers as G17 and S23, the effect is not significant and good results are obtained.

For the case UI-1007, the same effect as in UI-2005 can be seen (See Fig. 14(a) and (b)), but only in the results of T3 and T4. Since the wave energy is very small compared to the case UI-2005 and the wave is trapped below about 27 km, the effect of sparcity of layers in the stratosphere is not so serious as before. Therefore, very good profiles are obtained in the other models.

Next we examine the solutions in the stratosphere. The results are shown in Fig. 15(a), (b) and (c). The model S18, S23 and S33 generally give good results, though for the case UII-2007, the amplitudes near 50 km in S18 and S23 reduces to 60 or 70%. The model G17, the top of which is placed at 35 km, gives very good results for the cases UII-2007 and UIII-2007 (In these figures, we cannot distinguish the pattern in G17 from the control solution). However in the case UIII-2007, the reflection at the top makes nodes at 28 km and 15 km. Therefore, G17 is not adequate for modelling waves in the stratosphere, when the waves can propagate into the mesosphere.

Fig. 14(a) Same as Fig. 8(a), but for the case UI-1007.

Fig. 14(b) Same as Fig. 8(b), but for the case UI-1007.
Fig. 15(a) Same as Fig. 8(a), but in the stratosphere in models, the control solution (solid lines), S33 (dotted lines), S23 (dashed lines), S18 (chained lines) and G17 (doubled lines for the case UI-2007).

Fig. 15(b) Same as Fig. 15(a), but for the case UII-2007.

Fig. 15(c) Same as Fig. 15(a), but for the case UIII-2007.
7. Time integrations with the initial data obtained from the control solution

In the discussion on the steady state of planetary waves, we have shown that the steady state solution obtained by a low resolution model is considerably different from the corresponding control solution. If numerical forecast is made with a low resolution model from the initial data which are obtained from the control solution, what will happen to the waves in the model? In principle, the waves should stay at their initial positions as they are stationary in the control solution. However, since the steady state solution in a low resolution model is different from the one given as the initial data, the waves may move erroneously. Obviously, the real numerical weather prediction should contain errors of the same nature.

The time integrations are performed up to 40 days. The $p$-velocity $\omega_m$ caused by surface topography is fixed during the integration as same as the steady state case. The results in the troposphere are shown in Fig. 16 through Fig. 20. Time changes of the phase difference from the initial ($t=0$ day) and the amplitudes calculated by use of the various models are shown as to several levels in the troposphere. Generally until about 5 day the phase and the amplitude of the waves change greatly and they become stationary after about 20 day. It seems that the waves gradually transform into the models own stationary state which is maintained by $\omega_m$.

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**Fig. 16(a)** Time changes of amplitudes and phases of geopotential heights in the troposphere at the level about 450 mb, from the initial data obtained from the control solution in models, T18 (solid lines), T4 (dashed lines) and T3 (dotted lines) for the case UI-2007. Phases are shown with the difference from the initial.

**Fig. 16(b)** Same as Fig. 16(a), but in models, G17 (solid lines), S23 (dashed lines) and G7 (dotted lines).

**Fig. 17(a)** Same as Fig. 16(a), but for the case UI-2007 near 750 mb.

**Fig. 17(b)** Same as Fig. 16(b), but for the case UI-2007 near 750 mb.
latory noise can be seen together with this trend. This noise arises from false generation of barotropic Rossby waves retrograding with the period of about 5 to 10 days. In all cases, both the amplitude and the phase change significantly in $T_3$ and $T_4$. The amplitude variation is the same order of magnitude as the initial value and the phase shifts as much as $90^\circ$. In particular for the case $UII-2007$ in $T_3$, the phase reverses at 18 days at the level 150 mb. $G_7$ is not so bad as $T_3$, but the amplitude increases twice as much as the initial (Fig. 17(b)). Thus we understand that it is not adequate to use these models for the long-range forecasting i.e., time integration up to one week or longer.

As most of numerical prediction models currently used have 5 or 6 layers at most, there must exist the same problem in them. Since they are used only for one or two day forecasting, the problem has no fatal influence on the forecasting, partly because in such a short period planetary waves do not move so much. In the long-range forecasting, it is important to simulate the behaviour of planetary waves correctly. Therefore, we need a model which has sufficiently high resolution such as the model $G17$ for example. In $G17$ and $S23$ the initial states remain considerably steady with small errors. The errors are less than $15^\circ$ about the phase change and $10 \text{ m}$ about the amplitude, which may be acceptable for a long range forecasting. But the errors in the other low resolution models are much larger than the values.

If we modify the data to fit to the stationary state of a low resolution model, the waves will be stationary and the false retrotressing of the ultra long waves will not occur in the model.

For examining the performance of the models in the stratosphere, the case $UI-2007$ and $UII-2007$ are shown in Fig. 21 through Fig. 24. Apparently the results of $S33$ show features of the steady state very well. $S23$ and $S18$ also give
good results, though not so good as S33. It is mentioned in the former section, G17 is an excellent model except for the case UII-2007. For this case, the amplitude reduces to a half of the initial value and the phase shifts 60°, when time integration is performed (Fig. 23 and Fig. 24).

Fig. 21 Time changes of amplitudes and phases of geopotential heights in the stratosphere at the level about 15 km from the initial data obtained from the control solution, in models, S33 (solid lines), S23 (dashed lines) and S18 (dotted lines). Phases are shown with the difference from the initial and amplitudes are normalized with the initial one.

Fig. 22 Same as Fig. 21, but at about 25 km.

Fig. 23 Same as Fig. 21, but for the case UII-2007, at about 15 km. Chained lines denotes the model G17.
8. Concluding remarks

We examined how the vertical resolution and the upper boundary of a numerical model affect the vertical structure of the stationary planetary waves forced by surface topography using simple one-dimensional equations.

The influence of the upper boundary is negligible in the lower atmosphere if a model has enough number of layers and damping effects such as Newtonian cooling, which act on the wave reflected by the upper boundary, is properly incorporated. If the upper boundary is located at about 35 km and there are several layers in the stratosphere, the influence does not reach the troposphere. But if the values used in this paper \( \tau = 10 \) day is reduced by a factor 3 or 4, the damping effects become smaller and the upper boundary has influence on the troposphere.

The sponge layer may be useful to prevent the unnatural reflection at the top. But it should be placed at the sufficiently high level where no significant reflection of waves of physical reality occurs. For example, if the sponge layer is placed at 100 mb, the wave reflection by the stratospheric jet as appeared in the case UI-2007 can never be expressed properly. Hence it will be necessary for the simulation of planetary waves in the troposphere to place the top in the middle stratosphere even if we use the sponge layer.

For the purpose of simulation of climatological mean state it is permissible to use a model with 5 or 6 layers if one concerns with the results in a qualitative sense. However, it may be necessary that the grid intervals are sufficiently small, as \( \Delta z = 1 \sim 2 \) km in the troposphere and \( \Delta z = 2 \sim 3 \) km in the stratosphere for the simulation of the waves in the troposphere if we attempt to express the complex structures of planetary waves with a fair accuracy.

When time integration is performed with a model which has a few layers starting from the real data, the errors are serious. To reduce the errors to be tolerable for a long-range forecasting, we must use a high resolution model such as mentioned above.

In the real atmosphere or in three dimensional numerical models, planetary waves propagate in the lateral direction as well as in the vertical. So the upward waves may be smaller in the amplitude. It is supposed that this effect may reduce the influence of the reflected waves at the upper boundary, compared with the present results.

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大気数値モデルでプラネタリー波を表現することに関する問題

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プラネタリー波の構造に対する数値モデルの鉛直分解能と上層の境界の影響を、準地衡風、β面近似の簡単な一次元モデルを用いて考察した。鉛直分解能と上層の高さが色々異なったモデルで得られた結果を、格子間隔が250mと非常に細かく、90kmを上端とし、付近に非常に大きな摩擦のある層を二つおいた“Control solution”と比較した。上層が低い影響は、もし中部成層圏に上層をおけば対流圈の波の構造には重大なことではなかった。ニュートン冷却や他の摩擦効果が、現実にある位の強さでも十分に上層での反射波を減衰してしまうためである。

分解能の悪いモデル（5, 6層）で得られる定常プラネタリー波は真の解（control）と見かけでは似ている。しかし、定量的にも正確な解を得るには格子間隔は対流圏で1～2km、成層圏で2～3kmにし、上層は中部成層圏におくべきである。

真の解（control）を初期値として分解能の悪いモデルで時間積分を行なった場合、波はもはや定常ではなくって、モデル自身の定常状態へと変化してしまう。この問題を解決するためには定常の場合と同様な精度の良いモデル使わねばならない。

同様な結果は成層圏でのプラネタリー波の表現についても得られた。