On the Vertical Diffusion from a Ground Level Source in the Atmospheric Boundary Layer

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Abstract

Vertical diffusion from a ground level line source was estimated using the structure model of the atmospheric boundary layer proposed by Yokoyama et al. (1977a, b, c). The two-dimensional differential equation of diffusion was computed numerically for stationary and horizontally homogeneous conditions. Making the equation dimensionless, we showed that the characteristic spread ($Z\sigma$) divided by the height of the boundary layer (h), is a function of two dimensionless lengths, $x/R_e h$ and z_0/h , in the mixing and neutral layers (x : downwind distance from the source, R_e : the quantity formed from the characteristic scale of wind speed, eddy diffusivity and the height of the boundary layer, z_0 : the roughness length of the ground surface.) In the stable layer, another dimensionless length, h/L (L : Monin-Obukhov length), is needed in addition to the above two parameters.

From the relationship between $Z\sigma/h$ and $x/R_e h$, it may be shown that in the mixing layer, $Z\sigma$ is a function of traveling time of the smoke. In the neutral and stable layers, however, $Z\sigma$ is independent of the mean wind speed and depends only on the downwind distance from the source.

As for the computed concentrations, vertical spread $Z\sigma$ in the mixing layer increases rapidly in the early stage of the diffusion and tends to vary slowly when the upper part of the layer affects the diffusion. Profiles of vertical concentration in the mixing layer are nearly exponential when $Z\sigma/h \ll 1$ and approach a Gaussian distribution gradually with an increase in the downwind distance. On the other hand, in the stable layer, these profiles are nearly Gaussian even at short downwind distances and become more rounded at long distances. The estimated vertical spread $Z\sigma$ and concentrations at the ground surface are discussed, comparing them with diffusion experiments and the Pasquill-Gifford chart.

1. Introduction

Theoretical investigations concerning the diffusion process above the atmospheric surface layer or constant flux layer are generally based on a hypothetical distribution of the exchange coefficient. These investigations were not confirmed by observation of the turbulence above the surface layer. Recently, on the basis of airplane observations performed by Gamo et al. (1976), Yokoyama et al. (1977a, b, c) proposed a structure model of the turbulence in the atmospheric boundary layer. This model was later examined by Gamo and Yokoyama (1979) in the mixing layer and Yamamoto et al. (1979) in the stable

layer and appeared to be reasonable for most boundary layers except extremely stable layers. In the extremely stable layer, they suggested that more careful observation be conducted to develop the structure model.

Here, our interest is to show how the diffusion process may be described by their model. As a preliminary study, we considered the vertical diffusion from a ground level source using a Lagrangian similarity method (Yokoyama and Mizuno, 1978). By qualitative analysis, we concluded that in the mixing layer, wind speed does not contribute to the vertical diffusion but merely transports the plume in the downwind direction.

In this paper, a two-dimensional differential

equation for diffusion from a ground level source will be computed numerically and the characteristics of the vertical profile of the concentration and its dependency on the stratification of the layer will be studied using their structure model.

2. The equation of diffusion and the model of the atmospheric boundary layer

The semi-empirical steady state equation for diffusion from a continuous, ground level, line source is given by

$$U \frac{\partial C}{\partial x} = \frac{\partial}{\partial z} \left(K_z \frac{\partial C}{\partial z} \right) + Q \quad (1)$$

where C is the concentration, U the wind speed, K_z the vertical eddy diffusivity and Q the emission strength per unit length. Diffusion in the x direction was neglected compared to the advection term in equation (1). Also, the Coriolis force was ignored since only the vertical diffusion from an infinitely long line source was considered here. The following boundary condition is applied to equation (1) at the ground surface and at the top of the boundary layer.

$$\frac{\partial C}{\partial z} = 0 \quad \text{at } z=0 \text{ and } h \quad (2)$$

We assume that the eddy diffusivity K_z is proportional to the eddy diffusivity of momentum K_m , or $K_z = \alpha K_m$. Here, α is a dimensionless constant of the order of 1 and should be determined by comparing calculations and experiments. K_m is defined as

$$K_m = u_*^2 / \frac{dU}{dz} \quad (3)$$

where u_*^2 is the momentum flux per unit mass.

Profiles of U and K_m can be obtained from the Monin-Obukhov similarity hypothesis for the surface layer and the model of Yokoyama et al. (loc. cit.) for the upper part of the boundary layer.

(i) Mixing layer (free convective layer)

The mixing layer will be divided into two sublayers; the surface layer and the upper layer. In the surface layer, Monin-Obukhov similarity yields

$$K_m(z) = C_1 w_* h \left(\frac{z}{h} \right)^{4/3} \quad (4)$$

where C_1 is a universal constant and w_* is defined by

$$w_* = \left(\frac{gh}{T_0} \frac{q_0}{C_p \rho} \right)^{1/3} \quad (5)$$

where g is the acceleration of gravity, T_0 the

ambient air temperature, q_0 the latent heat flux, C_p the specific heat of air at constant pressure and ρ is the air density. Above the surface layer or in the upper convective layer, Yokoyama et al. (1977a) proposed the eddy diffusivity in the form

$$K_m = C_1' w_* h \Phi(\zeta) \quad (6)$$

and

$$\zeta = 1 - \frac{z}{h} \quad (7)$$

where C_1' is a universal constant and $\phi(\zeta)$ is a nondimensional universal function of ζ . According to the observations by Gamo et al. (1976), $\phi(\zeta)$ is approximately given by

$$\Phi(\zeta) \doteq 1 \quad (8)$$

The boundary of the surface layer and the upper convective layer is determined from the height at which K_m of the surface layer (4) equals that of the upper layer (6). Putting $C_1 = 0.75$ (Panofsky, 1974) into (4) and $C_1' = 0.034$ (Yokoyama et al., 1977a), we find that the internal boundary height, h_0 , is approximately one-tenth of the whole height of the mixing layer h , i.e.,

$$h_0 \doteq 0.1h \quad (9)$$

We assume that from the ground surface to h_0 , K_m can be found from (4) and above h_0 , K_m can be found from (5).

The wind profile may be obtained by integrating equation (3)

$$U = \int_{z_0}^z \frac{u_*^2}{K_m(z)} dz \quad (10)$$

where z_0 is the roughness length of the surface. If u_* is assumed to equal the friction velocity at the surface (u_{*0}), we have from equations (4) and (10),

$$U = \frac{3u_{*0}^2}{C_1 w_*} \left\{ \left(\frac{z_0}{h} \right)^{-1/3} - \left(\frac{z}{h} \right)^{-1/3} \right\} \quad (11)$$

Assuming that u_* decreases with height and $dU/dz = 0$ at the top of the convective layer, Yokoyama and Yoshikado (1977c) obtained a wind profile of the form

$$U = \frac{3u_{*0}^2}{C_1 w_*} \left[\left(\frac{z_0}{h} \right)^{-1/3} - \left(\frac{z}{h} \right)^{-1/3} \right] + \frac{1}{2} \left\{ \left(\frac{z_0}{h} \right)^{2/3} - \left(\frac{z}{h} \right)^{2/3} \right\} \quad (12)$$

Since the additional term $1/2 \{ (z_0/h)^{2/3} - (z/h)^{2/3} \}$ in equation (12) is quite smaller than the term $[(z_0/h)^{-1/3} - (z/h)^{-1/3}]$ in equation (11) every where when $z_0 \ll h$, then the difference between the wind profile equations (11) and (12) is con-

sidered to have only a small effect on the diffusion process. Therefore, we will choose the wind profile (12) for the following calculations.

(ii) Neutral layer

According to Yokoyama et al. (loc. cit.), the eddy diffusivity in the neutral layer is given by

$$K_m(z) = k u_{*0} z \zeta \quad (13)$$

where k is von Karman's constant ($k=0.35$, (Businger et al., 1971)). The profile of the wind speed can be obtained by substituting equation (13) into (10);

$$U = \frac{u_{*0}}{k} \left(\ln \frac{z}{z_0} - \frac{z-z_0}{h} \right) \quad (14)$$

(iii) Stable layer

The eddy diffusivity in the stable layer is given by

$$K_m(z) = \frac{k u_{*0} z}{1 + \beta z/L} \cdot \zeta \quad (15)$$

where β is the universal constant and L the Monin-Obukhov length. If $z \ll h$ or $\zeta \approx 1$, $K_m(z)$ tends to the eddy diffusivity in the constant flux layer near the surface, where the value of β was estimated experimentally to be about 5 (Webb, 1970; Businger et al., 1971). Thus, we assume that β equals 5 in the following calculation.

Substituting (15) into (10) we have

$$U = \frac{u_{*0}}{k} \left[\ln \frac{z}{z_0} - \frac{z-z_0}{h} + \beta \frac{h}{L} \frac{z-z_0}{h} - \frac{1}{2} \beta \frac{h}{L} \left\{ \left(\frac{z}{h} \right)^2 - \left(\frac{z_0}{h} \right)^2 \right\} \right] \quad (16)$$

3. Normalization

Before dealing with equation (1), we will make it dimensionless. Characteristic scales of the variables in equation (1) may be chosen as

h : thickness of the boundary layer

U_h : mean wind speed at $z=h$

K_{max} : maximum value of the eddy diffusivity
 k_z

The following dimensionless quantities will be introduced

$$\left. \begin{aligned} \hat{U} &= U/U_h \\ \hat{z} &= z/h \\ \hat{z}_0 &= z_0/h \\ \hat{K}_z &= K_z/K_{max} \\ \hat{x} &= \alpha x/(Reh) \end{aligned} \right\} \quad (17)$$

where Re is defined by

$$Re = \alpha h \cdot U_h / K_{max} \quad (18)$$

Dimensionless concentration C will also be introduced as

$$\hat{C} = C \cdot h \cdot U_h / Q \quad (19)$$

Substituting relationships (17) through (19) into equation (1) we obtain

$$\hat{U} \cdot \frac{\partial \hat{C}}{\partial \hat{x}} = \frac{\partial}{\partial \hat{z}} \left(\hat{K}_z \frac{\partial \hat{C}}{\partial \hat{z}} \right) + 1 \quad (20)$$

It appears from equation (20) that $\hat{C}$ must be of the form

$$\hat{C} = f_c(\hat{U}, \hat{K}_z, \hat{x}, \hat{z}) \quad (21)$$

Figures 1-a to 1-c are the profiles of $\hat{U}$ and $\hat{K}_m$ in the mixing, neutral and stable layers, respectively. Since $\hat{U}$ depends on $\hat{z}$, $\hat{z}_0$ and h/L , and $\hat{K}_z$ depends on $\hat{z}$ and h/L , equation (21) may be rewritten as

$$\hat{C} = f_c'(\hat{x}, \hat{z}, \hat{z}_0, h/L) \quad (22)$$

where h/L is necessary only for the stable layer. Equation (22) implies that at a distance $\hat{x}$, the vertical distribution of dimensionless concentration $\hat{C}$ is determined by the two parameters $\hat{z}_0$ and h/L . In other words, if we define the characteristic length $\hat{Z}\sigma (= Z\sigma/h)$ for the vertical spread at $\hat{x}$, $\hat{Z}\sigma$ should be written in the form

$$\hat{Z}\sigma = f\left(\hat{x}, \hat{z}_0, \frac{h}{L}\right) \equiv f\left(\frac{\alpha x}{Reh}, \frac{z_0}{h}, \frac{h}{L}\right) \quad (23)$$

Here, the first term of the right-hand side of equation (23) represents the dependency of $\hat{Z}\sigma$ on the normalized downwind distance. The other terms $\hat{z}_0$ and h/L indicate that the profile of the vertical concentration is affected by these two factors. The qualitative features of the vertical spread can be investigated using the general equation (23) for $\hat{Z}\sigma$.

(i) Mixing layer

In this layer, K_z was assumed to be constant in the upper convective layer, so that K_{max} is given by

$$K_{max} = \alpha C_1' w_* h \quad (24)$$

Substituting equation (24) into (18), we have

$$Re = \frac{U_h}{C_1' w_*} \quad (25)$$

From equation (23) and (25), $\hat{Z}\sigma$ can be written in the form

$$\hat{Z}\sigma = f\left(\frac{\alpha w_*}{U_h} \cdot \frac{x}{h}, \frac{z_0}{h}\right) \quad (26)$$

Thus, for the mixing layer, $w_*/U_h \cdot x/h$ appears

a) mixing layer

c) stable layer

Fig. 1 Profiles of normalized wind speed for $z_0=10^{-4}$ and eddy diffusivity. a) mixing layer, b) neutral layer, c) stable layer.

b) neutral layer

to be one of the characteristic factors for the development of $\hat{Z}\sigma$. The other factor, z_0/h , represents the contribution of the surface roughness on the profile of vertical concentration.

Deardorff and Willis (1975) have shown that the appropriate dimensionless time for diffusion in the mixing layer is given by

$$\hat{t} = \frac{w_*}{h} \cdot t = \frac{w_*}{h} \cdot \frac{x}{\bar{U}} \tag{27}$$

where $\bar{U}$ is the uniform mean wind speed in the mixing layer. In the limit $z_0 \rightarrow 0$, $\bar{U}$ tends to U_h , so that our $\hat{x}$ becomes equivalent to $\hat{t}$ in (27). Since w_* and $\bar{U}$ are given independently, the essential parameter for the development of the smoke spread is not the downwind distance itself but the traveling time of the smoke.

(ii) Neutral layer

The eddy diffusivity in this layer takes a maximum value at $z=1/2$ and $K_{\max}$ becomes

$$K_{\max} = \frac{\alpha}{4} k u_* o h \tag{28}$$

Substituting equation (28) into (18) and putting

$z=h$ in equation (14), we have

$$Re = \frac{4}{k} \cdot \frac{U_h}{u_{*0}} = \frac{4}{k^2} \left(\ln \frac{h}{z_0} + \frac{z_0}{h} - 1 \right) \\ \doteq \frac{4}{k^2} \left(\ln \frac{h}{z_0} - 1 \right) \quad (29)$$

Thus, from equations (23) and (29), we obtain

$$\hat{Z}_\sigma = \frac{Z_\sigma}{h} = f_z \left(\frac{u_{*0}}{U_h} \cdot \frac{\alpha x}{h}, \frac{z_0}{h} \right) \\ = f_z \left(\frac{x}{h} \cdot \frac{\alpha k^2}{\ln \left(\frac{h}{z_0} \right) - 1}, \frac{z_0}{h} \right) \quad (30)$$

As shown in equation (30), u_{*0}/U_h is only a function of $\hat{z}_0$. Therefore, $\hat{Z}_\sigma$ should be given as a function of the downwind distance once $\hat{z}_0$ is fixed. In the mixing layer, however, $\hat{Z}_\sigma$ depends on the traveling time $\hat{t}$ as mentioned above.

(iii) Stable layer

In the stable layer, the height at which $\hat{K}_z$ is at its peak value is

$$\hat{z}_{\max} = \frac{z_{\max}}{h} = \frac{L}{\beta h} \left(\sqrt{1 + \beta \frac{h}{L}} - 1 \right) \quad (31)$$

Apparently $\hat{z}_{\max}$ decreases with increasing h/L (see Figure 1-c). Substituting equation (31) into (15) we have

$$K_{\max} = \alpha k u_{*0} h \left(\frac{L}{\beta h} \right)^2 \left(\sqrt{1 + \beta \frac{h}{L}} - 1 \right)^2 \quad (32)$$

Thus, from equation (16), (18) and (32) we obtain

$$Re = \frac{1}{k^2} \left(\ln \frac{h}{z_0} - 1 + \frac{\beta h}{2L} \right) \left(\sqrt{1 + \beta \frac{h}{L}} + 1 \right)^2 \quad (33)$$

where we neglected $\hat{z}_0$ compared to 1. Since Re in the stable layer does not include the wind speed explicitly as in the neutral layer, Z_σ should only depend on the downwind distance.

4. Calculations

Numerical calculations were carried out using the Gauss-Seidel method with a central difference approximation in the vertical direction and a upwind difference in the mean wind direction. The grid length $\Delta z (= \Delta z/h)$ in the vertical direction was taken to be 0.005 and the grid length in the mean wind direction was chosen so that the ratio $\Delta z/\Delta x$ satisfies the computational stability condition. Near the source, a smaller grid was used to minimize the discretization error. The calculated concentration was evaluated by

the characteristic spread Z_σ , which is defined by the distance between the height of the maximum concentration and the height at which the concentration decreases to 60% of maximum.

The parameter

$$\bar{z} = \int_0^\infty C z dz / \int_0^\infty C dz$$

or

$$\sigma_z = \int_0^\infty C z^2 dz / \int_0^\infty C dz$$

is often used to characterize the development of vertical spread; $\bar{z}$ is meaningful for diffusion from the surface, but it cannot be a measure of spread from elevated sources. Another parameter, σ_z , is useful for either elevated or ground level sources. But if the vertical distribution of the smoke is not axisymmetric, we have to introduce one more parameter to characterize the nonsymmetry. We are presently investigating the diffusion from elevated sources and making comparisons with that from surface sources (Mizuno et al., 1980). For the vertical diffusion from elevated sources, nonsymmetry of the concentration is expected, so that above-defined Z_σ is considered suitable to represent and compare the vertical spread of diffusion from various sources heights. Z_σ is approximately equivalent to the standard deviation of the concentration, σ_z , if the distribution function is Gaussian. The profile of the vertical distribution of concentration is often approximated in the following form

$$C = C_0 \exp \left(-A \left[\frac{z}{\sigma_{zs}} \right]^s \right) \\ = \frac{Q \sigma_{zs}^s \Gamma(1/s)^{3/2}}{2U \Gamma(3/s)^{1/2}} \exp \left(-A \left[\frac{z}{\sigma_{zs}} \right]^s \right) \quad (34)$$

where

$$A = \left(\frac{\Gamma(3/s)}{\Gamma(1/s)} \right)^{s/2} \quad (35)$$

and s is a nondimensional constant (shape factor), σ_{zs} is the standard deviation of the smoke spread and C_0 is the concentration at the ground surface. From equation (34) we can derive that Z_σ is related to σ_{zs} by

$$\frac{\sigma_{zs}}{Z_\sigma} = A^{1/s} (-\ln 0.6)^{-1/s} \quad (36)$$

As shown in (36), the relation between σ_{zs} and Z_σ depends on the shape factor s .

(i) Mixing layer

Calculations carried out for four cases of $\hat{z}_0$ are shown in Fig. 2, where the relation of $\hat{z}_0$ and $\hat{C}_0$ versus $\hat{x}$ are shown. First, we compare the results with the O'Neill data (Barad, 1958). Nieuwstadt (1979) rearranged the data of the O'Neill diffusion experiments and plotted the variation of $C_0\bar{U}h/Q$ against $w_*x/\bar{U}h$, where C_0 was estimated from the integrated concentration to the lateral direction to the wind direction measured at $z=50$ cm (Fig. 3). Chaudhry and Meroney (1973) and Horst (1979) applied Lagrangian similarity models to the diffusion from a ground level source using the dimensionless diffusivity of heat $\phi_h=ku_*z/K_h$ and showed that their calculations agreed well with the O'Neill data. K_h/K_m in an extremely unstable layer is not known precisely yet but estimated to be about 2 to 3 (see Monin and Yaglom, 1971). In our study, we assume that $\sigma=K_z/K_m \simeq K_h/K_m \simeq 2$, in the mixing layer.

For most of the unstable layers of the O'Neill experiments, $\hat{z}_0$ is of the order of 10^{-5} . The solid lines in Fig. 3 are the calculations with $\alpha=2$. The dashed line is the laboratory data of Deardorff and Willis (1975). The calculated concentrations agree well with the field experiment, except for large $w_*x/\bar{U}h$ where the experimental data decrease more rapidly than the calculations. One of the reasons for this discrepancy is that

Fig. 2 Dimensionless vertical spread and normalized concentration at a ground surface as a function of dimensionless downwind distance and $\hat{z}_0$ (mixing layer).

Fig. 3 Dimensionless cross-wind integrated concentration at a ground surface as a function of $\hat{t}$ in the mixing layer. Solid lines are the calculations with $\alpha=2.0$.

the error in estimating C_0 becomes larger with increasing downwind distance, since the observed concentrations become very low. Also, as indicated by Deardorff and Willis (loc. cit.), the effect of the elevated maximum concentration might appear after long traveling time. This effect is not accounted for in our model.

$\bar{z}$ at the downwind distance $x=100$ m was estimated using the vertical concentration distribution (Nieuwstadt and Ulden, 1978). The comparison of $\bar{z}$ is shown in Fig. 4. the experimental results are somewhat smaller than the calculations. As shown in Monin and Yaglom (1971), in the limit of the free convective condition in the surface layer, $\bar{z}$ should be related to the traveling time, t , of the smoke plume by

$$\bar{z}=a\cdot\left(\frac{w_*t}{h}\right)^{3/2}\cdot h \tag{37}$$

The broken line in Fig. 4 represents (37) with the universal constant $a=1.3$.

Assuming a Gaussian distribution of material in the vertical direction, the standard deviation of spread is often estimated using measured concentration at the ground surface:

$$\sigma_{z2}=\sqrt{\frac{2}{\pi}}\frac{Q}{C_0U_e} \tag{38}$$

Fig. 4 Dimensionless mean height of diffusing material, $\bar{z}/h$, as a function of $\hat{t}$. Solid lines are the calculations with $\alpha=2.0$. Dashed line corresponds to $1.3\hat{t}^{2/3}$.

where suffix 2 indicates the shape factor $s=2$ and U_e is the effective transport wind speed. If the real profile of the concentration is not Gaussian, σ_{z2} is not equal the standard deviation σ_{zs} of the smoke. The relation between σ_{z2} and σ_{zs} is given from (34) by

$$\frac{\sigma_{z2}}{\sigma_{zs}} = \sqrt{\frac{2}{\pi}} \frac{1}{s} \left(\frac{\Gamma(1/s)^3}{\Gamma(3/s)} \right)^{1/2} \quad (39)$$

Thus from (36) and (39) we have

$$\frac{\sigma_{z2}}{Z\sigma} = \frac{2}{\sqrt{\pi}} \frac{\Gamma(1/s)}{s} \quad (40)$$

The value $\sigma_{z2}/Z\sigma$ is 1.12 when $s=1$ and 1.01 when $s=3$, so that if $1 \leq s \leq 3$, $Z\sigma$ can be approximately regarded as σ_{z2} estimated from relation (38).

The St. Louis experiment (McElroy and Pooler, 1968) gives σ_{z2} as a function of the traveling time of the diffusing matter. In the experiment, z_0 was approximately 3 m. Neither the height of the mixing layer nor the heat flux near the surface were obtained. We assume, as possible values, $w_* \approx 2$ m/s to 2.5 m/s for the unstable case and $h=2,000$ m. Fig. 5 shows the comparison of the experiments and the calculations. In the figure, the runs which contain $\sigma_{z2} > 2,000$ m were excluded. The calculated $Z\sigma$ agree fairly well with the experiments, considering the uncertainty of the convection and the scatter of the data.

Finally we compare the calculations with the Pasquill-Gifford chart, which was drawn on the

Fig. 5 Comparison of calculated $Z\sigma$ with experimental vertical spread σ_{z2} (all the data belong to modified gustiness factor classes B_1' and B_2' defined by McElroy and Pooler, 1968). Blocked-circles are the data considered to be significantly affected by a lid. Solid line $w_t=2$ m/s, dashed line $w_t=2.5$ m/s.

bases of diffusion experiments in the atmosphere. This chart is widely utilized for atmospheric environment problems and familiar to environmentalists. The characteristics of the present study can be clarified by comparison with the chart. According to equation (26), $Z\sigma$ may be obtained when h , w_*/U_h and z_0/h are fixed. Since h and w_* are given once the heat flux q_0 is fixed, then q_0 , U_h and z_0 become independent parameters. As for Pasquill's stability categories, surface wind speed and insolation are independent parameters. Thus we need to change U_h and q_0 to U_{10} (wind speed at 10 m height) and I (insolation near the surface). U_h is related to U_{10} through equation (12). The relation between the heat flux and the insolation has not been given theoretically yet. Empirically, Gamo et al. (1976) found that q_0 is approximately 40% of the insolation. Therefore, we assume $q_0=0.4 I$. Table 1 shows typical values of the stability parameters and corresponding Re numbers.

The vertical spread of the Pasquill-Gifford chart was originally given by the height (H) at which the concentration falls to 10% of the

maximum ground level concentration. The relation between H and Z_σ can be given using equations (34) and (36).

$$\frac{H}{Z_\sigma} = \left(\frac{\ln 0.1}{\ln 0.6} \right)^{1/s} = 4.6^{1/s} \tag{41}$$

The Pasquill-Gifford chart is represented by $\sigma_{z,p} \cong H/2.14$ instead of H . $\sigma_{z,p}$ equals the standard deviation of the vertical spread, if the distribution is Gaussian. From (41) we have

$$\frac{\sigma_{z,p}}{Z_\sigma} = \frac{4.6^{1/s}}{2.14} \tag{42}$$

In order to compare our results with the Pasquill-Gifford chart, it is necessary to convert Z_σ to $\sigma_{z,p}$ through (42).

Fig. 6 shows the variation of Z_σ , where as typical values, we put $h=1,000$ m and $q_0=70$ cal/m²s, referring to Table 1. Since the shape factor s is estimated to be nearly 1 for short diffusion time (see Table 3), from (42) we can infer that $2Z_\sigma$ should corresponding to $\sigma_{z,p}$. In the very unstable layer, Nieuwstadt and Ulden (1978) have shown that s is nearly 1, which is the same result as our calculation. In Fig. 6, if z_0 is in the range of 1 cm to 10 cm, our results roughly correspond to classes A to C at short distance except for $U_{10}=1$ m/s. With the increase of wind speed, $2Z_\sigma$ may become smaller than class C. If the wind speed increases in the thermally convective layer, forced convection should be induced near the ground surface. Probably this effect causes vertical diffusion to develop and Z_σ at short distances may approach the spread of the neutral condition. In our model, such a transition layer was not taken into account.

(ii) Neutral layer

In the neutral layer, $\hat{Z}_\sigma$ can be calculated once the parameter $\hat{z}_0$ is given. Fig. 7 shows $\hat{Z}_\sigma$ and

Fig. 6 Variation of vertical spread with wind speed U_{10} for $z_0=10$ cm (—) and $z_0=1$ cm (---); $h=1,000$ m and $q_0=70$ cal/m²s. (---) represents Pasquill-Gifford chart.

$\hat{C}_0$ plotted against $\hat{x}$ for three cases of $\hat{z}_0$. The relation between $\hat{Z}_\sigma$ and $\hat{x}$ is converted into the real scales of Z_σ and x , when Re and α are fixed. Re numbers in the neutral and stable layers depend on $\hat{z}_0$ and h/L , as shown in equation (33), where the neutral layer is a special case of $h/L=0$. Fig. 8 shows the variation of Re .

According to the Lagrangian similarity model (Chaudhry and Meroney, 1972; Horst, 1979), calculations with the nondimensional eddy diffusivity of heat proposed by Businger et al. (1971) fit the O'Neill experiments in the near-neutral and slightly stable layers, as well as in the unstable layer. Businger et al. (1971) found that in neutral and stable layers, $K_h/K_m=1.35$, so that at first we assumed $\alpha=K_z/K_m \cong K_h/K_m=1.35$.

Table 1 Re numbers in the mixing layer

Insolation	(ly/hr)	>50		50~25		<25	
(representative Ins.)	(ly/hr)	62.5		37.5		12.5	
equivalent heatflux q_0	(cal/m ² s)	70		42		14	
height of boundary layer	(m)	1000	500	1000	500	1000	500
wind velocity at $z=10$ m (m/s)	(representative wind velocity m/s)						
~2	1	A	17 21	A-B	21 25	B	30 36
2~3	2.5	A-B	43 52	B	51 61	C	74 89
3~5	4	B	60 72	C	72 86	C	104 125
5~6	5.5	C	86 103

Fig. 7 Dimensionless vertical spread and normalized concentration at a ground surface as a function of dimensionless downwind distance (neutral layer).

Fig. 8 Variation of Re number as a function of z_0^{-1} and h/L . (neutral and stable layers).

However, in comparison with the experimental data, it was found that somewhat larger value is preferable. Thus, we assume $\alpha = 1.55$ in near-neutral to stable layers.

Relations between Z_σ and x are shown in Fig. 9 for some typical cases of z_0 and h . We may see that at short downwind distance, Z_σ depends almost entirely on z_0 and the effect of the boundary layer thickness h can be neglected. When $Z_\sigma \geq h/2$ the reflection of the plume by the upper boundary begins to affect the development of Z_σ .

The comparison with the St. Louis experiments is shown in Fig. 10, where σ_{z2} (experiment) and Z_σ (calculation) are shown. In the experiments, only near-neutral cases are chosen. In order to compare the σ_{z2} of the experiments, the calcu-

lated Z_σ should be converted to σ_{z2} through equation (40). Since the shape factor s in the neutral layer is $1 \leq s \leq 2$, from equation (40) we may expect $Z_\sigma \approx \sigma_{z2}$. Taking into account the error of the data, the calculations agree well with the experiments.

In Fig. 9, the Pasquill-Gifford class D is also shown. In order to compare our results with the chart, we have to change Z_σ to $\sigma_{z,p}$ through equation (42) as discussed before. In the neutral layer, the shape factor s is nearly 1 at short distance and gradually increases with distance (see Table 3), so that we may expect that near the source $\sigma_{z,p} \approx 2Z_\sigma$ and $\sigma_{z,p} \approx 1.3Z_\sigma$ at long distances. The calculated $\sigma_{z,p}$ is not shown here, but qualitatively our results with z_0 1 cm to 10 cm correspond to the Pasquill-Gifford chart. One should note that the vertical profile of the concentration is not given by a simple Gaussian distribution function in the neutral layer.

The comparison with the O'Neill data will be discussed in the section on the stable layer.

(iii) Stable layer

In the stable layer, $\hat{Z}_\sigma$ may be calculated given h/L and z_0 . According to Yamamoto et al.'s observation (1979), h/L takes a value of about 1 to 5 in the stable layer. As a representative value, therefore, we put $h/L = 1$ for a slightly stable layer, $h/L = 2$ for a moderately stable layer and

Fig. 9 Variation of vertical spread as a function of roughness length of a ground surface and the height of neutral boundary layer. Solid lines without 'h' are the calculations for $h = 1,000$ m. Dashed line corresponds to Pasquill-Gifford class D.

Fig. 10 Comparison of calculated Z_σ with experimental vertical spread σ_{z2} . Points are from St. Louis study by McElroy and Pooler (1968), near-neutral cases. No. 16($\circ$), 24($\square$), 37($\triangle$), 42($\blacktriangle$), 43($\bullet$).

$h/L=5$ for a strongly stable layer.

Fig. 11 shows the resulting relations of $\hat{Z}_\sigma$ and $\hat{C}_0$ versus $\hat{x}$. Using R_e numbers shown in Fig. 8 and $\alpha (=1.55)$, we get the relations of the real scale of Z_σ and x . The case where h/L changes but h and z_0 are fixed is shown in Fig. 12. This situation corresponds to a hypothetical stable layer in which h/L increases but h remains constant. As is apparent from Fig. 12, the slope of Z_σ decreases with increasing h/L . The case where h/L and z_0 are fixed but h changes is shown in Fig. 13. We can see that Z_σ is suppressed when h decreases, whereas in the neutral layer Z_σ hardly depends on h except in the region where Z_σ becomes larger than $h/2$. This is because K_{max} decreases when h/L increases and the height of K_{max} becomes much lower than that found in the neutral layer, then the diffusion in the upper part of the layer should be much suppressed.

Now, we consider the case where only z_0 varies with h and h/L constant. The increase of Z_σ with z_0 is as large as in the neutral layer (Fig. 14). In comparison with Figs. 13 and 14, it appears that near the source, for instance $x < 0.1$ km, Z_σ depends mostly on the roughness length z_0 , is not dependent on h nor h/L .

Here we again use the O'Neil data to compare with the calculations. Since $C_0 U_{10}/Q$ can be obtained with little error from the data, we compare our results using the concentration $C_0 U_{10}/Q$ instead of Z_σ .

The height of the inversion layer was inferred from the intersection of the temperature profiles observed by the tower measurement below 16 m

Fig. 11 Dimensionless vertical spread and normalized concentration at a ground surface as a function of dimensionless downwind distance (stable layer). Solid line: $z_0=10^{-4}$, dashed line: $z_0=10^{-3}$.

Fig. 12 Variation of vertical spread with h/L for $h=200$ m and $z_0=10$ cm. Dashed lines represent Pasquill-Gifford classes E and F.

and by airborne measurement in the upper layer. Two examples of the potential temperature profiles and estimated h are shown in Fig. 15. The data of the O'Neil experiments are listed in Table 2. As for Monin-Obukhov length L , we used the values estimated by Nieuwstadt and

Fig. 13 Variation of vertical spread with the height of stable layer for $h/L=2$ and $z_0=10$ cm.

Fig. 14 Variation of vertical spread with the roughness length of a ground for $h=200$ m and $h/L=2$.

Ulden (1978). The data are classified into two cases, case 1 ($h \geq 200$ m) and case 2 ($h < 200$ m), in order to compare with the calculations.

The solid lines in Fig. 16 are the calculated relations between $C_o U_{10}/Q$ and x with $z_o=0.6$

Table 2 O'Neill data, nearly-neutral to stable cases

	RUN	h (m)	L^{-1} (m^{-1})	h/L
case 1 ($h \geq 200$ m)	21	250	0.006	1.5
	22	280	0.005	1.4
	24	330	0.004	1.3
	46	(200)	0.039	1.8
	57	—	-0.005	—
case 2 ($h < 200$ m)	14	(25)	0.600	15
	32	(20)	0.120	2.4
	36	110	0.127	14
	37	60	0.010	0.6
	59	(60)	0.095	5.7

h in parentheses are not reliable. L are the values estimated by Nieuwstadt and Ulden (1978).

Fig. 15 Potential temperature profile and estimated height of the surface inversion layer.

cm and $h=300$ m. This situation roughly corresponds to case 1. The calculations with $z_o=0.6$ cm and $h < 200$ m are shown in Fig. 17, which are to compare with case 2. Experimental results with $h/L \leq 2$, and $-1/L < 0.01$ (nearly neutral layer) fall in a rather narrow range, almost independent of h , and a good agreement with the calculations is seen. For the strongly stable layer, where $h/L \geq 5$, calculated concentration with $h \leq 60$ m becomes much larger than that with

Fig. 16 Comparison of calculated concentration at a ground surface with experimental cross-wind integrated concentration at $z=50$ cm (O'Neill data). Solid lines are calculated concentration for $h=300$ m and $z_0=0.6$ cm. No. 21($\circ$), 22($\bullet$), 24(∇), 46($\blacktriangledown$), 57($\square$). Refer to case 1 in Table 3.

Fig. 17 The same as Fig. 16 except for $h<200$ m. Refer to case 2 in Table 3.

$h(m)$	h/L	Run No.
110	14	36($\bullet$)
60		37($\blacktriangledown$), 59(∇)
25	15	14($\circ$)
20	2.5	32($\square$)

We estimate the shape factor s using the relation

$$2s = \frac{\ln(\hat{C}(z_3)) - \ln(\hat{C}(z_2))}{\ln(\hat{C}(z_1)) - \ln(\hat{C}(z_2))} \tag{43}$$

where $z_3=2\cdot z_2=4\cdot z_1$ and z_3 is chosen as $\hat{C}(z_3)=1/10\cdot\hat{C}_0$. The shape factor s of the concentration profiles in Fig. 18a to c are listed in Table 3. It is clear that in the mixing layer, the vertical profile of the short range diffusion is far from Gaussian and s increases with x .

Incidentally the distribution function is not given by a simple exponential power law. For instance, if we chose z as the height where $\hat{C}(z_3)=\hat{C}(z_0)=0.6\cdot\hat{C}_0$, instead of $\hat{C}(z_3)=0.1\cdot\hat{C}_0$, the concentration distribution near the surface is emphasized and s becomes much smaller (see Table 3). This tendency can be found in any type of stratified layer.

In the neutral layer, s is again less than 2. Interestingly, the variation of s with x is smaller than that in the mixing layer, at least over the distance investigated. The shape factor s is also dependent on the roughness length.

$h=300$ m. The calculations in such a strongly stable layer also agree well with the experiments. Compared to the Pasquill-Gifford chart, our results fall into the class E when z_0 is in the range of 1 cm to 10 cm with $200\text{ m}\leq h\leq 500\text{ m}$ and $h/L\approx 1$. Here, we used the relation $Z_\sigma\approx\sigma_{z,p}$ because the shape factor $s\approx 2$ in the stable layer (Table 3). When h decreases or h/L increases beyond the above values, the calculated vertical spreads are much more suppressed.

5. Vertical concentration distribution

In the previous sections we have discussed the characteristics of the vertical spread Z_σ . The vertical concentration distribution is, however, not characterized by Z_σ alone. Fig. 18a, b and c show typical dimensionless concentration distributions in mixing, neutral and stable layers, respectively. The dashed lines in the figures correspond to the Gaussian distribution function which coincides with the calculated concentration at the ground ($\hat{C}_0$) and at the height where the concentration $\hat{C}$ decreases to one tenth of $\hat{C}_0$.

In the stable layer, we can expect the profile of the vertical concentration to have a roundish shape even near the source, since the diffusion in the upper part of the layer is strongly suppressed. In fact, the shape factor s in the stable layer is as large as 1.7 over the short range of diffusion and increases beyond 2.0 at long downwind distances. With an increase of the stability h/L , s also increases. Nieuwstadt and Ulden's

result (1978) about the dependency of the shape factor s on the stability is in qualitative agreement with our calculations. The dependency of s on the downwind distance and the roughness length, however, has not been shown experimentally.

6. Conclusions

Vertical spread from a ground level source has been studied using Yokoyama et al.'s turbulent structure model (1977a, b, c). The characteristic vertical spread Z_σ is, in general, written in the form

$$\hat{Z}_\sigma = \frac{Z_\sigma}{h} = f\left(\frac{\alpha x}{Reh}, \frac{z_0}{h}, \frac{h}{L}\right)$$

where h/L is necessary for the stable layer only.

Fig. 18 Profiles of normalized vertical concentration for $z_0=10^{-4}$. a) mixing layer, b) neutral layer, c) stable layer with $h/L=2$. Dashed lines are the Gaussian distribution functions which coincide with computed concentration at $z=0$ ($C=C_0$) and the height where $C=C_0/10$.

Table 3 Values of the shape factor s

<div><div><div><div></div><div>$\hat{z}_0$</div></div><div><div>$\hat{X}$</div><div>h/L</div></div></div></div>	unstable	neutral		stable	
	10^{-4}	10^{-4}	10^{-3}	10^{-4}	
		0	0	2	5
3×10^{-3}	—	—	—	1.7 (1.3)	2.2 (2.1)
10^{-2}	1.0 (0.9)	1.1 (1.0)	1.2 (1.2)	2.0 (1.5)	2.5 (2.2)
3×10^{-2}	1.7 (1.3)	1.3 (1.1)	1.5 (1.4)	2.4 (1.9)	—
10^{-1}	1.8 (1.6)	1.5 (1.2)	1.6 (1.4)	2.7 (2.4)	—

Values in parentheses are obtained using the concentration profile under the height of the characteristic vertical spread Z_σ .

Substituting the specific variables into Re , we may infer qualitative features of the vertical diffusion. For instance, it appears that Z_σ in the mixing layer depends on the traveling time of the plume, whereas in the neutral or the stable layer, Z_σ becomes a function of downwind distance. Some characteristics of the results are as follows.

In the mixing layer, Z_σ increases rather rapidly over the short range where the turbulence in the surface layer dominates the diffusion. But when Z_σ increases and the turbulence in the upper part of the layer becomes dominant, the development of Z_σ begins to slow down. In connection with this phenomenon, the concentration in the early stages of diffusion decreases very sharply up from the surface and at long downwind distance, the profile approaches a Gaussian distribution.

In the neutral layer, the roughness length of the surface is effective in the development of vertical spread. In the stable layer, since the eddy diffusivity in upper part of the layer decreases with an increase of h/L , the diffusion in the upper part of the stable layer may be strongly suppressed and the profile of the vertical concentration becomes rounded even at short distances. The development of Z_σ near the source is, however, governed mostly by the roughness of the surface as in the neutral layer. The effect of h/L becomes dominant with increasing downwind distance.

Results of the numerical calculations agree with experiments if the eddy diffusivity of material is assumed to be approximately $2 \cdot K_m$ in the unstable layer and $1.55 \cdot K_m$ in the near-neutral to stable layers.

The shape factor s defined by equation (35) is not uniquely determined in any of the stratifications. It increases with the downwind distance and the vertical extent which is taken into account to estimate s itself. In addition it depends on the stability of the layer and the roughness of the surface.

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大気境界層中における地上煙源からの鉛直拡散について

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数理計画

大気境界層に関する横山らの乱流構造モデルを用いて拡散の微分方程式を解き、地上煙源からの鉛直拡散について調べた。方程式を無次元することにより、代表的な鉛直拡散幅 ($Z\sigma$ で表わす) を境界層の高さ (h) で割った値 ($Z\sigma/h$) は混合層、中立層のとき X/Reh (X : 煙源からの風下距離, Re : 拡散係数と風速および高さについての代表値を組み合わせてできる無次元パラメータ) と z_0/h (z_0 : 地表面粗度長) で表わされることが導びかれる。安定層ではさらに h/L (L : Monin Obukhov 長さ) がもう一つの無次元パラメータになる。この結果を用いると混合層中の鉛直方向拡散幅は煙の輸送時間に依存し、中立層、安定層では風下距離だけの関数となることわかる。

計算した結果について特徴的なことは、混合層では拡散初期のころ、 $Z\sigma$ が急激に増加し後にゆるやかに増加するようになる。中立層、安定層ではこのような顕著な変化は認められない。また、混合層での濃度分布形は拡散初期に指数分布に近く、次第にガウス分布に近づくのに対し、安定層でははじめから既にガウス分布に近く、拡散するにつれて、益々丸みをおびた形になる。

モデルの妥当性は大気中における拡散実験および Pasquill-Gifford 線図と比較することにより検討した。