Dynamics of the Four-Day Circulation in the Venus Atmosphere

By Y. Matsuda

*Division of Astronomy & Earth Sciences, Tokyo Gakuai University
(Manuscript received 4 July 1980, in revised form 15 October 1980)*

Abstract

In order to make clear the problem of the four-day circulation, we construct a simple axisymmetric model. This model contains the mechanism first proposed by Gierasch (1975), *i.e.*, upward transports of angular momentum by a meridional circulation with the aid of very large horizontal viscosity which dissipates differential rotation. Further, a suppressing effect on this mechanism due to finiteness of the horizontal eddy viscosity is also involved. The velocity and the temperature field are represented by a few fundamental modes. Terms expressing the nonlinear interactions among the modes are explicitly written in the mode equations. Stationary solutions of this system are obtained mainly by a two-layer model, for both an infinite and a finite horizontal eddy viscosity.

First, we determine magnitude of the mean zonal flow (U) as a function of the meridional circulation (V) from angular momentum balance. In the case of an infinite horizontal viscosity, U is simply proportional to V . Its ratio (U/V) is given by the inverse ratio of period of planetary rotation (τ_Ω) to the time constant of vertical diffusion (τ_v) (*i.e.*, $U/V \approx \tau_\Omega/\tau_v$). In the case of a finite horizontal viscosity, U has a maximum value for a certain value of V . Its maximum value is determined by the ratio of horizontal viscosity to vertical one as well as τ_Ω .

Next, associating this U - V relation with the vorticity equation in the zonal direction, we classify types of solutions according to the effect which dominates and balances solenoidal term in the vorticity equation. The types of solution are as follows.

Thermal wind balance of the Venus type: The vertical gradient of centrifugal force due to atmospheric rotation dominates.

Thermal wind balance of the earth type: The vertical gradient of centrifugal force due to atmospheric rotation coupled with planetary rotation dominates.

Direct cell balance: The frictional force associated with the meridional circulation dominates.

Kinds of balance are determined on a two-dimensional parameter space of τ_Ω/τ_v and the latitudinal differential heating denoted by G_r . In an infinite horizontal viscosity case, thermal wind balance of the Venus type appears in the whole range of large G_r . In the case of finite viscosity, solutions of this balance can exist only in a more restricted domain in the τ_Ω/τ_v - G_r diagram. G_r of this domain has an upper limit depending τ_Ω/τ_v , and only a direct cell balance can correspond to a G_r value beyond the upper limit. In a portion of the domain where thermal wind balance of the Venus type is realized, solution of direct cell balance is also obtained as a stable solution. Thus for this parameter range, two utterly different states, a fast zonal motion accompanied by a slow meridional circulation and a strong meridional circulation associated with a slight zonal motion are possible as stable stationary states for the same differential heating. The former corresponds to the four-day circulation, while the latter means a direct cell between day side and night side in actual situation.

Results of the numerical experiments by Young & Pollack (1977) are discussed in the light of the present results.

1. Introduction

The four-day circulation of the Venus upper atmosphere is one of the prominent phenomena

which are not yet clearly understood in the field of dynamic meteorology in spite of a number of excellent studies.

Venus rotates so slowly (the period of the

rotation is 243 days) that a simple axisymmetric circulation about the subsolar-antisolar point line is expected to take place in that atmosphere. However, fast cloud motion indicating fast mean zonal motion was discovered by Boyer and Camichel (1961) and then a number of measurements have confirmed the existence of circulation which is quite different from the naïvely expected simple one. Namely, fast mean zonal motion, which is now known as the four-day circulation (*e.g.* Keldysh, 1977; Boyer *et al.*, 1978) is a main feature of the upper atmosphere. It is a well established fact that the retrograde rotation period in the equatorial region is about four days. Further, observations by Mariner 10 indicate that the angular velocity increases with latitude although angular momentum diminishes somewhat with latitude, and the rotation period at 50° latitude could be as short as two days. (see Murray *et al.* 1974)

The problem of the four-day circulation can be stated as follows;

1) How is the fast rotation generated and maintained?

2) What is the condition under which such an unexpected phenomenon necessarily takes place? In other words, what are the essential parameters which determine the system and what is the parameter range in which motion of this kind is selected?

Specifically, we raise following questions concerning effects of the planetary rotation.

3) Is the rotation of the planet essential to generation of the four-day circulation? If it is so, why can the slow rotation yield such a high rotation rate of the upper atmosphere?

4) If the rotation of the planet is not essential, a non-linear instability is thought to be most likely as the generating mechanism. In this case, are there multiple stationary solutions in the system describing the Venus upper atmosphere? If this is the case, it is natural to suppose that one of them is axisymmetric convection about the subsolar-antisolar point line (with slight modification due to the slow rotation of the planet) and the other is the one corresponding to the actual circulation of the Venus atmosphere. Are there stationary solutions except these? If so, what is the nature of the solutions? The purpose of the present study is to answer theoretically these questions.

So far many attempts have been made to answer these questions (*e.g.* Schubert and Whitehead, 1967; Thompson, 1970; Young and Schu-

bert, 1973; Fels and Lindzen, 1974; Gierasch, 1975; Kálnay de Rivas, 1975; Young and Pollack, 1977). Among them, the recent numerical study by Young & Pollack (1977) is the most remarkable one for its success of numerical simulation of the four-day circulation. According to the results of their calculations, the meridional circulation is the principal means by which zonal momentum is transported upward. Hence one must note the importance of the meridional circulation as a generating mechanism of the four-day circulation. However, it seems that their further interpretation of the results is somewhat obscure. They emphasized importance of the instability of the convection between subsolar and antisolar point, which had been proposed by Thompson (1970). From their explanation, however, we can not understand, at least clearly, the relation between this instability and the meridional circulation in generating the fast rotation. From figures presented in their article the acceleration of the mean zonal winds of upper levels seems to be mainly due to upward transport of angular momentum by the meridional circulation. In this case, why can tilting of the convection cell be so important for the generation of the fast mean zonal winds? Further it seems questionable that the upward transport of angular momentum by the meridional circulation is cooperative with the acceleration of the mean zonal flow by the tilted convection cell because the latter mechanism decelerates the lower layer. Accordingly their explanation seems insufficient even though they succeeded in simulation of the phenomenon.

This somewhat peculiar situation is considered to arise from the lack of a solid theoretical basis on which the complex numerical results could be analysed. Consequently, a theoretical study is obviously desirable at this stage for clear understanding of the generating mechanism of the four-day circulation. The present article is the result of such an effort.

In order to get a basic idea for developing our theory, it is advantageous to examine theoretical studies which have been proposed up to the present day. Most of the theoretical attempts thus far made are based on the following idea; since primary flow expected to occur in the Venusian atmosphere is a convection cell induced by differential heating between day side and night side it is most natural to consider that the mean zonal flow is produced by this convection in some way such as tilting of the axis of the convection

cell. In fact tilt of the axis of a convection cell causes non-vanishing Reynolds stress uw , and this Reynolds stress can yield the mean zonal flow $\bar{u}$. Concerning cause of the tilt of the axis, two postulates have so far been proposed. One is the so called "moving flame" mechanism which ascribes the tilt to the movement of the sun with respect to the Venus atmosphere and, the other is a mechanism which invokes an instability of the convection cell. The former was first proposed by Schubert and Whitehead (1967), and the latter by Thompson (1970). Many calculations along these lines have been performed using two dimensional models. Even now, however, it is still uncertain whether either of these mechanisms can actually produce mean zonal flow with comparable magnitude to that observed (For example, see a review by Stone, 1975). Especially, it is of importance to note that any mechanism of this kind for producing the strong mean zonal winds involves an inherent difficulty. Though it may be certainly possible that a mean zonal flow is induced to some extent by such a mechanism, it may be difficult for a very strong mean zonal flow to be maintained by the same mechanism. The reason for thinking so is as follows. The convection between day and night side is caused by differential heating between day side and night side. But if a strong mean zonal flow is induced, the flow transports heat and has a tendency to diminish the energy source for generating the convection. This means, in turn, that the strong mean zonal flow itself must fade out, for it is maintained by that convection. In short this mechanism has a difficulty that the mean zonal flow must be maintained by the convection which it will destroy. It is natural to consider that in the equilibrium state, at most the two circulations are able to have comparable magnitude. The generation of the strong zonal winds by this mechanism may not be completely impossible. But it is not likely to be the case. In fact, we can hardly find direct evidences that this mechanism is working in the numerical experiments of Young & Pollack. Moreover, explanations based on a mechanism of this kind involve a difficulty concerning the direction of the mean zonal flow. A simple "moving flame" mechanism produces the same direction of mean zonal flow as the motion of the sun, namely, the reverse direction to that observed, if the fluid layer is heated from above. In view of this situation, Young & Schubert (1973) have introduced the effect of stratification of the atmosphere into the

"moving flame" mechanism. Thus, explanation by the "moving flame" mechanism becomes similar to that invoking forced internal gravity waves. While the implications for Venus of the study by Young & Schubert (1973) are not clear (see the review by Stone (1975)), studies based on momentum transport by internal gravity waves made by Fels & Lindzen (1974) and Plumb (1975) may be relevant to Venus. Certainly, excitation of the mean zonal flow by waves may be effective if waves transport momentum from a lower dense layer to an upper rare layer and there accelerate fluids of the upper layer. Such an example is propagation from the troposphere to the stratosphere in the earth. However, in our problem the mean zonal flow must be accelerated by gravity waves in the layer where the waves are excited by heating. Accordingly, if the retrograde mean zonal flow is excited by gravity waves, at the same time a prograde mean zonal flow must be excited on both sides of that layer. Then the magnitude of the prograde mean zonal flow on the upper side of the layer would be larger than that in the layer heated, since air density decreases with altitude. However, this stronger prograde mean zonal flow has not been observed up to the present day at all. The generation of this counterflow is a difficulty of the explanation by gravity wave excitations. Fels and Lindzen (1974) have taken critical layer absorption into consideration and have reached a negative conclusion to the explanation by gravity wave forcing. This negative conclusion results fundamentally from the existence of the counter flows. The study by Plumb (1975) seems to be free from this difficulty. But, this is due to such an artificial upper boundary condition that vertical velocity of the disturbance is assumed to vanish there. With this condition necessity for existence of the counterflow is removed. Therefore, we can not regard this study as conclusive. After all, all studies based on gravity waves have not succeeded in an explanation of those observed.

Consequently it seems to be most promising to explain the four-day circulation based on some mechanism which invokes not the effects of diurnal heating but those of latitudinal heating. Apparently the latter can remain unaffected by a strong mean zonal flow. So far studies based on this idea have been made by Leovy (1973) and Gierasch (1975). Leovy (1973) was the first to point out that the Venus upper atmosphere is in a state of thermal wind balance or "cyclotrophic balance" where the vertical gradient of

centrifugal force due to a vertical shear of the zonal winds is balanced by meridional temperature gradient. However, he did not show how such a balance is realized and maintained. After this primary idea was proposed, Gierasch (1975) has shown that the strong mean zonal winds could be maintained by the upward transport of the angular momentum due to the meridional circulation with upward motion at the equatorial latitudes and downward motion at higher latitudes, provided that the horizontal eddy diffusion is large enough to equalize the angular velocity instantaneously. Though his results are essentially correct, his study seems insufficient in the following sense. In the first place, while he showed *diagnostically* that the fast atmospheric rotation is possible state of atmospheric motion, he did not show *prognostically* that this state must appear. In other words, he did not answer the question whether the fast zonal motion is a unique consequence of the given conditions or not. In the second place, the magnitude of the meridional circulation is directly given by meridional differential heating without considering the energy balance which is necessary to maintain the temperature field in thermal wind balance. However, the magnitude of the meridional circulation must be determined together with the magnitude of the mean zonal flow as a solution of the governing equations, so that it is desirable to incorporate effects of temperature field in the equations of heat balance and meridional circulation. In the third place, an infinite horizontal diffusion is assumed in his study. As a result, effect of differential rotation of the atmosphere can not be taken into consideration, in spite of the fact that this plays an important role in this problem, as will be shown later. In the present work we shall investigate the generation of fast rotation by angular momentum transport by the meridional circulation without these restrictions. In this sense our study can be regarded as a generalization and extension of Gierasch (1975).

It may be necessary to mention the results of numerical experiments performed by Kálnay de Rivas (1975), because they might be taken as a negative proof of the idea proposed by Gierasch. Certainly her calculations seem to have shown that mean zonal winds are dissipated by the large horizontal diffusion coefficient which is required for the present mechanism to work. However, we can infer that this dissipation results from irrelevant formulation of the horizontal diffusion term used there rather than largeness of the

coefficient itself. Namely, judging from the equation employed in these calculations, the horizontal diffusion term is designed to mix not angular velocity but vorticity (see Kálnay de Rivas, 1973). Apart from the problem of whether this assumption can be justified from a physical view point, this assumption is in one sense favourable for the Gierasch mechanism because the assumption implies a tendency to accelerate the zonal winds in the equatorial region at the expense of higher latitudes. But the diffusion term thus formulated failed to conserve total angular momentum which must be rigorously conservative. As a result, we may suppose that the angular momentum accumulated in the equatorial upper layer by this particular diffusion scheme will be dissipated out by the same scheme. We can find just this phenomenon in the result of the numerical experiments by Kálnay de Rivas. Therefore, an argument against the present idea deduced from the calculations by Kálnay de Rivas is not valid.^(*)

2. The model and basic equations

As stated in the introduction, we attempt to understand the cause of the four-day circulation by considering effects of the differential heating in the latitudinal direction. For this purpose, we construct our axisymmetric model as follows; At first, we assume a Boussinesq fluid. Next, we shall assume symmetry about the equatorial plane together with axisymmetry about the rotation axis of the planet. (For applicability of assumption of Boussinesq fluid and axisymmetry, see below) Since the velocity field is a solenoidal field because of Boussinesq fluid assumption, it can be described in terms of a toroidal vector ($\mathbf{T}$) and a

(*) After the completion of a final form of the present article the author noticed results of observations made by Pioneer Venus orbiter mission (see Science, Vol. 205, 6 July 1979, etc.) Among informations provided by Pioneer Venus, the analysis of clouds motion by Rossow *et al.* (to be published) is most informative for our present study. According to it, the midlatitude jet observed by Mariner 10 is not present, and solid body rotation of zonal winds is suggested. Further, it is also confirmed that meridional velocities are poleward in both hemispheres with speeds of several meters per second (in agreement with Mariner 10 results). In view of these observational results, Rossow *et al.* suggest that the meridional circulation (and eddy processes) are producing the four-day circulation.

poloidal vector ($\mathbf{S}$). Namely,
where

$$\mathbf{V} = \mathbf{T} + \mathbf{S}$$

$$\text{where } \mathbf{T} = \text{rot}\left(\frac{\mathbf{r}}{r} \Phi\right), \quad \mathbf{S} = \text{rot rot}\left(\frac{\mathbf{r}}{r} \Psi\right)$$

(Φ and Ψ are scalar functions.)

Next, we represent Φ , Ψ and the temperature field (Θ) by sums of spherical harmonic functions. (see Appendix A) From the symmetry assumed above, these expansions may be restricted;

$$\Phi = \sum_{l=1}^{\infty} T_{2l-1}^0(r) Y_{2l-1}^0(\theta, \varphi)$$

$$\Psi = \sum_{l=1}^{\infty} S_{2l}^0(r) Y_{2l}^0(\theta, \varphi)$$

$$\Theta = \sum_{l=0}^{\infty} \Theta_{2l}^0(r) Y_{2l}^0(\theta, \varphi)$$

For our theoretical study, it is suitable to treat only a few fundamental modes and examine their interactions. We employ the modes whose degree is smaller than three. Namely, we use a mode representing rigid rotation (T_1^0);

$$T_r=0, \quad T_\theta=0, \quad T_\varphi = -\frac{1}{R} \frac{\partial P_1^0(\cos \theta)}{\partial \theta} T_1^0(r)$$

a mode representing differential rotation (T_3^0);

$$T_r=0, \quad T_\theta=0, \quad T_\varphi = -\frac{1}{R} \frac{\partial P_3^0(\cos \theta)}{\partial \theta} T_3^0(r)$$

a mode representing meridional circulation (S_2^0);

$$S_r = \frac{2 \cdot 3 P_2^0(\cos \theta)}{R^2} S_2^0(r),$$

$$S_\theta = \frac{1}{R} \frac{\partial P_2^0(\cos \theta)}{\partial \theta} \frac{\partial S_2^0(r)}{\partial r}, \quad S_\varphi = 0$$

a mode representing average vertical temperature gradient;

$$\Theta_0^0 = \Theta_0^0(r) Y_0^0(\theta, \varphi) = \Theta_0^0(r)$$

and a mode representing meridional temperature contrast;

$$\Theta_2^0 = \Theta_2^0(r) Y_2^0(\theta, \varphi) = P_2^0(\cos \theta) \Theta_2^0(r)$$

For the purpose of our study, it is most likely to obtain explicit representations of nonlinear interactions of these modes which come from advection terms of Navier-Stokes equation. Procedures of the calculations of the nonlinear terms are given in Appendix A. According to these results, we obtain the following mode equations as the basic equations for our investigation.

$$\frac{\partial T_1^0}{\partial t} = \frac{6}{5} \frac{1}{R^2} \frac{\partial [S_2^0 \{T_1^0 + R^2 \Omega - (18/7) T_3^0\}]}{\partial z} + \nu \frac{\partial^2 T_1^0}{\partial z^2} \quad (2.1)$$

$$\frac{\partial T_3^0}{\partial t} = \frac{2}{5} \frac{1}{R^2} \left\{ -3 \frac{\partial [S_2^0 (T_1^0 + T_3^0 + R^2 \Omega)]}{\partial z} + 5 \frac{\partial S_2^0}{\partial z} \cdot (T_1^0 + T_3^0 + R^2 \Omega) \right\} + \nu \frac{\partial^2 T_3^0}{\partial z^2} - \frac{\nu_H}{R^2} T_3^0 \quad (2.2)$$

$$\frac{\partial S_2^{0*}}{\partial t} = \frac{2}{3} \frac{1}{R^2} (T_1^0 + R^2 \Omega) \frac{\partial (T_1^0 + R^2 \Omega)}{\partial z} + \frac{8}{7} \frac{1}{R^2} \frac{\partial (T_1^0 T_3^0)}{\partial z} + 4 \frac{T_3^0}{R^2} \frac{\partial T_3^0}{\partial z} + \nu \frac{\partial^2 S_2^{0*}}{\partial z^2} - \frac{\nu_H'}{R^2} S_2^{0*} + \alpha g \Theta_2^0 \quad (2.3)$$

$$\frac{\partial \Theta_0^0}{\partial t} = -\frac{6}{5} \frac{1}{R^2} \frac{\partial (S_2^0 \Theta_2^0)}{\partial z} - c_0 (\Theta_0^0 - \bar{\Theta}_0^0) \quad (2.4)$$

$$\frac{\partial \Theta_2^0}{\partial t} = -6 \frac{1}{R^2} \frac{\partial \Theta_0^0}{\partial z} S_2^0 - \frac{12}{7} \frac{1}{R^2} S_2^0 \frac{\partial \Theta_2^0}{\partial z} - \frac{6}{7} \frac{1}{R^2} \frac{\partial S_2^0}{\partial z} \Theta_2^0 - \tilde{c} \Theta_2^0 - \frac{\kappa_H'}{R^2} \Theta_2^0 + \kappa \frac{\partial^2 \Theta_2^0}{\partial z^2} + Q_2^0 \quad (2.5)$$

where

$$S_2^{0*} = -\frac{\partial^2 S_2^0(z)}{\partial z^2}$$

Q_2^0 is an external latitudinal differential heating corresponding to Θ_2^0 -temperature field, Ω is angular velocity of the planetary rotation, R is a radius of the planet, $\tilde{c}$ is a coefficient of Newtonian cooling, κ is a vertical eddy heat diffusion coefficient, ν is a vertical eddy diffusion coefficient, κ_H' is a horizontal eddy thermal diffusion coefficient and ν_H , ν_H' are horizontal eddy diffusion coefficients. We define c as $c = \tilde{c} + \kappa_H'/R^2$ for simplicity. $\bar{\Theta}_0^0$ in (2.4) is a mean equilibrium temperature field which is determined by radiative process or small scale convections. $1/c_0$ is a relaxation time by which the mean temperature field perturbed by other causes reaches the equilibrium state. (2.4) shows that a mean temperature field, namely, stratification of the atmosphere is influenced by the meridional circulation (S_2^0) coupled with meridional temperature contrast (Θ_2^0). However, since the phenomenon confronting us takes place in the stratosphere of Venus and a stable stratification is expected to

be predominant, a mean temperature field must be determined mainly by radiative process. Hence, we could neglect the first term in the right hand side of (2.4) in comparison with the second term. Thus we find $\Theta_0^0 = \bar{\Theta}_0^0$ and as a result, the Θ_0^0 mode will be separated out from the above associated equations. Next, we put

$$6 \frac{\partial \Theta_0^0}{\partial z} = 6 \frac{\partial \bar{\Theta}_0^0}{\partial z} \equiv \Gamma$$

and regard stratification, Γ as constant for the sake of simplicity. Further, we neglect the second and the third term in the right hand side of (2.5) because heat transport owing to the assumed strong stratification (the first term) is considered to be predominant over the two terms. Thus (2.5) is reduced to

$$\frac{\partial \Theta_2^0}{\partial t} = - \frac{\Gamma}{R^2} S_2^0 - c \Theta_2^0 + \kappa \frac{\partial^2 \Theta_2^0}{\partial z^2} + Q_2^0 \quad (2.6)$$

Evidently, we should impose "stress free" as the upper boundary condition of our model. However, as for the lower boundary condition some explanation may be needed. The fluid layer to be discussed in the present study is, of course, the layer of the Venus upper atmosphere where solar energy is absorbed by the cloud and the four-day circulation is observed. Since we could not extend our model to the Venus surface, the lower boundary of our model is far distant from the surface. Notwithstanding, we cannot apply "stress free" to the lower boundary of our model. In fact, if "stress free" were assumed, an absolute value of rigid rotation (T_1^0) could not be determined at all. Actually, the "rigid" condition at the Venus surface makes atmospheric rigid rotation at the lower boundary of our model vanish by the mediation of internal viscosity of the fluid between the Venus surface and the lower boundary of our model, if there is no mechanism to produce the atmospheric rotation in the fluid layer.*

Because we assume an ordinary positive eddy diffusion, the rigid rotation is not influenced by eddy diffusion at all. We write $\nu_H = \nu_H^* + \bar{\nu}_H$, $\nu_H' = \bar{\nu}_H$ where $\bar{\nu}_H$ means usual eddy diffusion coefficient, while ν_H^* is macroscopic eddy diffusion associated with T_3^0 and acts on only T_3^0 . One may doubt the validity of the assumption

of positive viscosity, because negative viscosity is most familiar phenomenon in the earth. However, negative viscosity is nothing but a result of behaviours of geostrophic motion, which is two-dimensional due to very fast rotation of the planet earth. Hence, we have no reason for applying the idea of negative viscosity to the Venus atmosphere where motion of atmosphere must not be geostrophic. Thus, it is rather natural to assume positive viscosity for the Venus atmosphere. Further, we have assumed that ν_H^* acts only on T_3^0 mode. This idea is based on the following facts. A pattern of horizontal motion of S_2^0 is essentially different from that of T_3^0 . The flow pattern of T_3^0 is a parallel flow having shear whose stability has been often discussed in the context of the stability theory of fluid dynamics. Hence, it may be allowable to expect the occurrence of its instability. Then, T_3^0 mode is diminished by the Reynolds stress $u'v'$ (u' : eddy eastward velocity, v' : eddy northward velocity, bar means zonal average) due to the macro-eddy caused by the instability. But, horizontal motion of S_2^0 can not be affected by this Reynolds stress at all. Namely, influence of macro-eddy is considerably different for the different modes. So, it may be rather natural to assume selective eddy viscosity ν_H^* . Here, we can not verify the necessity of the assumed selective macro-eddy viscosity, but we think that this assumption is probable enough to be worth examining its consequences.* Hence, we attempt to explain the four-day circulation based on this assumption. κ_H means an ordinary horizontal eddy thermal diffusion coefficient. Thus in the present study we shall investigate the following three cases.

$$(I) \quad \nu_H^* = \infty, \quad \bar{\nu}_H = 0$$

$$(II) \quad 0 < \nu_H^* \leq \infty, \quad \bar{\nu}_H = 0$$

$$(III) \quad \nu_H^* = 0, \quad \bar{\nu}_H \neq 0$$

In the first and the second case, we assume selective eddy diffusion coefficients which parameterize large scale processes. The second case includes the first case as a limiting one. But the first case is easy to treat, so that in the first place we shall inquire into this case thoroughly, apart from the second case. In the third case we assume no particular large scale process which results in a difference in magnitude of the eddy diffusion coefficient acting on the different modes.

* If the rigid atmospheric rotation (Q_1) is generated by some cause in that fluid layer, we ought to interpret Q involved in (2.1)~(2.3) as $Q + Q_1$, for applying our model to this situation.

* Rossow and Williams (1979) shows that this ν_H^* can result from barotropic vorticity mixing.

3. Infinite macro-eddy viscosity

The aim of the present section is to consider the case (I). In this case, we can neglect T_3^0 mode because the infinite diffusion acting on this mode dissipates it immediately. Since we are concerned with the stationary state, we assume: $\partial/\partial t \equiv 0$ throughout this article. Thus, we obtain the simplified equations as follows.

$$\frac{\partial T_1^0}{\partial t} = \frac{6}{5} \frac{1}{R^2} \frac{\partial S_2^0 (T_1^0 + R^2 \Omega)}{\partial z} + \nu \frac{\partial^2 T_1^0}{\partial z^2} = 0 \quad (3.1)$$

$$\begin{aligned} \frac{\partial S_2^{0*}}{\partial t} &= \frac{2}{3} \frac{1}{R^2} (T_1^0 + R^2 \Omega) \frac{\partial T_1^0}{\partial z} \\ &\quad - \nu \frac{\partial^4 S_2^0}{\partial z^4} + \alpha g \Theta_z^0 = 0 \end{aligned} \quad (3.2)$$

$$\frac{\partial \Theta_z^0}{\partial t} = - \frac{\Gamma}{R^2} S_2^0 - c \Theta_z^0 + \kappa \frac{\partial^2 \Theta_z^0}{\partial z^2} + Q_z^0 = 0 \quad (3.3)$$

Integrating equation (3.1), we obtain the equation representing flux of total angular momentum:

$$- \frac{6}{5} \frac{1}{R^2} S_2^0 (T_1^0 + R^2 \Omega) - \nu \frac{\partial T_1^0}{\partial z} = 0 \quad (3.4)$$

In this equation, the first term expresses net transport of total angular momentum $T_1^0 + R^2 \Omega$ by the meridional circulation S_2^0 , and the second term expresses diffusive flux of the total angular momentum. The integration constant has been set to be zero, because the left hand side of (3.4) is zero at the upper boundary from the boundary condition:

$$S_2^0 \propto w = 0 \quad \text{and} \quad \frac{\partial T_1^0}{\partial z} \propto \frac{\partial u}{\partial z} = 0 \quad (\text{stress free}).$$

It is important to note that the transport of angular momentum expressed by the first term is due to the mechanism proposed by Gierasch (1975). Namely, if $S_2^0 < 0$, $T_1^0 > 0$ ($S_2^0 < 0$ means a motion rising in the equatorial regions and sinking in the polar regions), in the equatorial regions angular momentum is transported upward by the rising motion of the meridional circulation, while in the polar regions angular momentum is transported downward by the sinking motion. Since the T_1^0 mode is a zonal flow with the same angular velocity at all latitudes, the sum of the two opposite effects results in net upward transport of angular momentum due to the difference in the lever-arm length in the equatorial and polar regions.

Further consideration of this problem will be given by the use of the two-layer model shown

Fig. 1 Configuration of T_1^0 and S_2^0 in the two-layer model.

in Fig. 1. (for applicability of the two-layer model, see below) Namely, concerning the toroidal T_1^0 mode, the atmosphere is divided into two layers, while the S_2^0 mode has values at the middle level, the upper boundary and the lower boundary. According to boundary conditions, we can put S_2^0 at the upper and the lower boundary equal to zero. According to the "rigid" condition at the lower boundary and the steady state condition, T_1^0 in the lower layer must also be zero. Thus, we can write equation (3.4) for the two-layer model as follows;

$$T_1^0 = 0 \quad (3.5)$$

$$- \frac{6}{5} \frac{1}{R^2} S_2^0 (T_1^0 + R^2 \Omega) - \nu \frac{T_1^0 - T_1^0}{H/2} = 0 \quad (3.6)$$

where H is the depth of the atmosphere, T_1^0 , T_1^0 represent T_1^0 in the lower and upper half of the fluid, respectively. The expression of the first term in equation (3.4) in terms of the two-layer model needs some careful consideration. Equation (3.6) is an expression of equation (3.4) at the middle level, so that one may think that it is natural to express the first terms as

$$- \frac{6}{5} \frac{1}{R^2} S_2^0 \left(\frac{T_1^0 + T_1^0}{2} + R^2 \Omega \right).$$

But this formulation can lead to serious errors in a qualitative sense in our situation. As is already seen, the net total angular momentum is transported upward by this advection term (if $S_2^0 < 0$). However this formulation is such that the quantity of total angular momentum transported upward is governed by T_1^0 which should be determined from it. This formulation has a similar feature to the centred difference scheme in the finite differencing problem. It is well known that this scheme gives more accurate solutions if the grid interval is sufficiently small, that is, if the value of the variable changes sufficiently gradually (this condition is equivalent to large ν in this problem), and this scheme gives a completely unrealistic solution unless this condition is satisfied. But in our crude two-layer

model it is necessary to treat not only the case satisfying this condition but also the case which is far from satisfying this condition. Thus, we can understand that this formulation is inadequate to our problem which includes the case of small ν . On the other hand the formulation (3.6) gives correct results, at least, in a qualitative sense. This formulation corresponds to a forward difference or so-called "upstream difference" scheme. This scheme is less accurate but known to be stable. The stability of this scheme is assured by the fact that, in this formulation, the quantity of upward transported flux does not contain information on the upper layer. The replacement of $(T_1^0 + T_1^0)/2$ by T_1^0 means underestimation of upward transport of total angular momentum, so that it is impossible in this formulation to obtain overestimated false T_1^0 . Hence this formulation is adequate to our problem where the case of large change of T_1^0 is most important for the purpose of our study.

From (3.5), (3.6), we have

$$T_1^0 = \frac{3}{5} |S_2^0| \frac{H\Omega}{\nu} \quad (3.7)$$

Magnitude of the mean zonal wind at the equator and a representative velocity of the meridional circulation are expressed by T_1^0 and S_2^0 , respectively as follows;

$$U(z) = -\frac{T_1^0(z)}{R} \frac{\partial Y_1^0}{\partial \theta} \bigg|_{\theta=\pi/2} = \frac{T_1^0(z)}{R} \quad (3.8)$$

$$V(z) = \frac{1}{R} \frac{\partial S_2^0(z)}{\partial z} \frac{\partial Y_2^0}{\partial \theta} \bigg|_{\theta=(1/2)\sin^{-1}(1/3)} \\ = -\frac{1}{2} \frac{1}{R} \frac{\partial S_2^0(z)}{\partial z} \quad (3.9)$$

These quantities are expressed in the two-layer model as follows;

$$U = T_1^0/R, \quad V = |S_2^0|/(R \cdot H)$$

Hence, (3.7) is rewritten as

$$\frac{U}{V} = \frac{3}{5} \Omega \frac{H^2}{\nu} = \frac{6}{5} \pi \frac{\tau_\nu}{\tau_\Omega} \quad (3.10)$$

where $\tau_\Omega \equiv 2\pi/\Omega$ is the period of the planetary rotation and $\tau_\nu \equiv H^2/\nu$ is relaxation time of vertical diffusion.

Equation (3.10) shows that magnitude of the mean zonal flow is proportional to that of the meridional circulation and the planetary rotation. Hence the rotation of the planet is found to be essential for the rotation of the upper atmosphere. The difficulty in the explanation of the generation of the four-day circulation based on Venus rotation exists in the fact that effects of

the planetary rotation are supposed to be negligible on account of its slow rotation. But, here, we should ask; what is the time constant with which Venus rotation is compared and said to be slow? Concerning the magnitude of the four-day circulation compared with that of the meridional circulation, equation (3.10) indicates that τ_Ω , the period of the planetary rotation must be compared with relaxation time of vertical diffusion, τ_ν . Hence, despite the fact that the Venus rotation is certainly slow if compared to the turnaround of the meridional circulation or the rotation of the earth, Venus' rotation can be sufficiently fast in comparison with the relaxation time of vertical diffusion (see below), and only this is required for the mean zonal circulation to be faster than the meridional circulation. However, one may conjecture that the effect of the planetary rotation should be estimated according to a comparison of the period of the planetary rotation with the overturning period of the meridional circulation, because the zonal flow is produced by Coriolis effect on the meridional circulation. Indeed, this conjecture may be correct, if we are concerned with atmospheric motion in a limited time, say, a time comparable with the period of overturning of the meridional circulation. But the solution to be discussed here is one in the stationary state. The discrepancy between this conjecture and the solution in a stationary state suggests that the stationary solution represents an equilibrium state reached after a long time and that the fast mean zonal flow is the result of accumulation of angular momentum acquired from the slow planetary rotation during the course of a long time. On the other hand, if we compare U with the Venus rotation itself, we find,

$$\frac{U}{U_{\text{rot}}} = \frac{3}{5} \frac{\tau_\nu}{\tau_m} \quad (3.11)$$

Here τ_m is a measure of the overturning time of the meridional circulation defined as $\tau_m = R/V$. This equation indicates that large super rotation of the atmosphere can occur if the relaxation time of the vertical diffusion is much longer than the meridional circulation time. This condition has been pointed out by Gierasch (1975).

Next we attempt to make rough estimates of the number by substituting tentative values into these equations. For example, $\nu \sim 10^4 \text{ cm}^2/\text{s}$ gives 1000 days as τ_ν if we assume 10 km as H . In this case we obtain $U/V \sim 10$, and if we employ

$V=10$ m/s namely $\pi_m=6$ days according to observations (Murray *et al.* (1974)), $U/U_{\text{rot}} \sim 100$. These ratios are comparable with those observed.* This estimation indicates that we can match results of observations based on this mechanism. Accordingly, the consideration along this line seems very promising.

The above result is indeterminate, for only the relation between the magnitude of the meridional circulation and that of the mean zonal circulation is obtained. Thus, combining the equation of angular momentum balance with that of S_2^0 and Θ_2^0 , we must determine the magnitude of the meridional circulation and that of the mean zonal circulation as solutions of simultaneous equations. For that purpose, it is convenient to employ the following assumptions;

$$(i) \quad \frac{\partial^4 S_2^0}{\partial z^4} \rightarrow \frac{l^4}{H^4} S_2^0$$

$$(ii) \quad c^* \equiv c - \kappa \frac{\partial^2}{\partial z^2} \rightarrow c - \kappa \frac{(-l^2)}{H^2}$$

These replacements are completely justified by putting l as π in the case where solutions are harmonic type. This is the case if the equations can be linearized and Q_2^0 is of harmonic type. On the other hand, as will be seen below, the term which is modified by these assumptions will not be important in the nonlinear case. Consequently, these assumptions are considered to make no serious errors. By this assumption, Θ_2^0 is expressed from equation (3.3) as follows.

$$\Theta_2^0 = \frac{1}{c} \left(-\frac{\Gamma}{R^2} S_2^0 + Q_2^0 \right) \quad (3.12)$$

Here, we write c instead of c^* . Substituting the above formula into equation (3.2) and using (3.10), the following important equation is obtained if we express it in terms of the two-layer model.

$$U^2 + 2R\Omega U + \frac{5}{2} \nu \frac{l^4}{H^4} \frac{\nu R}{H^4} U$$

$$= -\frac{5}{2} \frac{\alpha g}{c} \frac{\Gamma}{R^2} \frac{\nu R U}{\Omega} - \frac{3}{2} \frac{\alpha g Q_2^0 H}{c} \quad (3.13)$$

Before solving (3.13) to determine U it is useful to get an insight into qualitative characteristics of these solutions by a simple consideration. Equation (3.13) comes from the one expressing acceleration of S_2^{0*} namely accelera-

tion of the longitudinal component of vorticity so that this equation represents those effects which are balanced by the torque due to the latitudinal temperature difference. The effect of the latitudinal temperature difference is expressed by the right hand side, there are three terms which should be balanced with it on the left hand side. The third term on the left hand side comes from $\nu(\partial^2 S_2^{0*}/\partial z^2)$, and represents diffusion of vorticity. If this term is balanced with the right hand side, a direct meridional circulation cell dominates and the loss of vorticity of that cell due to the vertical diffusion is compensated by the gain due to a torque arising from the latitudinal temperature difference. On the other hand, the first and the second terms on the left hand side express torque due to a gradient of centrifugal force which consists of vertical gradient of the square of the sum of the atmospheric rotation and the planetary rotation. The difference between them is due to the next point. That is, while in the first term the gradient of centrifugal force is due to the zonal winds and its vertical shear, the second term is due to the vertical shear of the zonal winds coupled with the planetary rotation. Accordingly the former is expressed as $U(U/H)$ and the latter is expressed as $\Omega(U/H)$ in our two layer model. Hence, if these terms dominate, the latitudinal temperature gradient is mainly maintained by the vertical gradient of centrifugal force based on a vertical shear of zonal winds. The situation in which the second term dominates is well known as thermal wind balance in the field of dynamic meteorology, and it is also well known that the atmosphere of our planet is approximately in such a state. Since Venus itself rotates so slowly and the upper atmosphere of Venus rotates so rapidly (about 60 times), the upper atmosphere of Venus is considered to belong not to this case but to the case in which the first term predominates, as was pointed out first by Leovy (1973). In the following discussion we shall refer to each state where the third, the second and the first term dominates as direct cell balance, thermal wind balance of the earth type and thermal wind balance of the Venus type, respectively.* Accordingly equation (3.13) can be regarded as one

* The present condition concerning ν required for matching observations is obliged to be too strong due to crudity of the two-layer model. See below.

* We shall use the term "thermal wind balance of the earth type" and "thermal wind balance of the Venus type" instead of "geostrophic" and "cyclotrophic", because we note the relation of winds with temperature field rather than that with pressure field in this article.

determining the type of balance of the atmosphere under discussion.

Next, we attempt to determine the conditions of predominance of each balance. For this purpose, it is convenient to rewrite the coefficients of that quadratic equation in terms of four time constants and a non-dimensional number;

$$\begin{aligned}\tau_\Omega &= 2\pi/\Omega, \quad \tau_\nu = H^2/\nu, \quad \tau_N = 1/c, \\ \tau_{gr} &= (2\pi/\sqrt{\alpha g \Gamma}) \cdot R/H \quad \text{and} \\ G_r &= \frac{\alpha g |Q_2^0|}{\nu^3} \cdot \frac{H^7}{R^2} = \frac{\alpha g |Q_2^0| (H^2/\nu) H^3}{\nu^2} \left(\frac{H}{R}\right)^2\end{aligned}\quad (3.14)$$

The meanings of the first three are apparent. τ_{gr} represents the period of a gravity wave which has the same size as the whole atmosphere under discussion. G_r may be interpreted as the Grashof number multiplied by the square of the inverse of the aspect ratio if $|Q_2^0|(H^2/\nu)$ is regarded as a representative temperature difference, and we shall refer to this non-dimensional number as the "Grashof number" in the following part of this article. Dividing equation (3.13) by R^2 , we can write this equation in the following form;

$$(U/R)^2 + \beta(U/R) + A_1(U/R) = -A_2(U/R) + \tilde{Q} \quad (3.15)$$

where

$$\begin{aligned}\beta &= 4\pi/\tau_\Omega, \quad A_1 = \frac{5\pi}{(2\pi)^2} \frac{\tau_\Omega}{\tau_\nu^2}, \\ A_2 &= 5\pi \frac{\tau_N}{\tau_\nu} \cdot \frac{\tau_\Omega}{\tau_{gr}^2} \quad \text{and} \quad \tilde{Q} = \frac{3}{2} \frac{\tau_N}{\tau_\nu^3} G_r\end{aligned}\quad (3.16)$$

For further treatment of the problem it is convenient to classify our problem into two cases. Namely,

- (i) $A_1 \gg A_2$ (i. e. $2\pi\sqrt{\tau_\nu\tau_N} < \tau_{gr}$)
- (ii) $A_1 \ll A_2$ (i. e. $2\pi\sqrt{\tau_\nu\tau_N} > \tau_{gr}$)*

In the former case the geometrical average of time constants of the eddy diffusion and the Newtonian cooling is smaller than the period of the gravity wave, consequently dissipative effects are predominant. On the contrary, in the latter case, effects of the stratification are dominant. In the first place we shall consider the former case.

- (i) $2\pi\sqrt{\tau_\nu\tau_N} < \tau_{gr}$

In this case we can neglect $A_2(U/R)$ in

equation (3.15). Since $A_2(U/R)$ comes from the first term on the right hand side of (3.12), we can understand that heat transport by the circulation is neglected in this case. As a result, Q_2^0 is balanced by the terms expressing dissipation of Θ_2^0 , so that Θ_2^0 is determined solely by Q_2^0 and a latitudinal temperature difference is proportional to a heating difference in the latitudinal direction. Thus, this case is just opposite to that treated by Gierasch (1975). Accordingly, the predominant term on the right hand side of (3.15) indicates directly the type of balance. Then, referring to (3.15) and (3.16), the conditions for each balance are obtained in terms of the time constants and the non-dimensional number as follows;

direct cell balance:

$$\begin{aligned}\frac{5}{2\sqrt{6}} \frac{1}{2\pi} \frac{\tau_\Omega}{\sqrt{\tau_\nu\tau_N}} &> \sqrt{G_r} \\ \text{and} \quad \frac{\tau_\Omega}{\tau_\nu} &> \frac{4\pi}{\sqrt{5}}\end{aligned}$$

thermal wind balance of the earth type:

$$\sqrt{\frac{\tau_N}{\tau_\nu}} \frac{\tau_\Omega}{\tau_\nu} < \frac{4\pi/\sqrt{6}}{\sqrt{G_r}} \quad \text{and} \quad \frac{\tau_\Omega}{\tau_\nu} < \frac{4\pi}{\sqrt{5}}$$

thermal wind balance of the Venus type:

$$\begin{aligned}\sqrt{\frac{\tau_N}{\tau_\nu}} \frac{\tau_\Omega}{\tau_\nu} &> \frac{4\pi/\sqrt{6}}{\sqrt{G_r}} \\ \text{and} \quad \frac{5}{2\sqrt{6}} \frac{1}{2\pi} \frac{\tau_\Omega}{\sqrt{\tau_\nu\tau_N}} &< \sqrt{G_r}\end{aligned}$$

According to the above conditions, we can determine the kind of balance from those basic parameters of the atmosphere. However, these conditions are rather complicated so that some simplification is desired for an intuitive understanding of the relation between conditions and kinds of balance. For this purpose it is suitable to restrict our discussion to the cases where τ_ν and τ_N are of the same order, and refer to these as τ_d , or consider τ_d as an average of τ_ν and τ_N . This simplification would be qualitatively justified in view of the fact that both τ_ν and τ_N are time constants of diffusion process. By this simplification, our system can be described by only two parameters, namely, τ_Ω/τ_d and G_r . The former parameter indicates the speed of the planetary rotation and the latter parameter indicates magnitude of the differential heating. Thus, we can draw a diagram which classifies conceptually the kind of balance in terms of the above two parameters. This diagram is drawn in Fig. 2. For drawing this diagram, numerical factors

* In the present article we shall use the notation " $\ll$ " following the next rule: $A \ll B \leftrightarrow \sqrt{A} < \sqrt{B}$

Fig. 2 Diagram illustrating the regime of balance in the case $2\pi\sqrt{\tau_v\tau_N} < \tau_{Gr}$. **D** is direct cell balance, **E** is thermal **V** is thermal wind balance of the Venus type.

Fig. 3 Schematic contour maps illustrating magnitudes of U , V and U/V in the $\tau_\Omega/\tau_d - Gr$ coordinate for the case of (i).

involved in the conditions are set to unity for the sake of simplicity. It should be noted that the balance has a mixed character in the vicinity of a boundary between two regimes in this figure.

Further, referring to equation (3.10), (3.15) and (3.16), in Fig. 3 we can draw contours of U , V and U/V which illustrate distribution of magnitude of the mean zonal flow and the meridional circulation in the parameter space of τ_Ω/τ_d and Gr_r . From these figures we can find the following results.

U , magnitude of the mean zonal flow remains small in the range of small Gr_r . A direct cell balance or a thermal wind balance of the earth type corresponds to this range. Further, in this case a direct cell balance appears if the rotation of the planet is slow compared to the diffusion time, and a thermal wind balance of the earth type appears if the planet rotates rapidly. In fact, from Fig. 3 we see that V is relatively large in the former case and U is relatively large in the latter case. On the other hand, if the magnitude of the latitudinal difference in heating is sufficiently large, U becomes large and as a result thermal wind balance of the Venus type necessarily appears.

(ii) $2\pi\sqrt{\tau_v\tau_N} > \tau_{Gr}$

In the present case, the period of the gravity wave, τ_{gr} appears as a new parameter. This case differs from case (i) in that temperature difference in the latitudinal direction is not simply proportional to heating difference in the latitudinal direction but proportional to $(-A_2(U/R) + \tilde{Q})$. Hence it is necessary to pay special attention to the case that $A_2(U/R)$ is predominant and mainly balances $\tilde{Q}$ in the equation. Physically this corresponds to the effect of differential heating being much reduced by the effect of vertical motions in a strongly stratified atmosphere (small τ_{gr}), so that the residual latitudinal temperature difference becomes very small. If it is so, we obtain $U/R \approx \tilde{Q}/A_2$ to the first approximation, and then substituting this approximate value into each term in the left hand side, we can determine the kind of balance by the comparison of the magnitude of these terms. Accordingly, there are two categories in each thermal wind balance of both types. One is the above case, and we shall refer to this special case as (A). The other is the case that the predominant term on the left hand side is larger than that of $A_2(U/R)$ also and directly balances $\tilde{Q}$, and we shall refer to this case as (B). The case treated by Gierasch (1975) is involved in (A) of thermal wind balance of the Venus type. The conditions of each balance are depicted in Fig. 4. This figure indicates that

process mentioned above is not assumed, but eddy viscosity in the usual sense which acts equally on S_2^0 and Θ_2^0 as well as on T_3^0 is assumed. Even such a viscosity can transfer angular momentum back to the equatorial region against the meridional flow. Simultaneously the diffusion has a destructive influence upon S_2^0 and Θ_2^0 . Comparison of results of the two cases will provide us with information about the role of the selective macro-eddy diffusivity which represents large scale disturbances. We shall consider the former in the present section and the latter in the following section.

a) Basic equations

At first we attempt to represent the system of equations (2.1), (2.2), (2.3) and (2.6) in terms of the two-layer model. For doing so, it is necessary to consider a lower boundary condition for the T_3^0 mode. If we naïvely apply a rigid boundary condition, T_3^0 in the lower-half must be zero as is T_1^0 . But, since the existence of a boundary layer is possible, it is more reasonable to consider that a motion expressed by T_3^0 exists in the lower layer but it is subject to surface friction. Now that total flux of angular momentum through the boundary surface must vanish in a stationary state (We are concerned only with stationary state), the lower-half of T_1^0 must be zero, because the frictional torque acting on the T_1^0 mode has non-vanishing total angular momentum flux. On the contrary, all toroidal modes except T_1^0 have no component of total angular momentum, that is, integration of angular momentum of each toroidal mode over the whole sphere is zero. Hence it is evident that each mode except T_1^0 can neither gain nor lose total angular momentum through boundary friction, if we adopt a linear friction law, and hence they need not to be zero in the lower half layer. Thus we shall retain T_3^0 in the lower layer as a variable and include a surface friction term in its equation.

Integrating equation (2.1) we obtain

$$\frac{6}{5} \frac{1}{R^2} S_2^0 \left(T_1^0 - \frac{18}{7} T_3^0 + R^2 \Omega \right) + \nu \frac{\partial T_1^0}{\partial z} = 0 \quad (4.1)$$

as the equation for angular momentum flux.

For the purpose of expressing equation (2.1)~(2.3) and (2.6) in terms of the two-layer model variable, it is necessary to consider the expressions of each term by examining its implication. In (2.2) there is a term which implies advection of T_3^0 , i.e.,

$$\frac{2}{5} \frac{1}{R^2} \left\{ -3 \frac{\partial}{\partial z} (S_2^0 T_3^0) \right\}.$$

Note that T_3^0 is transported downward by a negative S_2^0 , in contrast to that T_1^0 (total angular momentum) is transported upward by the same meridional circulation. Since the term is advective in nature we may have spurious pair generation of T_3^0 at two levels, if we adopt the usual centred difference scheme. In the present case, however, the difficulty may not be so serious as in the case of T_1^0 , because the $\partial T_3^0 / \partial t$ equation contains not only a flux divergence term (the first term in brace) and a vertical eddy diffusion term but also other terms such as

$$\frac{\partial S_2^0}{\partial z} (T_1^0 + R^2 \Omega), \quad \frac{\partial S_2^0}{\partial z} T_3^0 \quad \text{and} \quad -\frac{\nu_H}{R^2} T_3^0.$$

From a close examination of the effects of these terms, we see that T_3^0 is primarily produced by a coupling between S_2^0 and T_1^0 with larger magnitude in the upper layer and transported downward. In this situation there is very little possibility that a spurious negative value appears, because the transport is from the layer of a larger value to that of a smaller value. Further the dissipation of the mode due to a large horizontal eddy diffusion ($-(\nu_H/R^2)T_3^0$) may reduce the error. Thus we may adopt usual centred difference-type expressions for the terms involving T_3^0 in (2.2).

Next we shall consider the roles of T_3^0 in the flux equation (4.1) and also in the equation for T_1^0 , (2.1). The term $S_2^0(-T_3^0)$ means suppression of the acceleration of the mean zonal flow due to the existence of a positive T_3^0 mode. Associated with transports of angular momentum by vertical motions of the meridional circulation, S_2^0 (we consider only the case $S_2^0 < 0$ here), horizontal motions of the meridional circulation transports angular momentum horizontally. Namely, in the upper layer pole-ward motion of the meridional circulation results in an increase of angular velocity in the polar regions while in the lower layer equatorward motion of the meridional circulation results in a decrease of angular velocity in the equator regions. These processes are well known effect of Coriolis force in dynamic meteorology. In the present model, these deviations of angular velocity from rigid rotation are expressed by a positive T_3^0 in both layers. (Note that the T_3^0 mode is positive in high latitudes and negative in low latitudes.) In this case upward transport of angular momentum by the meridional circulation is decreased, because the

upper layer loses air masses with relatively large angular velocity in the polar regions by the sinking motion, at the same time the upper layer gains air masses with relatively small angular momentum in the equatorial regions by the rising motion. Thus the upper layer loses angular momentum both near the poles and near the equator as the result of the existence of a positive T_3^0 . These processes are implied by the term $S_2^0(-T_3^0)$. Since (2.1) is the equation for $\partial T_1^0/\partial t$, the term $\partial(S_2^0 T_3^0)/\partial z$ cannot be regarded as an advection and hence we have no reason to adopt any form other than a simple average of T_3^0 's at the two layers for expressing the quantity at the middle level. From the previous discussions we have ample reason to expect that T_3^0 does not become negative spuriously, so that its effect to suppress the upward transport of T_1^0 may well be incorporated. From a similar consideration we shall adopt centred-difference type expressions for terms in (2.3).

After eliminating Θ_2^0 , we obtain the following equations,

$$\frac{6}{5} \frac{1}{R^2} S_2^0 \left(-\frac{18}{7} \frac{T_3^0 u + T_3^0 u}{2} + R^2 \Omega \right) + \nu \frac{2}{H} T_1^0 u = 0 \quad (4.2)$$

$$-\frac{6}{5} \frac{S_2^0}{R^2} \frac{T_3^0 u + T_3^0 u}{2} + \frac{2}{R^2} \frac{S_2^0}{H/2} T_3^0 u \cdot \frac{H}{2} + \frac{4}{5} \Omega S_2^0 + \nu \frac{2}{H} (T_3^0 u - T_3^0 l) - \nu \frac{2}{H} \gamma' T_3^0 l - \frac{\nu_H}{R^2} \frac{H}{2} T_3^0 l = 0 \quad (4.3)$$

$$\frac{6}{5} \frac{S_2^0}{R^2} \frac{T_3^0 l + T_3^0 u}{2} + \frac{2}{R^2} \frac{-S_2^0}{H/2} T_3^0 u \cdot \frac{H}{2} - \frac{4}{5} \Omega S_2^0 - \frac{2S_2^0}{R^2} T_1^0 u - \nu \frac{2}{H} (T_3^0 u - T_3^0 l) - \frac{\nu_H}{R^2} \frac{H}{2} T_3^0 u = 0 \quad (4.4)$$

$$\frac{4}{3} \frac{1}{R^2 H} \left(\frac{T_1^0 l + T_1^0 u}{2} + R^2 \Omega \right) (T_1^0 u - T_1^0 l) + \frac{1}{R^2 H} \frac{16}{7} T_1^0 u T_3^0 u + \frac{1}{R^2 H} 8 \frac{T_3^0 l + T_3^0 u}{2} (T_3^0 u - T_3^0 l) - \tilde{\lambda} S_2^0 + \frac{\alpha g Q_2^0}{c} = 0 \quad (4.5)$$

where

$$\gamma' \equiv \left. \frac{\partial T_3^0}{\partial z} \right|_{\text{boundary layer}} \cdot \frac{H/2}{T_3^0},$$

$$\tilde{\lambda} \equiv \nu \frac{l^4}{H^4} + \frac{\alpha g}{c} \frac{\Gamma}{R^2} = \frac{2}{5} \frac{\Omega}{\nu} \lambda$$

b) $S_2^0 - T_1^0$ relations

If we regard S_2^0 as a parameter, (4.2), (4.3) and (4.4) form simultaneous linear equations in three unknowns, T_1^0 , $T_3^0 l$ and $T_3^0 u$.

Putting

$$\bar{T}_3^0 \equiv \frac{1}{2} (T_3^0 u + T_3^0 l)$$

$$T_3^{0'} \equiv T_3^0 u - T_3^0 l$$

the simultaneous equations for these variables are solved, with approximations as will be explained later (see Appendix B);

$$T_1^0 = R^2 \Omega \frac{\frac{3}{5} \sigma - \frac{96}{7} \sigma^3}{\delta + \frac{108}{35} \sigma^2 + \frac{432}{35} \sigma^3} \quad (4.6)$$

$$\bar{T}_3^0 = R^2 \Omega \frac{\frac{4}{5} \gamma \delta^2 \sigma + \frac{6}{5} \sigma^2 + \frac{24}{5} \sigma^3}{\delta + \frac{108}{35} \sigma^2 + \frac{432}{35} \sigma^3} \quad (4.7)$$

$$T_3^{0'} = R^2 \Omega \frac{\frac{16}{5} \delta \sigma + \frac{12}{5} \sigma^2 + \frac{96}{7} \sigma^3}{\delta + \frac{108}{35} \sigma^2 + \frac{432}{35} \sigma^3} \quad (4.8)$$

where $\gamma = 4\gamma'$

$$\sigma \equiv \frac{(-S_2^0)}{\nu_H H} \quad (4.9)$$

$$\delta \equiv \left(\frac{R^2}{\nu_H} \right) \left/ \left(\frac{H^2}{\nu} \right) \right. = \frac{\tau_{\nu H}}{\tau_{\nu}} \quad (4.10)$$

where $\tau_{\nu H} \equiv R^2/\nu_H$ expresses the relaxation time of horizontal viscosity. Recalling

$$\tau_m = \frac{R}{V} = \frac{R}{(-S_2^0)/RH},$$

σ is rewritten as

$$\sigma = \frac{R^2/\nu_H}{R/(-S_2^0)/RH} = \frac{\tau_{\nu H}}{\tau_m} \quad (4.11)$$

Namely σ is the ratio of a meridional overturning time to the relaxation time of macroscopic horizontal eddy viscosity. In Gierasch (1975) theory and also in the previous section of this study, this ratio was presumed to be much smaller than unity. In the present sections we are investigating effects of a finite ν_H so that we must consider the whole range of σ . δ is the ratio of the horizontal to vertical relaxation time due to respec-

tive eddy viscosities, and this is an important parameter for determining realizability of super-rotation. Both Gierasch theory and the previous discussions in the study are concerned with the case $\delta=0$. In the frame work of the present section, we could consider general values of δ . However, we can not get a fast zonal motion for a large δ , since there is no scope for Gierasch's mechanism to work in this case. Thus, we shall confine our discussion to the case $\delta \ll 1$. In fact, in deriving (4.6) through (4.8) this approximation has already been made. In short, while Gierasch's (1975) theory required

$$\tau_\nu \gg \tau_m \gg \tau_{\nu H} \quad (4.12)$$

and treat the case $\tau_{\nu H}=0$ (namely $\sigma=\delta=0$), we retain only the inequality $\tau_\nu \gg \tau_{\nu H}$ (namely $\delta \ll 1$) and relax the other relation. First, we examine characteristics of T_1^0 given by (4.6). T_1^0 as a function of σ is conceptually depicted in Fig. 5-a. The most remarkable point of this figure is that T_1^0 has a maximum at a certain value of σ and T_1^0 can not exceed this maximum value. If σ is small, that is $\sigma \ll \sqrt{\delta}$ (4.16) reduces to

$$T_1^0 = R^2 \Omega \frac{(3/5)\sigma}{\delta} = \frac{3}{5} \frac{\Omega H}{\nu} (-S_2^0) \quad (4.13)$$

This is just the equation (3.7) which is the result in the case of an infinite horizontal viscosity. Accordingly, we can understand that complete neglect of T_3^0 mode is justified only when $\sigma \ll \sqrt{\delta}$. With the increase of σ , T_1^0 increases but it is

smaller than the corresponding one in (3.7), and reaches a maximum for a certain value of σ . This result is entirely different from the one in the preceding section where T_1^0 could grow without limit with an increase of σ . Evidently this difference comes from the finiteness of ν_H which allows the existence of the T_3^0 mode. Indeed, although $(-S_2^0)$ is absolutely necessary for the excitation of T_1^0 , a large $(-S_2^0)$ has an effect of suppressing the existence of T_1^0 , because, it interacts with T_1^0 , to produce T_3^0 which, in turn, destroys T_1^0 through a coupling with $(-S_2^0)$. Apparently the latter effect is second order in $(-S_2^0)$ and hence predominates over the former when $(-S_2^0)$ becomes very large as depicted in Fig. 5-a. Using the approximation of $\delta \ll 1$, the maximum value of $U(=T_1^0/R)$ is calculated from (4.12) as follows.

$$U_{\max} = \frac{\sqrt{105}}{60} \frac{\Omega R}{\sqrt{\delta}} \quad \text{at } \sigma = \sigma_M \equiv \frac{\sqrt{105}}{18} \sqrt{\delta} \quad (4.14)$$

(4.14) shows that superrotation of the atmosphere can be realized in the case of $\delta \ll 1$ if suitable σ is selected, and the magnitude of $U_{\max}/R\Omega$ is determined only by the parameter δ .^{*} For a large σ (4.6) becomes negative. The condition of positive T_1^0 is $\sigma < 1$, except for a numerical factor. By (4.11), this means

$$\tau_m > \tau_{\nu H} \quad (4.15)$$

This is nothing but the condition that the meridional overturning be slower than the horizontal diffusion, which was a prerequisite for the Gierasch mechanism to work (Gierasch (1975)). We should not confuse this condition with the condition for neglecting the existence of the T_3^0 mode. The latter condition is already seen to be $\sigma \ll \sqrt{\delta}$, namely

$$\sqrt{\delta} \tau_m \gg \tau_{\nu H} \quad (4.16)$$

Hence the domain of σ where the former condition is satisfied but the latter condition is not satisfied corresponds to such that the Gierasch mechanism is working but the existence of T_3^0 is important. Our main concern in the present section lies in this state.

Next, we examine $\bar{T}_3^0$ and $T_3^{0'}$ as functions of σ , which are conceptually illustrated in Fig. 5-b and Fig. 5-c, respectively. Approximate

Fig. 5 Schematic illustration of T_1^0 , $\bar{T}_3^0$ and $T_3^{0'}$ as functions of σ in the case of $\delta \ll 1$.

* The condition of fast super-rotation based on (4.14) requires an excessively small δ , because the two-layer model tends to underestimate U , as has been already discussed.

formulae for T_1^0 , $\bar{T}_3^0$ and $T_3^{0'}$ in each range of σ , which are derived from (4.6)–(4.8), are given in Table 1. From Fig. 5 as well as from Table 1, we can understand that $\bar{T}_3^0$ and $T_3^{0'}$ can not exceed $R^2\Omega$. For a small σ ($\ll 1$) T_1^0 is predominant over $\bar{T}_3^0$, $T_3^{0'}$ (or T_3^{0u} , T_3^{0u}) by a factor including $\delta^{-1/2}$. On the other hand, in the case of $\sigma \geq 1$, both of the magnitudes of rigid rotation T_1^0/R and differential rotation, T_3^{0u}/R and T_3^{0l}/R become comparable to the planetary rotation, $R\Omega$. The case $\sigma \geq 1$ is such that the meridional overturning is not slower than horizontal diffusion (see (4.11)), and as the result the T_3^0 mode can not be effectively eliminated. Therefore in this case, the mechanism of accelerating the rigid rotation of the upper atmosphere does not work, hence super rotation can not appear.

c) Solutions

So far we have been concerned with the behaviour of T_1^0 as a function of S_2^0 . The result indicates that fast atmospheric rotation can be present for a meridional circulation of suitable strength, provided that δ is sufficiently small. However this result implies only possibility of having a fast zonal flow, because the value of S_2^0 is not yet determined as a solution of the simultaneous equations for a prescribed differential heating Q_2^0 . If we make a simplifying assumption that S_2^0 is proportional to Q_2^0 , as has been done by Gierasch (1975), i.e., if we assume $c=0$ in (3.12) the $S_2^0-T_1^0$ relation is equivalent to the $Q_2^0-T_1^0$ relation, except for a constant factor. Then we see that in this case the zonal mean wind velocity increases with the increase of the external heating to reach a maximum T_1^0 for a certain strength of Q_2^0 . For further increase

of Q_2^0 , the zonal flow decreases and the circulation goes back to a direct cell type. These behaviours are quite different from those in the infinite ν_H case, where T_1^0 grows without limit and a Venus type circulation predominates always for a sufficiently large Q_2^0 . However, the result is not surprising if we remember that the Gierasch's mechanism does not work for a relatively small ν_H , as explained previously (see also Gierasch, 1975). Then it is most interesting to investigate how the $Q_2^0-T_1^0$ relation becomes when we use (3.12) without the approximation. Equation (4.5) is rewritten in the following form;

$$\begin{aligned} & (T_1^0/R^2)^2 + 2 \cdot \frac{2\pi}{\tau_\Omega} (T_1^0/R^2) \\ & + \frac{24}{7} (T_1^0/R^2)(T_3^{0u}/R^2) + 12(\bar{T}_3^0/R^2)(T_3^{0'}/R^2) \\ & + \frac{3}{2} \frac{1}{\delta} \left(\frac{1}{\tau_v^2} + (2\pi)^2 \frac{\tau_N}{\tau_v} \frac{1}{\tau_{gr}^2} \right) \sigma = \frac{3}{2} \frac{\tau_N}{\tau_v^3} G_r \end{aligned} \quad (4.17)$$

where the notations are the same as the previous sections. This equation is an extension of (3.15) to the case of finite ν_H . In the above equation, the left hand side involves T_1^0 , $\bar{T}_3^0$ and $T_3^{0'}$. Since T_1^0 , $\bar{T}_3^0$ and $T_3^{0'}$ are already expressed by σ as (4.6)–(4.8), the left hand side can be regarded as a function of σ , that is, (4.17) is an algebraic equation with respect to σ . However, it may be practically impossible to obtain explicit solutions of this equation. Then, we take the following way. (4.17) is a modified form of the equation for the zonal component of vorticity, so that we can classify the modes of general circulation according to this equation, as we did in the preceding section. Comparing (4.17) with the corresponding ones in the preceding section, we can easily see that predominance of the last term means a direct cell balance, and that of the second term means a thermal wind balance of the earth type, and further that of the first term means a thermal wind balance of the Venus type. The third and the fourth terms have no counterparts in the corresponding equations in the preceding section. The third term represents the latitudinally variable part of the vertical gradient of centrifugal force arising from vertical shear of the T_1^0 mode and the T_3^0 mode, while the fourth term represents that due to vertical shear of only the T_3^0 mode. However, our following consideration will show that these two terms cannot dominate over the other terms, so that

Table 1. Approximate functions of T_1^0 , $\bar{T}_3^0$ and $T_3^{0'}$ in various domains of σ for the case of $\delta \ll 1$.

σ	$\sigma \ll \sqrt{\delta}$	$\sqrt{\delta} \ll \sigma \ll 1$	$1 \ll \sigma$	
T_1^0	$\frac{3}{5} R^2 \Omega \frac{1}{\delta} \sigma$	$\frac{7}{36} R^2 \Omega \frac{1}{\sigma}$	$\frac{10}{9} R^2 \Omega$	
σ	$\sigma \ll \delta^2$	$\delta^2 \ll \sigma \ll \sqrt{\delta}$	$\sqrt{\delta} \ll \sigma$	
T_3^0	$\frac{4}{5} r R^2 \Omega \delta \sigma$	$\frac{6}{5} R^2 \Omega \frac{\sigma^2}{\delta}$	$\frac{7}{18} R^2 \Omega$	
σ	$\sigma \ll \delta$	$\delta \ll \sigma \ll \sqrt{\delta}$	$\sqrt{\delta} \ll \sigma \ll 1$	$1 \ll \sigma$
$T_3^{0'}$	$\frac{16}{5} R^2 \Omega \cdot \sigma$	$\frac{12}{5} R^2 \Omega \frac{\sigma^2}{\delta}$	$\frac{7}{9} R^2 \Omega$	$\frac{5}{9} R^2 \Omega$

it is not necessary to consider a new kind of balance; balance in this section. For the purpose of determining which term predominates on the left hand side, it would be necessary to examine behaviours of each term as a function of σ .

The behaviours of T_1^0 , $\bar{T}_3^0$ and $T_3^{0'}$ as functions of σ have already been examined in detail. The magnitudes of the first, the second and the last terms as functions of σ in various cases are drawn in Figs. 6a~6c. (Concerning the explanation of this figure see below.) (4.6)~(4.8) show that all the terms except the last one on the left hand side of (4.17) are proportional to $(R\Omega)^2$. On the other hand the last term does not involve any effect of the planetary rotation. Hence fast planetary rotation is favourable for the predominance of the first four terms over the last one. Further comparison among the four terms can be made without referring to the magnitude of Ω . From Table 1, we can determine the order of magnitude of the four terms as follows;

$$\begin{aligned}\sigma \ll \delta &\longrightarrow \textcircled{2} \gg \textcircled{1} \gg \textcircled{3} \gg \textcircled{4} \\ \delta \ll \sigma \ll 1 &\longrightarrow \textcircled{1} \ll \textcircled{2} \ll \textcircled{3} \ll \textcircled{4} \\ 1 \leq \sigma &\longrightarrow \textcircled{1} \sim \textcircled{2} \sim \textcircled{3} \sim \textcircled{4} \sim \Omega^2\end{aligned}$$

Next, we compare the magnitudes of the four terms especially the first and the second terms, with that of the fifth term. The fifth term expresses effects of dissipation of the meridional circulation, and it is simply proportional to σ . Since we are most interested in the neighbourhood of σ_M where T_1^0 attains a maximum, we compare the maximum values of the first and the second terms with the value of the last term at $\sigma = \sigma_M$;

(values of the fifth term at $\sigma = \sigma_M$)/(the maximum value of the first term) =

$$\begin{aligned}&\frac{20\sqrt{105}}{7} \frac{1}{(2\pi)^2} \left(\frac{\tau_\Omega}{\tau_\nu} \right)^2 \sqrt{\delta} \quad (\text{if } 2\pi\sqrt{\tau_\nu\tau_N} < \tau_{gr}) \\ &\frac{20\sqrt{105}}{7} \frac{\tau_N}{\tau_\nu} \left(\frac{\tau_\Omega}{\tau_{gr}} \right)^2 \sqrt{\delta} \quad (\text{if } 2\pi\sqrt{\tau_\nu\tau_N} > \tau_{gr})\end{aligned}\quad (4.18)$$

and

(values of the fifth term at $\sigma = \sigma_M$)/(the maximum value of the second term) =

$$\begin{aligned}&\frac{5}{8\pi^2} \left(\frac{\tau_\Omega}{\tau_\nu} \right)^2 \quad (\text{if } 2\pi\sqrt{\tau_\nu\tau_N} < \tau_{gr}) \\ &\frac{5}{2} \left(\frac{\tau_\Omega}{\tau_{gr}} \right)^2 \quad (\text{if } 2\pi\sqrt{\tau_\nu\tau_N} > \tau_{gr})\end{aligned}\quad (4.19)$$

Thus, we are led to distinguish the following cases for the sake of classification of states of

$$\begin{aligned}&(\text{i}) \quad 2\pi\sqrt{\tau_\nu\tau_N} < \tau_{gr} \\ &(\alpha) \quad \frac{\tau_\Omega}{\tau_\nu} < \sqrt{\frac{2}{5}} \cdot 2\pi, \quad \left(\frac{\tau_\Omega}{\tau_\nu} \right)^2 \sqrt{\delta} < \frac{7}{20\sqrt{105}} (2\pi)^2 \\ &(\beta) \quad \frac{\tau_\Omega}{\tau_\nu} > \sqrt{\frac{2}{5}} \cdot 2\pi, \quad \left(\frac{\tau_\Omega}{\tau_\nu} \right)^2 \sqrt{\delta} < \frac{7}{20\sqrt{105}} (2\pi)^2 \\ &(\gamma) \quad \frac{\tau_\Omega}{\tau_\nu} > \sqrt{\frac{2}{5}} \cdot 2\pi, \quad \left(\frac{\tau_\Omega}{\tau_\nu} \right)^2 \sqrt{\delta} > \frac{7}{20\sqrt{105}} (2\pi)^2 \\ &(\text{ii}) \quad 2\pi\sqrt{\tau_\nu\tau_N} > \tau_{gr} \\ &(\alpha) \quad \frac{\tau_\Omega}{\tau_{gr}} < \sqrt{\frac{2}{5}}, \quad \left(\frac{\tau_\Omega}{\tau_{gr}} \right)^2 \sqrt{\delta} < \frac{7}{20\sqrt{105}} \\ &(\beta) \quad \frac{\tau_\Omega}{\tau_{gr}} > \sqrt{\frac{2}{5}}, \quad \left(\frac{\tau_\Omega}{\tau_{gr}} \right)^2 \sqrt{\delta} < \frac{7}{20\sqrt{105}} \\ &(\gamma) \quad \frac{\tau_\Omega}{\tau_{gr}} > \sqrt{\frac{2}{5}}, \quad \left(\frac{\tau_\Omega}{\tau_{gr}} \right)^2 \sqrt{\delta} > \frac{7}{20\sqrt{105}}\end{aligned}$$

In the above classification of parameter ranges, the division into (i) and (ii) are made to distinguish the two different internal states of the fifth term. Namely, the fifth term is composed of two parts $(1/\tau_\nu^2)\sigma$ and $(2\pi)^2(\tau_N/\tau_\nu)(1/\tau_{gr}^2)\sigma$ and they represent different physical effects as discussed in Section 3. The physical implication of the fifth term and as the result a state of balance change depending on which of the above two components is more important in this term. However, at first we shall be concerned with balance of terms in (4.17) and disregard the above difference.

Behaviours of the three terms as functions of σ , in each of the three cases are depicted in Figs. 6a~6c. In these diagrams we omit the third and fourth terms, because they are much smaller than the first two for $\sigma \ll 1$ and become comparable only in the range $\sigma \geq 1$, where the fifth term decisively dominates over the others.

Now we are ready to determine solutions T_1^0 and σ which satisfy (4.17) for a given G_r , with the aid of the diagrams. In Figs. 6a~6c, σ as a solution of (4.17) can be determined as the abscissa of an intersection of a horizontal line which represents the right hand side and a dashed curve which is the sum of the five (substantially three) terms on the left. From this solution σ thus obtained, we can determine T_1^0 as the value of the second term at this σ , apart from a multiplicative constant. Note that the second term is proportional to T_1^0 . More descriptions about solutions of individual cases will be given later. On the other hand, once the intersection is determined, the major contributor to the dashed curve seen at the point can be found and consequently

Fig. 6 Illustration of the terms in equation (4.17) as functions of σ in the case of $\delta \ll 1$. Logarithmic scales are used for the abscissa and the ordinate. In these figures $\delta = 10^{-3}$ is employed. A curve having a steep peak, a curve having a moderate peak and a straight line represent the first, the second and the fifth term in the left hand side of equation (4.17). The third and the fourth term are not drawn. The right hand side is indicated by a horizontal bar. The sum of terms on the left hand side is shown by a dashed line. Solutions of this equation are indicated by open circles. (a), (b) and (c) illustrate the case of fast rotation, moderate rotation and slow rotation of the planet respectively. (See text). The greek characters with suffix correspond to those in Fig. 7 and 8.

we see what physical effect in the left of (4.17) is the predominant one on the left hand side of equation, and expresses the type of balance of the circulation. In Fig. 7 and 8 magnitudes of the mean zonal velocity U (derived from T_1^0) and those of the meridional velocity V (derived from σ) are schematically shown for the case (i), respectively. The classification diagram for the case (i) is shown in Fig. 9. Both Figs. 7, 8 and 9 are drawn on a two dimensional parameter space $\tau_\Omega/\tau_d - G_r$. Namely, as the preceding section, we have regarded τ_ν and τ_N as equal and denoted both by τ_d in preparing these figures. As will be discussed later in detail, U and V become multivalued functions in some portion of the $\tau_\Omega/\tau_d - G_r$ space. Therefore U -surfaces and V -surfaces in Figs. 7 and 8 are overlapped in such a place.

Fig. 6a illustrates how solutions of (4.17) are determined in the case of a moderate or fast planetary rotation ($\tau_\Omega/\tau_d < 1$, distinguished as case α). Values of U and V of this case are illustrated in the panel designated by α in Figs. 7 and 8. In this case the sum of the five terms (shown by a dashed line) on the left hand side has a maximum at a certain finite value of σ . Now, if the latitudinal differential heating is weak (G_r is small), the right hand side is small, so that we have only one intersection corresponding to a small σ , as shown by α_1 . Since the major contributor to the left hand side is the second term, we can consider that this state is in a thermal wind balance of the earth type. Fig. 7 and 8 show that U and V remain small in this case.

Next we shall consider solutions for a moderate value of G_r . This case is designated as α_2 . From Fig. 6a, we can find an important fact that the horizontal line intersects the dashed curve at three points, that is, three solutions exist in the present case. The problem of the multiple solutions and physical implications of these solutions will be discussed in the next subsection in detail.

In the case of a sufficiently large G_r (designated by α_3), only one solution can exist. From Fig. 9 and also from Figs. 7 and 8 we see that this solution is in a direct cell balance and has a very large V and a very small U . This means that sufficiently large differential heating can be associated only with very fast meridional circulation.* This is an utterly different feature from the results obtained in Section 3. In the previous

* As will be explained, this situation is considered to correspond to the convection between day side and night side in the actual Venus situation.

Fig. 7 Contour maps of U in the τ_Ω/τ_d - G_r coordinate in the case of $\delta \ll 1$. $\delta=10^{-3}$ is employed in these figures. Values attached to contour lines in (a) are those of U (normalized by R/τ_v). In some portion of these diagrams U is multi-valued. Sections of this contour maps at fixed τ_Ω/τ_d (designated by α , β and γ) are given in (b), which is schematically depicted in arbitrary scales. Figure (c) is a schematic three-dimensional diagram showing U as a function of τ_Ω/τ_d and G_r . Notice the portion of multi-valued. Kind of balance is indicated in this figure (see Figs. 8 and 9 also).

treatments any effect to suppress the Gierasch acceleration mechanism was not considered so that large G_r was always favourable for high atmospheric rotation. In the present case, the very large meridional temperature gradient can not be maintained by the vertical shear of the zonal winds. For attaining such a balance a large vertical shear of the zonal winds must be generated by the fast meridional circulation. But too fast meridional circulation violates the condition under which the present mechanism for acceleration of the mean zonal flow can work as long as ν_H is finite. Hence such a balance can not exist and as a result only the direct cell balance can exist under a very large differential heating. For the cases β and γ interpretation of solutions could be given in the same way as in the case α .

Fig. 8 Contour maps of V . As in Fig. 7 except for V .

Fig. 9 Schematic diagram illustrating regime of each balance in the case of $\delta \ll 1$. Thermal wind balance of the Venus type I (V-I) is the region where there is only one solution, while thermal wind balance of the Venus type II (V-II) is the region where there is a solution of direct cell balance also, together with solutions of thermal wind balance of the Venus type. D and E indicate the same as in Fig. 2. Coordinates of some points are written, except for numerical factors, in the figure.

Fig. 9 illustrates the regions of each balance in the parameter space. The interesting point found in Fig. 9 is the fact that the regime of the thermal wind balance of the Venus type has an upper boundary in G_r , depending on τ_Ω/τ_d . The extent of the region of this balance is determined by δ and, of course, it increases if δ is decreased. From (4.14), it has been already required that δ should be small for the mean zonal flow to be large. Now small δ is required again for the region of large mean zonal flow to be large. Fig. 7 and Fig. 8 indicate that in some portion of the region of thermal wind balance of the Venus type, U and V are multivalued, (Hence, the solution of an alternative balance, namely, that of direct cell balance, also exists in this region although we refer to this region only as "thermal wind balance of the Venus type" for the sake of simplicity.)

For the case of $2\pi\sqrt{\tau_v\tau_N} > \tau_{gr}$, the situation may not be fundamentally altered. It is because the procedures and results mentioned above remain applicable to this case too, if we replace τ_v by τ_{gr} in the coefficient of the last term on the left hand side of (4.17). Hence, we omit further explanation of this case.

d) Nature of solutions

First it is necessary to examine stability of three stationary solutions obtained in some range of τ_Ω/τ_d - G_r space. This examination may be made by calculating eigenvalues of linearized equations perturbed about each stationary state. However, without invoking such a calculation, we can determine the stability from a fairly general view point. The results of this general consideration indicates that a solution having the largest σ (namely having the smallest U) and one having the smallest σ (Namely having largest U) are stable and the intermediate solution is unstable. Here, leaving a development of this theory to a subsequent article, it is suitable to confine our discussion to a following remark. From V -surface in Fig. 8, we can see that among three solutions, the solution having the largest σ continues to the unique solution in the range of sufficiently large G_r , while the solution having the smallest σ continues to the unique solution in the range of small G_r . Each unique solution is considered to be stable and not to change its stability regardless of the appearance or disappearance of other solutions. Accordingly it may be inferred that the solution having the largest σ and the one having the smallest σ are

stable.

Next, we discuss the properties of the two stable solutions. One of them which has the smaller values of σ is shown to be in the state of a thermal wind balance of the Venus type because the first term predominate over the others. On the other hand, the one having the largest σ is found to correspond to direct cell balance, as is indicated by the predominance of the last term. Indeed, Figs. 6(a) show that the former solution has a large U and a small V , while the latter has a large V and a small U . Thus there exist two solutions of very different kinds as stationary solutions of the present problem for the same external heating; the one being characterized by fast mean zonal flow and slow meridional circulation, while the other being characterized by fast meridional circulation and slow mean zonal flow.

The physical mechanism which leads to the multiplicity of stationary states may be explained in the following way. For an external heating of a proper intensity which allows the two solutions, a circulation of the Venus type can be maintained if the initial zonal flow is strong enough to sustain the large pressure gradient (arising from a large temperature gradient*) and hence to keep the meridional circulation very slow. On this occasion a macro-eddy viscosity of a moderate magnitude is able to diffuse the angular momentum transported to the polar region by the meridional flow back towards the equatorial region. In short, the mechanism proposed by Gierasch (1975) can work. On the contrary, if a very weak zonal flow is given initially, the meridional pressure gradient of the same intensity cannot be balanced either with the centrifugal force or the Coriolis force and therefore a strong meridional circulation will result. Once a fast meridional motion appears it is impossible for the macro-eddy viscosity (of moderate magnitude) to diffuse back the angular momentum against the transport by the meridional flow. Thus the angular momentum may remain small and hence the strong meridional flow will continue. It is obvious that this state is also a self-consistent balanced state. Thus we see that the two stationary states are possible for the same external heating.

At the end of this section, we must discuss the implication of the two stable solutions to the

* For definiteness we may consider a radiative equilibrium temperature.

actual Venus atmosphere. For this purpose, it is necessary to note the applicability of our axisymmetric model to the actual three dimensional atmosphere. First, the solution of fast zonal flow can be regarded as a self-consistent solution of three dimensional atmosphere for the following reason. Namely, once fast rotation of the atmosphere is built up, then the longitudinal variation of any field, especially the differential heating will be considerably diminished. As a result an axisymmetric solution can be justified as a self-consistent solution. On the other hand, if a meridional direct cell predominates, the assumption is inadequate. In this case we must take the differential heating in the longitudinal direction into consideration. Then, we should superimpose a direct cell induced by longitudinal differential heating on a meridional direct cell. Indeed, this superposition is possible because the equations describing the system become then approximately linear. Thus, "meridional direct cell" in the axisymmetric model is considered to represent a direct cell in general. According to these considerations, we can understand that the solution characterized by fast mean zonal flow and slow meridional circulation corresponds to the four-day circulation while the solution characterized by the predominance of the meridional circulation corresponds to direct circulation between day side and night side. Hence, existence of multiple solution in our model means that both the four-day circulation and direct circulation between day side and night side are a stable state for the parameter range V-II in Fig. 9. Therefore, if we neglect the region V-I, in which the zonal wind is comparatively weak, appearance of the former state in the actual Venus atmosphere means that the Venus atmosphere belongs to the range V-II and as a result the latter solution is also a possible state in the Venus atmosphere. By the way, the latter solution is just what we expect to be the essential pattern of the Venus atmospheric circulation from simple considerations. Hence, our naïve expectation of the direct cell turns out to be correct in this sense. Thus, the relation between the four-day circulation and the direct cell is made clear.*

* It is noticeable that the direct cell solution could not be obtained before the consideration of this section. Indeed, Gierasch's study (1975) and the study of the preceding section could not answer whether our naïve expectation of the direct cell was correct or not.

5. Circulation in the case of non-selective eddy viscosity

In the previous sections we have assumed that the macro-eddy viscosity affects preferentially the T_3^0 mode to reduce its amplitude or to eliminate it completely. Thus the aim of the present section is to investigate the case where the same ν_H acts on the T_3^0 and S_2^0 modes and on the temperature field Θ_2^0 . Arguments in the preceding section remain almost valid. What is to be altered in the arguments is only the coefficient of the last term of the left hand side of equation (4.17), because only the addition of a horizontal diffusion term $-\nu_H S_2^{0*}/R^2$ to equation (2.3) is needed. One of the most important results in the preceding section has been that a small $\delta = \nu R^2/\nu_H H^2$, namely a large ν_H was necessary for the existence of fast rotation of the upper layer. But, in the present case, ν_H acts at the same time to destroy the S_2^0 mode whose existence is indispensable for our generation mechanism of the mean zonal flow. Hence, what we would like to know here is the consequence of the two opposing effects of ν_H .

Since the procedures required to obtain an equation involving the single variable, σ are the same as those in the preceding section, we present immediately a term to be altered in the final result. Namely the left hand side of equation (4.17) should be replaced by

$$\frac{1}{\delta} \left(\frac{1}{\tau_\nu^2} + \frac{1}{\delta \tau_\nu^2} + (2\pi)^2 \frac{\tau_N}{\tau_\nu} \frac{1}{\tau_{gr}^2} \right) \sigma$$

$$\begin{aligned} & \frac{\text{value of the last term at } \sigma = \sigma_M}{\text{maximum value of the first term}} \\ &= \frac{360}{7} \sqrt{\frac{35}{108}} \left\{ \frac{1}{(2\pi)^2} \sqrt{\delta} \left(\frac{\tau_\Omega}{\tau_\nu} \right)^2 \right. \\ & \quad \left. + \frac{\tau_N}{\tau_\nu} \left(\frac{\tau_\Omega}{\tau_{gr}} \right)^2 \sqrt{\delta} \right\} \end{aligned} \quad (5.1)$$

on account of $\delta \ll 1$. So that $\tau_\Omega/\tau_\nu < 1$ is a necessary condition for the predominance of the first term. If

$$\tau_{gr}/\tau_\nu > \sqrt{\tau_N/\tau_\nu} \sqrt{\delta}$$

we can neglect the term including τ_{gr} . Then equation (6.1) could be estimated to be

$$\sim \frac{20\sqrt{105}}{7} \left(\frac{\tau_\Omega}{\tau_\nu} \right)^2 \frac{1}{\sqrt{\delta}}$$

so that the condition of the predominance of the first term turns out to be $\tau_\Omega/\tau_\nu < \delta^{1/4}$.

Fig. 10 Diagram illustrating the regimes of balance in the case that the same magnitude of horizontal eddy diffusion acting on T_3^0 and S_2^0 is assumed. Concerning further explanation, see that of Fig. 9.

A diagram showing classification of type of balance in the two parameters space in this case is given by Fig. 10. From this figure, we can see that the regime of thermal wind balance of the Venus type certainly exists but its extent is considerably diminished if compared to the case of the preceding section. Hence we can conclude that the assumption of employing the same ν_H for the S_2^0 mode as that for the T_3^0 mode makes an existence of a thermal wind balance of the Venus type considerably more difficult. We have treated two limiting cases in the preceding and present sections. From results of the two limiting cases we can easily guess that an intermediate assumption concerning ν_H such that $\nu_H > \nu_H'$ but $\nu_H' \neq 0$ would lead to an intermediate result. Thus it would be natural to conclude that the larger the difference between ν_H and ν_H' becomes, the more easily a thermal wind balance of the Venus type can exist.

6. Discussion

The present section is mainly devoted to re-examination of the results of numerical experiments by Young & Pollack (1977) in the light of the results of the present study. Apparently, their results seem to be not entirely compatible with ours. The two results agree with each other in that the acceleration of the mean zonal flow is due to upward transport of angular momentum

by the meridional circulation. However there is a critical discrepancy in the role of the planetary rotation. Indeed, their results indicate that the planetary rotation is not necessary for the generation of fast mean zonal flow except in the determination of the direction of flow, while the planetary rotation is indispensable for the existence of fast mean zonal flow in the results of the present study. Further, in their explanation, the nonlinear instability, as proposed by Thompson (1970), connected with the meridional circulation was required, while there is no need for such instability in our results. Thus it is necessary to examine more carefully the results of calculations from which their conclusion is drawn. According to their article, the calculation supporting their conclusion is as follows. In order to show that the fast mean zonal winds can be maintained without planetary rotation, they have performed calculations in two ways; (1) by simply making both the planet and the sun stationary after performing time-integrations under the ordinary condition for a while and (2) by testing the stability of the convection between subsolar and antisolar points giving mean zonal wind perturbations. The results of the calculations show that the mean zonal flow decays if its initial magnitude is small, but the mean zonal flow grows if its initial magnitude is large. These results are common to calculations in the two ways (1) and (2). Thus they have confirmed that amplification of the mean zonal flow in the upper layer takes place even in the case of no planetary rotation. From this fact they have conjectured that planetary rotation is not necessary for the maintenance of the mean zonal flow. Further, from the results that the subsequent evolution of the vertical profile of the mean zonal flow (that is, steepening or damping) depends on the initial state, they have guessed that, together with the meridional circulation, a nonlinear instability such as proposed by Thompson must be involved for the excitation of the mean zonal flow.

Next we discuss the results of their numerical experiments and their interpretation based on the results of our study. At first we should note that they did not continue the calculations long enough to obtain a completely settled stationary solutions. Hence what we can say surely from the calculations performed is not the existence of the fast mean zonal flow in a stationary state but the possibility of its temporary sharpening in the case without the planetary rotation. The two things are not the same. Indeed, the present

mechanism of upward transports of angular momentum can work even in the case without planetary rotation if there exists initially an atmospheric rotation, in the lower layer due to some other causes. In this case, the rotation of the lower atmosphere plays the same role as the planetary rotation for the upper atmosphere. Namely, the upper atmosphere can be accelerated owing to the rotation of the lower layer which is initially given.

Thus, steepening of the profile of the mean zonal flow with the large initial perturbation is explained only by the effect of the meridional circulation. However, if this is so, why does not the meridional circulation yield steepening of the profile of the mean zonal flow also in the case of small initial perturbation? We have already pointed out that rotation of the layer at a certain level plays the role of a quasi-planetary rotation for the layers which are higher than that level. Hence, the weak perturbation of the T_1^0 mode means a slow quasi-planetary rotation for the upper layer. According to the results of the present study, if planetary rotation is weak, the vertical gradient of the mean zonal flow is small ($\partial U/\partial z = (U - 0)/H \propto \Omega$, see (3.10)), and further the thermal wind balance of the Venus type is difficult to support. Therefore we can predict damping of the small initial perturbation in this case. On the contrary, a large initial perturbation means a large quasi-planetary rotation for the upper layer, so that large vertical gradient of the mean zonal flow and temporary appearance of a thermal wind balance of the Venus type is expected. Thus we can explain the two different evolutions, *i.e.*, damping or steepening, depending on the different initial state in the framework of the present study. Hence, we can predict that in any case without planetary rotation the mean zonal flow disappears, if calculations are continued until the whole of the atmosphere becomes stationary due to the effect of viscosity (note that a true stationary state can be realized due to effects of viscosity).

By the way, necessity for "the nonlinear instability" in Young & Pollack's explanation comes from the fact that the above different behaviours were thought not to be explained only by the effect of the meridional circulation. Hence, "the nonlinear instability" such as proposed by Thompson is not necessary at all, because we have been able to explain the different behaviours without this instability. In fact, there is no direct evidence supporting the existence of this mecha-

nism, for example, a tilting of the convection cell between subsolar and antisolar points in the results of their calculations. Moreover, the compatibility of a tilting of the convection cell with upward transports of angular momentum by the meridional circulation is questionable. Lastly, we would like to point out that nonunique stable stationary solutions cannot be explained by an instability such as that proposed by Thompson. In an instability of this kind, if the parameter governing the system exceeds the critical value, a direct cell balance between subsolar and antisolar points becomes unstable and even an infinitesimal perturbation to it grows. Hence non-uniqueness of the stable stationary solution means only that there are two stable solutions with different directions of the mean zonal flow, and not that both a solution of a direct cell and solutions having a mean zonal flow are at the same time stable solutions. Thus, not only can we not find any need for invoking the nonlinear instability mechanism but also we find no possibility of cooperation of this mechanism in the calculations by Young & Pollack. From the above discussion, we may conclude that the essential feature of the calculations by Young & Pollack including the one without the planetary rotation can be explained in the framework of the results obtained by the present study. Lastly, it is of interest to pay attention to the fact that the large mean zonal flow formed during the course of a long time, namely, ~ 10 solar days in their calculations. We can regard this fact as supporting our suggestion that the fast mean zonal flow is a result of accumulation effect of the slow planetary rotation in the upper layer during a long time.

7. Conclusion

In the present study, we have examined our model and its consequences in detail. According to it, the fast zonal flow can actually appear under certain conditions by upward transports of angular momentum by the meridional circulation. Among the conditions required for its appearance, firstly, small $\delta (= \nu R^2 / \nu_H H^2)$, *i.e.*, large ν_H is required because it is a premise for Gierasch's mechanism to work. Moreover our study shows that maximum value of zonal winds velocity and the extent of parameter range of thermal wind balance of the Venus type is mainly controlled by δ . In this sense, δ is most fundamental parameter governing our system and small δ is most important condition required. In addition

to this condition, fast or moderate planetary rotation (*i.e.*, small or moderate τ_Ω/τ_ν) and moderate magnitude of differential heating (G_r) is necessary for fast zonal flow to appear. When these conditions are satisfied, fast zonal flow can actually appear. However, together with this solution, the direct cell can also exist as a stable state just for the same set of parameters. This multiplicity of solution is most remarkable and important consequence of our study. Namely, the direct cell between day side and night side is also a possible state of the Venus upper atmosphere, together with the four-day circulation. The starting point of the problem concerning the four-day circulation lay in the question: why a direct cell does not predominate in the Venus upper atmosphere? Therefore it is a unique point of our study to answer this question by showing that the direct cell is also possible state of Venus atmosphere. After all, the problems concerning the four-day circulation have been consistently explained based on our theoretical study. Nevertheless, this success of our study can not immediately exclude possibilities of other explanation. Indeed, we can not yet identify the maintaining mechanism of the four-day circulation from observations so far made. However, the existence of mean meridional circulation is confirmed by both Mariner 10 and Pioneer Venus. This fact is favourable to our explanation because the existence of the meridional circulation is a premise for our explanation. Further, zonal flow is approximately rigid rotation according to the analysis of Pioneer images (see Rossow *et al.* (1980)). This fact seems suggest validity of our explanation, because, if our mechanism is effectively working, distribution of zonal winds must be approximately rigid rotation (namely, $T_3^0 \ll T_1^0$). Indeed, these observational facts can not directly exclude other explanations. However, these facts could not be explained by other explanations. Hence it can be said that our explanation for the four-day circulation is most promising one, at least in the present stage.

However, even if our theory of the four-day circulation turns out to be justified by observations in future, our theory remains insufficient for the following reasons. As already described, large ν_H is required for our mechanism to work. However, elucidation of the origin of eddies yielding large ν_H is not yet made. (Large eddies observed by Pioneer Venus, whose r.m.s. velocity is about 10 m/s, might correspond to these eddies. See Rossow *et al.* (to be published)) Then, we

must inquire into origin and properties of these eddies in a subsequent work.

Secondly, our study is insufficient for the following reason. Certainly, we have shown that a stationary state with fast zonal flow exists together with that of direct cell as a stable solution. However, this means only that, if the former solution is once chosen, then this solution continues to exist stably. But we do not know why the state corresponding to the former solution rather than the latter solution is realized in the actual Venus upper atmosphere. What is the condition under which the former solution is preferred? This very important problem is beyond the scope of the present article. We would like to attempt to answer this question in a subsequent study.

Acknowledgements

The present article is essentially the same as the author's doctoral thesis submitted to Geophysical Institute, University of Tokyo, December 1978. I would like to express my sincere thanks to Prof. Gambo for the encouragement during the course of this work and the appreciation of the present study. It is my pleasure to thank Prof. Matsuno who properly criticized my attempts and guided me to the present results. Thanks are extended to Prof. Asai, Prof. Kimura and Prof. Sato for discussions and criticisms. The author is grateful to Prof. Leovy for many valuable comments on the draft of this article. The author would like to thank Dr. Rossow. Dr. Rossow kindly provided the author with their unpublished papers. The author was much encouraged by their results of analysis of Pioneer's data. The author thanks Mrs. Kudo for her typing.

Appendix A: Derivation of basic equations

Outlines of the derivation of our basic equations are as follows. Since a Boussinesq fluid is assumed, we can start from the Navier-Stokes equation in the following form

$$\frac{\partial \mathbf{V}}{\partial t} = -\frac{1}{\rho} \text{grad } p - \frac{1}{2} \text{grad } V^2 + \mathbf{V} \times \text{rot } \mathbf{V} + \alpha g \theta \cdot \frac{\mathbf{r}}{r} + \text{div (stress tensor)} \quad (\text{A.1})$$

It must be noted that the viscosity term cannot be written as

$$\nu \Delta \mathbf{V} = -\nu \text{rot rot } \mathbf{V} + \nu \text{grad (div } \mathbf{V})$$

because of the property that eddy viscosity is not homogeneous between the vertical and horizontal

directions.

Taking rot of (A.1), we obtain

$$\frac{\partial \text{rot } \mathbf{V}}{\partial t} = \text{rot}(\mathbf{V} \times \text{rot } \mathbf{V}) + \alpha g \text{rot} \left(\frac{\mathbf{r}}{r} \theta \right) + \text{rot div (stress tensor)} \quad (\text{A.2})$$

Because of $\text{div } \mathbf{V} = 0$, $\mathbf{V}$ is expressed as a sum of $\mathbf{T}$: toroidal mode and $\mathbf{S}$: poloidal mode.

$$\mathbf{T} = \text{rot} \left(\frac{\Phi}{r} \mathbf{r} \right), \quad \mathbf{S} = \text{rot rot} \left(\frac{\Psi}{r} \mathbf{r} \right)$$

Further, we develop Φ and Ψ in a series of spherical harmonic functions;

$$\Phi = \sum_{l,m} \{ T_l^{mc}(r) Y_l^{mc}(\theta, \varphi) + T_l^{ms}(r) Y_l^{ms}(\theta, \varphi) \}$$

$$\Psi = \sum_{l,m} \{ S_l^{mc}(r) Y_l^{mc}(\theta, \varphi) + S_l^{ms}(r) Y_l^{ms}(\theta, \varphi) \},$$

where

$$Y_l^{mc}(\theta, \varphi) = P_l^m(\cos \theta) \cos m\varphi$$

$$Y_l^{ms}(\theta, \varphi) = P_l^m(\cos \theta) \sin m\varphi$$

We use Y_l^m instead of Y_l^{mc} and Y_l^{ms} for the sake of simplicity. Hence,

$$\mathbf{T} = \sum_{l,m} \text{rot} \left(\frac{T_l^m Y_l^m}{r} \mathbf{r} \right) \equiv \sum_{l,m} \mathbf{T}(T_l^m(r))$$

$$\mathbf{S} = \sum_{l,m} \text{rot rot} \left(\frac{S_l^m Y_l^m}{r} \mathbf{r} \right) \equiv \sum_{l,m} \mathbf{S}(S_l^m(r))$$

(Concerning details of the mathematics see Bullard & Gellman (1954) and Chandrasekhar (1961)).

Then the advection term of (A.2) is written as follows

$$\begin{aligned} \text{rot}(\mathbf{V} \times \text{rot } \mathbf{V}) &= \sum \text{rot} \{ [\mathbf{T}(T_l^m) + \mathbf{S}(S_l^m)] \\ &\quad \times [\mathbf{S}(T_{l',m'}) + \mathbf{T}(S_{l',m'})] \} \\ &= \sum [\text{rot} \{ \mathbf{T}(T_l^m) \times \mathbf{S}(T_{l',m'}) \} \\ &\quad + \text{rot} \{ \mathbf{T}(T_l^m) \times \mathbf{T}(S_{l',m'}) \} \\ &\quad + \text{rot} \{ \mathbf{S}(S_l^m) \times \mathbf{S}(T_{l',m'}) \} \\ &\quad + \text{rot} \{ \mathbf{S}(S_l^m) \times \mathbf{T}(S_{l',m'}) \}] \end{aligned}$$

The mode equations of $T_l^m(r)$, $S_l^m(r)$ are obtained by calculating

$$\iint \{ \text{both sides of A.2} \} \times \mathbf{T} \{ 1 \cdot Y_l^m(\theta, \varphi) \} \sin \theta d\theta d\varphi$$

and

$$\iint \{ \text{both sides of A.2} \} \times \mathbf{S} \{ 1 \cdot Y_l^m(\theta, \varphi) \} \sin \theta d\theta d\varphi$$

From the former integration,

$$\begin{aligned} \frac{\partial S_l^*}{\partial t} &= - \sum_{\alpha, \beta} \{ (\mathbf{T}(T_\alpha) \mathbf{S}(T_\beta) \mathbf{T}_l) \\ &\quad + (\mathbf{T}(T_\alpha) \mathbf{T}(S_\beta^*) \mathbf{T}_l) + (\mathbf{S}(S_\alpha) \mathbf{S}(T_\beta) \mathbf{T}_l) \\ &\quad + (\mathbf{S}(S_\alpha) \mathbf{T}(S_\beta^*) \mathbf{T}_l) \} + \alpha g \Theta_l \\ &\quad + (\text{diffusion term}) \end{aligned} \quad (\text{A.3})$$

where $S_l^* = -d^2 S_l / dr^2$, α, β, γ means a pair of suffixes (l_α, m_α) , (l_β, m_β) , (l_γ, m_γ) , respectively, and $\mathbf{T}_l \equiv \mathbf{T}(1 \cdot Y_l^{m_\gamma}(\theta, \varphi))$. In the above equation

$$\begin{aligned} (\mathbf{S}(S_\alpha) \mathbf{S}(T_\beta) \mathbf{T}_l) &\equiv - \frac{1}{N_l} \iint \mathbf{T}_l \cdot \text{rot} \{ \mathbf{S}(S_\alpha) \times \mathbf{S}(T_\beta) \} \\ &\quad \times \sin \theta d\theta d\varphi \end{aligned} \quad (\text{A.4})$$

where

$$N_l = N_{l_\gamma m_\gamma} = \begin{cases} \frac{2\pi l_\gamma (l_\gamma + 1)}{2l_\gamma + 1} \frac{(l_\gamma + m_\gamma)!}{(l_\gamma - m_\gamma)!} & (\text{if } m_\gamma \neq 0) \\ \frac{4\pi l_\gamma (l_\gamma + 1)}{2l_\gamma + 1} & (\text{if } m_\gamma = 0) \end{cases}$$

Definitions of other terms expressing mode coupling are similar to (A.4). From the latter integration,

$$\begin{aligned} \frac{\partial T_l}{\partial t} &= - \sum_{\alpha, \beta} \{ (\mathbf{T}(T_\alpha) \mathbf{S}(T_\beta) \mathbf{S}_l) \\ &\quad + (\mathbf{T}(T_\alpha) \mathbf{T}(S_\beta^*) \mathbf{S}_l) + (\mathbf{S}(S_\alpha) \mathbf{S}(T_\beta) \mathbf{S}_l) \\ &\quad + (\mathbf{S}(S_\alpha) \mathbf{T}(S_\beta^*) \mathbf{S}_l) \} + (\text{diffusion term}) \end{aligned} \quad (\text{A.5})$$

Selection rules and explicit representations of nonlinear coupling such as (A.3) are given in Bullard & Gellman (1954) (in a slightly different form). In the calculations of the present case, we assume $H/R \rightarrow 0$ where R is a radius of the planet and H is a depth of the fluid. The equation for the temperature field may be derived in the similar way.

Appendix B: Derivation of simplified $S_2^0 - T_1^0$ ($\bar{T}_3^0, T_3^{0'}$) relation in the case of $\delta \ll 1$

By the use of equation (4.2), (4.3) and (4.4), we can express T_1^0 , $\bar{T}_3^0$ and $T_3^{0'}$ as a function of σ ;

$$T_1^0 = R^2 \Omega \frac{\left(\frac{3}{5} + \frac{3}{5} \gamma \delta + \frac{24}{5} \delta + \frac{12}{5} \gamma \delta^2 \right) \sigma - \frac{102}{35} \gamma \delta \sigma^2 - \frac{96}{7} \sigma^3}{\{ \delta + (\gamma + 8) \delta^2 + 4 \gamma \delta^3 \} - \frac{14}{5} \gamma \delta^2 \sigma + \frac{108}{35} \left(1 + \gamma \delta - \frac{160}{27} \delta \right) \sigma^2 + \frac{432}{35} \sigma^3} \quad (\text{B.1})$$

$$\bar{T}_3^0 = \frac{1}{2}(T_3^0{}_u + T_3^0{}_l) = R^2\Omega \frac{\frac{4}{5}\gamma\delta^2\sigma + \left(\frac{6}{5} + \frac{6}{5}\gamma\delta + \frac{8824}{315}\delta\right)\sigma^2 + \frac{24}{5}\sigma^3}{\{\delta + (\gamma + 8)\delta^2 + 4\gamma\delta^3\} - \frac{14}{5}\gamma\delta^2\sigma + \frac{108}{35}\left(1 + \gamma\delta - \frac{160}{27}\delta\right)\sigma^2 + \frac{432}{35}\sigma^3} \quad (\text{B. 2})$$

$$T_3^{0'} = T_3^0{}_u - T_3^0{}_l = R^2\Omega \frac{\left[\left\{ \frac{8}{5}\delta + \frac{8}{5}(\gamma + 8)\delta^2 + \left(\frac{32}{5}\gamma + \frac{2}{5}\gamma^2\right)\delta^3 \right\} \sigma + \left(\frac{6}{5} + \frac{9}{5}\gamma\delta + \frac{48}{5}\delta + \frac{3}{5}\gamma^2\delta^2 - \frac{64}{35}\gamma\delta^2 \right) \sigma^2 + \left(\frac{48}{7} + \frac{24}{7}\gamma\delta - \frac{352}{49}\delta \right) \sigma^3 \right]}{\{\delta + (\gamma + 8)\delta^2 + 4\gamma\delta^3\} - \frac{14}{5}\gamma\delta^2\sigma + \frac{108}{35}\left(1 + \gamma\delta - \frac{160}{27}\delta\right)\sigma^2 + \frac{432}{35}\sigma^3} \quad (\text{B. 3})$$

In the case of $\delta \ll 1$, we can neglect terms except one of the lowest power of δ in each coefficient of powers of σ in the numerators and denominators of (B.1), (B.2) and (B.3), we find the following approximate equations.

$$T_1^0 = R^2\Omega \frac{\frac{3}{5}\sigma - \frac{102}{35}\gamma\delta\sigma^2 - \frac{96}{7}\sigma^3}{\delta - \frac{14}{5}\gamma\delta^2\sigma + \frac{108}{35}\sigma^2 + \frac{432}{35}\sigma^3} \quad (\text{B. 4})$$

$$\bar{T}_3^0 = R^2\Omega \frac{\frac{4}{5}\gamma\delta^2\sigma + \frac{6}{5}\sigma^2 + \frac{24}{5}\sigma^3}{\delta - \frac{14}{5}\gamma\delta^2\sigma + \frac{108}{35}\sigma^2 + \frac{432}{35}\sigma^3} \quad (\text{B. 5})$$

$$T_3^{0'} = R^2\Omega \frac{\frac{16}{5}\delta\sigma + \frac{12}{5}\sigma^2 + \frac{96}{7}\sigma^3}{\delta - \frac{14}{5}\gamma\delta^2\sigma + \frac{108}{35}\sigma^2 + \frac{432}{35}\sigma^3} \quad (\text{B. 6})$$

Further, $(102/35)\gamma\delta\sigma^2$ can be neglected in the numerator of (B.4) since this term is always negligible. This is because, if $\sigma \ll 1/\delta$, then $3/5\sigma \gg (102/35)\gamma\delta\sigma^2$ and on the other hand if $\sigma \gg \delta^2$, then $(102/35)\gamma\delta\sigma^2 \ll (96/7)\sigma^3$. By applying the approximation of this kind to other places also, we obtain further simplified equations (4.6)~(4.8) in the text.

References

- Belton, M. J. S., G. R. Smith, D. A. Elliot, K. Klaassen, and G. E. Danielson, 1976a: Space-time relationships in the UV markings on Venus. *J. Atmos. Sci.*, **33**, 1383-1393.
- , G. Schubert and A. D. Del Genio, 1976b: Cloud patterns, waves and convection in the Venus atmosphere. *J. Atmos. Sci.*, **33**, 1394-1417.
- Boyer, C., and H. Camichel, 1961: Observations photographiques de la planète Vénus, *Annls. Astrophs.*, **24**, 531-535.
- , and ———, 1975: Etude photographique de la rotation de Vénus. *C.R. Acad. Sci. Paris t. 260*, 809-810.
- , and ———, 1967: Determination de la vitesse de rotation des Taches de Vénus. *C.R. Acad. Sci. Paris t. 264*, 990-992.
- , and P. Guerin, 1966: Mise en évidence directe, par la photographie, d'une rotation rétrograde de Vénus en 4 jours. *C.R. Acad. Sci. Paris t. 263*, 253-355.
- , and ———, 1969: Etude de la rotation rétrograde, en 4 jours, de la couche extérieure nuageuse de Vénus. *Icarus*, **11**, 338-355.
- , G. Coupinot and J. Hecquet, 1978: La rotation de la haute atmosphère de Vénus. Nouvelles déterminations. *C.R. Acad. Sci. Paris t. 2817*, 45-48.
- Bullard, E. C., and H. Gellman, 1954: Homogeneous dynamos and terrestrial magnetism. *Phi. Trans. Roy. Soc.*, **A247**, 213-278.
- Busse, F. H., 1967: On the stability of two-dimensional convection in a layer heated from below. *J. Math. Phys.*, **46**, 140-150.
- , 1972: On the mean flow induced by a thermal wave. *J. Atmos. Sci.*, **29**, 1423-1429.
- Chalikov, D. V., A. S. Monin, A. S. Safray, V. G. Turikov, and S. S. Zilitinkevich, 1975: Numerical simulation of the general circulation of the Cytherean lower atmosphere. *Icarus*, **26**, 178-208.
- Chadrsekhar, S., 1961: *Hydrodynamic and Hydro-magnetic Stability*. Clarendon Press, 645 pp.
- Elliasen, A., 1950: Slow thermally or frictionally controlled meridional circulation in circular vortex. *Astrophys. Norv.*, **5**, 19-60.
- Fels, S. B., and R. S. Lindzen, 1974: The interaction of thermally excited gravity waves with mean flows. *Geophys. Fluid Dyn.*, **6**, 149-191.
- Gierasch, P., 1975: Meridional circulation and the maintenance of the Venus atmospheric rotation. *J. Atmos. Sci.*, **32**, 1038-1044.
- Golitzyn, G. S., 1970: A similarity approach to the general circulation of planetary atmospheres. *Icarus*, **13**, 1-24.
- Kalnay de Rivas, E., 1973: Numerical models of the circulation of the atmosphere of Venus. *J. Atmos. Sci.*, **31**, 763-779.
- , 1975: Further numerical calculations of the circulation of the atmosphere of Venus. *J. Atmos. Sci.*, **32**, 1017-1024.

- Keldysh, M. V., 1977: Venus exploration with the Venera 9 and Venera 10 spacecraft. *Icarus*, **30**, 605–625.
- Leovy, C., 1964: Simple models of thermally driven mesospheric circulation. *J. Atmos. Sci.*, **21**, 327–341.
- , 1973: Rotation of the upper atmosphere of Venus. *J. Atmos. Sci.*, **30**, 1218–1220.
- Marov, M. Ya., V. W. Avduvsky, V. V. Kerzhanovich, M. K. Rozhdestvensky, N. F. Borodin, and O. L. Ryabov, 1973: Venera 8: Measurements of temperature, pressure, and wind velocity on the illuminated side of Venus. *J. Atmos. Sci.*, **30**, 1210–1214.
- Matsuda, Y. and T. Matsuno, 1978: Radiative-convective equilibrium of the Venusian atmosphere. *J. Meteor. Soc. Japan*, **56**, 1–18.
- , and ———, 1980: Four-day circulation in the Venus atmosphere. *Kagaku*, **50**, 285–293. (in Japanese)
- Murray, B. C., R. L. Wildey, and J. A. Westphal, 1963: Infrared photographic mapping of Venus through the 8–14 micron atmospheric window. *J. Geophys. Res.*, **68**, 4813–4818.
- , M. J. Belton, G. E. Danielson, M. E. Davies, D. Gault, B. Hopke, B. O’Leavy, R. G. Strom, V. Suomi and N. Trask, 1974: Venus: Atmospheric motions and structure from the Mariner 10 pictures. *Science*, **183**, 1307–1315.
- Plumb, R. A., 1975: Momentum transport by the thermal tide in the stratosphere of Venus. *Quart. J. Roy. Soc.*, **101**, 763–776.
- Pollack, J. B., and R. E. Young, 1975: Calculations of the radiative and dynamical state of the Venus atmosphere. *J. Atmos. Sci.*, **32**, 1025–1037.
- Rossow, W. B. and G. P. Williams, 1979: Large-scale motion in the Venus stratosphere. *J. Atmos. Sci.*, **36**, 377–389.
- , et al., 1980: Comments on “A three-dimensional model of dynamical processes in the Venus atmosphere.” *J. Atmos. Sci.*, **37**, 250–252.
- , et al., Cloud morphology and motions from Pioneer Venus images. (to be published)
- Sasamori, T., 1971: A numerical study of the atmospheric circulation of Venus. *J. Atmos. Sci.*, **28**, 1045–1057.
- Schubert, G. and J. A. Whitehead, 1967: Moving flame experiment with liquid mercury: Possible implications for the Venus atmosphere. *Science*, **163**, 71–72.
- , R. E. Young, and E. J. Hinch, 1971: Prograde and retrograde motion in a fluid layer: consequences for thermal diffusion in the Venus atmosphere. *J. Geophys. Res.*, **76**, 2126–2130.
- Stone, P., 1975: The dynamics of the atmosphere of Venus. *J. Atmos. Sci.*, **32**, 1005–1016.
- Thompson, R., 1970: Venus’ general circulation is a merry-go-round. *J. Atmos. Sci.*, **27**, 1107–1116.
- Traub, W. A., and N. P. Carleton, 1975: Spectroscopic observations of wind of Venus. *J. Atmos. Sci.*, **32**, 1045–1059.
- Turikov, V. J., and D. V. Chalikov, 1971: Computation of the general circulation of the Venusian atmosphere. *Izv. Atmos. Oceanic Phys.*, **7**, 705–722.
- Young, A. T., 1975: Is the four-day “rotation” of Venus illusory? *Icarus*, **24**, 1–10.
- Young, R. E., 1974: Finite amplitude thermal convection in a spherical shell. *J. Fluid Mech.*, **63**, 695–721.
- , and J. B. Pollack, 1977: A three-dimensional model of dynamical Processes in the Venus atmosphere. *J. Atmos. Sci.*, **34**, 1315–1351.
- and ———, 1980: Reply. *J. Atmos. Sci.*, **37**, 253–255.
- , and G. Schubert, 1973: Dynamical aspects of the Venus 4-day circulation. *Planet. Space Sci.*, **21**, 1563–1580.
- , ———, and K. E. Torrance, 1972: Non-linear motions induced by moving thermal waves. *J. Fluid Mech.*, **54**, 163–187.

金星大気の四日循環の力学

松 田 佳 久

東京学芸大学地学教室

金星の成層圏に見られる四日循環（四地球日で金星を一回りする高速の帯状流）の問題——いかにして高速の帯状流が生成維持されるのか、亦金星の自転は非常に遅いのにな何故夜昼間対流が卓越しないのか——を理論的に明らかにすることを試みた。その為に、Gierasch (1975) の提案した子午面循環に依る角運動量上方輸送に依る帯状流の加速のメカニズムを含んだ軸対称のモデルを作った。このメカニズムが働く為には水平粘性が十分大であることが必要であるが、水平粘性が有限であることによるこのメカニズムの阻害効果もこのモデルには含まれている。数学的には、速度場と温度場を少数の基本モードに展開して、モード間の相互作用を陽に非線型項として表現したモード方程式を作った。この非線型系の定常解を主として上下二層モデルに依り、水平粘性が無限大

の場合と有限の場合について求めた。

まず角運動量の釣り合いの式から、帯状流の流速 (U) と子午面循環の水平流速 (V) の関係を求めた。水平粘性無限大の場合は、 U は V に比例する。その比 U/V は惑星の自転周期 ($\tau\Omega$) と鉛直拡散の緩和時間 (τ_v) の比 $\tau_v/\tau\Omega$ で与えられる ($U/V \approx \tau_v/\tau\Omega$)。水平粘性が有限の場合は、 U は或 V の値に対して最大値を持ち、その最大値は水平粘性と鉛直粘性の比及び $\tau\Omega$ によって決まる。

それぞれの場合に得られた U と V の関係を経度方向の渦度方程式と連立させると解を定めることが出来る。その際、渦度方程式のソレノイダル項といかなる効果が釣り合うかに依り、解の型分類を試みた。それによると、大気の流れの型は次のように分類出来る。

金星型温度風バランス：大気の流れに依る遠心力の鉛直傾度が卓越し、それがソレノイダル項と釣り合う。

地球型温度風バランス：大気の流れに働くコリオリカの鉛直傾度が卓越し、それがソレノイダル項と釣り合う。

直接循環バランス：子午面循環に対する摩擦力が卓越し、それがソレノイダル項と釣り合う。

どの型の解が出現するかを、惑星の回転の効果を表わす $\tau\Omega/\tau_v$ と南北加熱差を表わす Gr の二つのパラメータ空間上において調べた。水平粘性が無限大の場合はどのような $\tau\Omega/\tau_v$ に対しても、 Gr が大きく成りさえすれば、金星型温度風バランスが出現する。(図2参照) 水平粘性有限の場合は、これとは違って、金星型温度風バランスの出現し得る Gr には $\tau\Omega/\tau_v$ に依る上限があり、それを越えると直接循環バランスしか存在し得ない。(図9参照) 金星型温度風バランスが解として存在する主なパラメータ領域では、同時に直接循環バランスも解として存在し得る。つまり、この領域では二つの全く異なった大気の流れ——弱い子午面循環に伴われた高速の帯状流と弱い帯状流を伴った高速の子午面循環——が同一の外的条件に対して安定な定常解として存在し得る。当然、 U, V はこの領域では多価函数となる(図7(c), 図8(c)参照)。現実の金星大気において、前者の状態は四日循環に相当し、後者の状態は夜昼間対流を意味する。

以上の結果に基づいて、Young と Pollach (1977) の数値実験の結果を再検討した。