NOTES AND CORRESPONDENCE

Influence of Electrostatic Forces on Riming
by Thin Ice Plates

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Abstract

Numerical investigations of riming by thin ice plates were conducted, including the effects of coulombic forces on collision efficiency. A superposition method was used to determine the trajectories of the ice crystal and aerosol particle. The results indicate that electrostatic forces may be important in the evolution of cloud microstructure because they reduce the critical size to which an ice crystal must grow before it can begin to rime.

In an environment of -10°C , 700 mb, the smallest uncharged ice plate capable of riming lies between 147 and 160 μm half-width. At -18°C , 400 mb, the critical size of the ice plate is reduced to between 103 and 133 μm for the uncharged case, and to less than 103 μ when moderate attractive electric charges are present on the ice plate and water droplet.

1. Introduction

Previous numerical investigations of riming by thin ice plates (Pitter and Pruppacher, 1974; Pitter, 1977a) examined the case of electrically neutral bodies interacting in the atmosphere. Those results found that thin ice plates, idealized as oblate spheroids of axis ratio 0.05 and density 0.92 g cm^3 , needed to reach a critical size (semi-major axis length) between 147 and 160 μm before they were capable of collecting water droplets of any size in an atmosphere of -10°C , 700 mb.

The numerical investigations agreed with field results of Ono (1969) and Wilkins and Auer (1970), both of which indicated that hexagonal plates less than 300 μm width were unrimed, but disagreed with field experiments of Reinking (1975) which reported riming of hex plates down to 150 μm width and branched plates down to 240 μm width. Reinking's data were obtained from Sierra Nevada clouds, which are characteristically more convective than those observed

by Ono or Wilkins and Auer. It was hypothesized that convective clouds might experience greater electrification which, in turn, might affect the critical size at which ice plates are capable of riming droplets. (In all studies, minimum droplet size found on rimed crystals was 10 μm .)

Two theories for determining collision efficiencies were investigated in the present study. Theory for electrostatic deposition from an air-stream was developed by Kraemer and Johnstone (1955), namely

$$E = -4K_e \quad (1)$$

where K_e is a dimensionless electric force number

$$K_e = \frac{C}{6\pi a\mu} \frac{qQ}{4\pi\epsilon_0 A^2 V} \quad (2)$$

Equations (1) and (2) apply only to oppositely charged bodies.

Theory for inertial impaction on ice crystals has been advanced by Slinn (1977). Slinn utilizes the Stokes number s and the critical Stokes number s^* ,

$$s = \frac{2a^2\rho_p(V-v)}{9\mu A} \quad (3)$$

and

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$$s^* = \frac{1.2 + \ln(1 + N_{Re})/12}{1 + \ln(1 + N_{Re})} \tag{4}$$

to determine the collision efficiency when $s > s^*$ by the relation

$$E = \left(\frac{s - s^*}{2/3 + s - s^*} \right)^{3/2} \tag{5}$$

2. Method

The numerical model reported by Pitter (1977b) was applied to riming by thin ice plates. The model is an extension of the method used by Pitter and Pruppacher (1974) and Pitter (1977a) to include coulombic forces of attraction or repulsion between the ice crystal and water droplet. Flow fields around the ice crystal were determined by Pitter *et al.* (1973) for several Reynolds numbers. Those Reynolds numbers, and the atmospheric conditions investigated (-18°C , 400 mb) determined the ice crystal size and terminal velocity. Table 1 presents the ice crystal parameters used in the study.

Flow fields around the droplet were determined by LeClair *et al.* (1969) for several Reynolds numbers. In the present study, only those fields for $N_{Re} = (0.01, 0.1, 1)$ were utilized. Drops with $N_{Re} < 0.031$ ($a < 11.7\text{ }\mu\text{m}$) were assigned the pattern associated with the flow field for $N_{Re} = 0.01$. Drops with $N_{Re} > 0.31$ ($a > 25.3\text{ }\mu\text{m}$) were assigned the pattern for $N_{Re} = 1$. Drops of intermediate size were assigned the pattern for $N_{Re} = 0.1$. The continuity of collision efficiency curves through these drop sizes is evidence that the approximation of drop flow field pattern does not cause serious error in results.

Electric charges on ice crystals and droplets were prescribed in the manner of Pitter (1977b) and Grover and Beard (1975). The charges, in coulombs, were specified for ice plates as

$$Q = \frac{2}{3} \times 10^{-9} A^2 \tag{6}$$

when A , the ice crystal half-width, is given in

Table 1. Ice crystal characteristics: -18°C , 400 mb. Numbers in parentheses indicate powers of 10.

Reynolds Number	Half-Width (μm)	Terminal Velocity (cm s^{-1})	Electric Charge (C)
1	103	9.9	7.0 (−14)
2	133	15.2	1.2 (−13)
5	193	26.3	2.5 (−13)
10	262	38.7	4.6 (−13)
20	366	55.4	8.9 (−13)

centimeters. For droplets, the charges were specified as

$$q = -\frac{2}{3} \times 10^{-9} a^2 \tag{7}$$

with the sign convention to allow for attractive electrostatic interactions. The charges on water droplets agreed with experimental data compiled by Takahashi (1973) for convective clouds. The charges on ice plates varied from 0.7 to 9×10^{-13} coulombs, which is comparable to the mean charges experimentally measured by Magono and Orikasa (1966) of 4 to 8×10^{-13} coulombs and by Burrows and Hobbs (1970) of 5 to 10×10^{-13} coulombs. Thus, the values are believed to be representative of charges on convective cloud hydrometeors. It is expected that charges on non-convective cloud hydrometeors would be significantly lower than those presented in this work.

Several ice crystal charging mechanisms have been identified. Some, such as diffusion growth in a bipolar ion atmosphere (Takahashi, 1970), do not require hydrometeor collisions to transfer charges. The precise nature of ice crystal charging in clouds remains a topic of research.

3. Results

Fig. 1 presents the results in terms of collision

Fig. 1 Efficiencies with which thin ice plates collect droplets. Solid lines denote present numerical results. Numbers identify ice crystal Reynolds numbers (see Table 1). a —attractive electrostatic forces; n —neutral interaction; t —Kraemer and Johnston electrostatic deposition theory.

efficiency of droplets with ice crystals of various Reynolds numbers investigated. Table 1 relates ice crystals Reynolds numbers to other parameters. The dotted lines labeled t identify results for Kraemer and Johnstone's electrostatic deposition theory, from Equation (1), utilizing the parameters identical to attractive electrostatic forces cases for the numerical study, denoted a . The agreement between the t and a lines indicate that the theoretical results may be used to estimate the riming efficiency of ice crystals when the bodies are oppositely charged. The lines labeled n were numerically determined with no electrostatic charges.

When $N_{RE} \geq 5$, the a lines merge with the n lines at drop radii slightly greater than the small cutoff size for the n lines. These results indicate that the expected magnitude of electrostatic (coulombic) forces in convective clouds does not significantly affect collision efficiency when inertial impaction is the dominant mechanism and larger ice crystals are riming.

For $N_{RE} = 2$, the a lines does not merge with the n line, indicating that, for $133 \mu\text{m}$ half-width ice crystals, the inertial forces are not so dominant as to effectively negate the effects of mutual electrostatic attraction. For the case of $N_{RE} = 1$,

only attractive electrostatic forces yielded nonzero collision efficiencies. This corresponds to ice crystals of $103 \mu\text{m}$ half-width, and indicates that inertial forces are insufficient for any collisions to occur with this size ice crystal. Thus, the electrical nature of the cloud can affect the critical ice crystal size necessary for riming to occur.

Fig. 2 presents the results for $N_{RE} = 5$. Included in the figure is the collision efficiency for inertial impaction theory, from Equation (5). The present results for neutral interactions follow the behavior of inertial impaction theory, although they lie slightly below the values predicted by theory. A possible weak point in the theory is the use of Equation (4) to determine the critical Stokes number. Equation (4) estimates a slightly lower critical Stokes number than the present results predict. In view of Slinn's comments, "...the suggestions are made not for the modelers to use but for the researchers to test," the present results have performed one such test, and verified the general applicability of his theory.

The general agreement between the a and t lines indicates that the sensitivity of the charge magnitude on the collision efficiency is approximately given by

$$E \sim -qQ \quad (8)$$

for attractively charged bodies when all other parameters are held constant and $s < s^*$.

Previous results of Pitter (1977a) found that, in an environment of -10°C , 700 mb, the smallest neutral ice crystals capable of riming with droplets was between 147 and $160 \mu\text{m}$ half-width (N_{RE} was between 2 and 2.5). Utilizing the same numerical model, the present results indicate that at -18°C , 400 mb, the critical ice crystal size is between 103 and $133 \mu\text{m}$ (N_{RE} between 1 and 2) for neutral interactions, and less than $103 \mu\text{m}$ when attractive electrostatic forces are considered. These results indicate that the critical ice crystal size for riming to commence is dependent on the electrical nature of the cloud and upon location within the cloud. Electrically active clouds and higher, colder regions of clouds allow riming to occur on smaller ice crystals, thereby shortening their deposition growth times.

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Fig. 2 Efficiencies with which $193 \mu\text{m}$ half-width ice plates collect droplets. Solid lines denote present results: x —attractive electrostatic forces; $\bullet$ —neutral interaction. Dashed line—Kraemer and Johnston electrostatic deposition theory. Dotted line—Slinn inertial impaction theory.

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Appendix

A	Ice crystal semi-major axis length
a	Droplet radius
C	Cunningham slip coefficient
E	Collision efficiency
K_e	Electric force number
N_{RE}	Ice crystal Reynolds number
N_{Re}	Droplet Reynolds number
Q	Ice crystal charge
q	Droplet charge
s	Stokes number
s^*	Critical Stokes number
V	Ice crystal velocity
v	Droplet velocity
μ	Dynamic viscosity of air
ρ_p	Droplet density

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薄い氷板による riming の静電気力の影響

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