Evidence of Ekman Pumping Working in a Small Scale Cyclonic Vortex

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Abstract

Theoretical considerations on the Ekman pumping actions in the planetary boundary layer are made for small scale cyclonic vortex with the horizontal dimension of about 100 km. These are applied to the nocturnal small-scale cyclonic vortex which is formed in the Ekman layer of the Kanto plains.

It is analytically shown that a secondary circulation is superposed on a primary one. An upward and a radial motion maximum of the secondary circulation are estimated 2.5 cm s$^{-1}$ and 1.5 m sec$^{-1}$ respectively. Although surface friction will cause a decrease in angular momentum of the circulation in the friction layer, the relative distribution of angular momentum shows an increase in the lowest layer and a decrease above the Ekman layer. This will be a result of the work done by the cross-isobaric flows due to the secondary circulation, and will be an indication of the action of Ekman pumping.

1. Introduction

It has been theoretically shown that the variation of wind with height in the planetary boundary layer is represented by the Ekman spiral. According to the theory, the air in the Ekman layer is transported toward lower pressure areas across the isobars, and the formula for the total air transport in the layer was given by Brunt (1934). The air in the planetary boundary layer receives the work done by the horizontal pressure forces which compensates partly for the dissipation of kinetic energy by surface friction. The flow across the isobars toward lower pressures will, owing to surface friction, produce upward motion and a slow compensating flow in the opposite direction above the friction layer, where the air loses kinetic energy due to the negative work done by the horizontal pressure forces. This is a mechanism by which the dissipating effect of surface friction can be transferred to the air above the friction layer.

This process, which transports air between the friction layer and the free atmosphere above it, is called Ekman layer injection (or suction), or Ekman pumping (Tritton 1977, Ogura 1978). The converging flow in the planetary layer and the diverging one above it with upward motion between the two make a flow which is called a secondary circulation (Holton 1972, Ogura 1978). It has been shown by Charney and Eliasen (1949) that the secondary circulation driven by the viscous forces in the friction layer is a very effective mechanism to dissipate synoptic scale disturbances in the free atmosphere. It takes about a week for the mechanism to dissipate the kinetic energy of synoptic scale disturbances. Otherwise it would take about one hundred days for the effect of eddy viscosity itself dissipate them.

Concerning the action of Ekman pumping, illustrations are usually taken from nature either on a synoptic scale or on an everyday life scale: a secondary circulation developed in such a large scale disturbance as a tropical depression or that formed in a stirred teacup. Presented in this report is an observational analysis of the action of Ekman pumping formed in a small scale cyclonic vortex in the atmosphere shown by the author (Harada 1981).

2. Theoretical considerations

The vertical variation of wind in the planetary boundary layer is given by integration of the
equations of motion in a steady state with respect to height. Taylor’s derivation, which is an extension of Ekman’s work, is presented in the textbooks (e.g. Brunt 1934). The variation of wind is called Ekman spiral which shows the vertical variation of wind associated with straight isobars. When we apply it to a small scale vortex with circular isobars, it may not provide the correct estimation of the variation of wind with height. Considering a circular vortex with a horizontal dimension of 100 km, the centrifugal force is equal to or greater than the Coriolis force. Therefore, if we are interested in small scale disturbances, equation expressing the Ekman spiral must be derived from the equations of motion in which centrifugal force is not ignored.

The Ekman spiral for a small scale vortex with circular isobars was first derived by Haurwitz (1941). The result shows that the angular velocity of the earth’s rotation in the equations expressing characteristic features in the Ekman layer may be substituted for the modified one. The modification is made by adding the angular velocity relative to the earth to the angular velocity of the earth’s rotation and by subtracting the former from the latter for a cyclonic and an anticyclonic vortex, respectively. Syono (1945) proposed a different approach to the problem. According to his derivation, the Ekman spiral developing in a disturbance with circular isobars is expressed by the equation

\[ V_r = V_{gr} \exp \left( -\frac{\pi}{D_e} z \right) \sin \left( \frac{\pi}{D_e} z \right) \]  
\[ V_\theta = V_{gr} \left[ 1 - \exp \left( -\frac{\pi}{D_e} z \right) \cos \left( \frac{\pi}{D_e} z \right) \right] \]  

where \( D_e \) is the depth of the Ekman layer, \( z \) height, \( V_{gr} \) the gradient wind speed, \( V_r \) the radial component of wind velocity, \( V_\theta \) is the azimuthal-wind velocity. The depth of the Ekman layer is

\[ D_e = \pi \left[ \frac{2K}{(f + \tilde{\zeta})} \right]^{1/2} \]  

where \( K \) is the eddy viscosity coefficient, \( \tilde{\zeta} \) mean vorticity in the Ekman layer and \( f \) the Coriolis parameter. If we replace the gradient-wind velocity with the geostrophic-wind one and ignore \( \tilde{\zeta} \), equations (1)-(3) are reduced to the ordinary ones expressing the Ekman spiral which develops in a disturbance with straight isobars. When the mean value of the vorticity of a disturbance is equal to or greater than that of the Coriolis parameter, \( \sim 10^{-4} \text{sec}^{-1} \), the Ekman spiral must be expressed by equations (1)-(3).

Equation (1) expresses a flow across the isobars in the Ekman layer. Putting it into the equation of continuity and integrating it with respect to height, we obtain the vertical velocity at the top of the Ekman layer

\[ \frac{D_e}{2\pi} \zeta_{gr} = \frac{1}{2} \frac{V_{gr}}{f + \tilde{\zeta}} \left( \frac{2K}{f + \tilde{\zeta}} \right)^{1/2} \]  

Here, \( \zeta_{gr} = 1/r \partial \zeta / \partial r (rV_{gr}) \) and \( r \) is the radial distance. Replacing \( (f + \tilde{\zeta}) \) with \( f \), the depth of the Ekman layer \( D_e \) reduces to \( (2K/f)^{1/2} \), which is the expression for disturbances with straight isobars. It is readily seen that the vertical velocity at the top of the Ekman layer is proportional to the geostrophic vorticity.

Let us estimate the order of magnitude of \( W_E \) for the large scale disturbance from (4). Putting \( \zeta_{gr}, \tilde{\zeta} \sim 10^{-5} \text{sec}^{-1}, f \sim 10^{-5} \text{sec}^{-1} \) and \( K \sim 3 \times 10^4 \text{cm}^2\text{sec}^{-1} \), \( W_E \) is of the order of a millimeter per second. Since \( D_e \) in equation (3) denotes the depth of the Ekman layer, we arrive at the same order of magnitude for \( W_E \) if \( \zeta_{gr} \) and \( \tilde{\zeta} \sim 10^{-5} \text{sec}^{-1} \) and \( D_e \sim 1000 \text{m} \). Meanwhile the magnitude of \( \zeta_{gr} \) and \( \tilde{\zeta} \) for a small scale disturbance, which we are concerned, is \( 10^{-4} \text{sec}^{-1} \) (Harada 1981). Then, \( W_E \) is of the order of a centimeter per second with \( K \sim 3 \times 10^4 \text{cm}^2\text{sec}^{-1}, f \sim 10^{-4} \) or with \( D_e \sim 500 \text{m} \) and \( \zeta_{gr} \sim 10^{-4} \). The value of \( W_E \) is greater by one factor than that of the synoptic scale disturbance.

Work done by the secondary circulation against the horizontal pressure forces causes dissipation of kinetic energy of the disturbance in the free atmosphere where viscosity may be ignored. The time required for this process is called the spin-down time, which is usually defined as the time which it takes for kinetic energy of a disturbance to become \( 1/e \) times its original value. Charney and Eliassen (1949) estimated the time at 6 days for synoptic scale disturbances by integration of the simplified linearized equation of motion.

It is necessary to derive the equation for the spin-down time of small scale disturbances. The vorticity equation reduces to

\[ \frac{d}{dt} (f + \tilde{\zeta}) + (f + \tilde{\zeta}) \frac{\partial w}{\partial z} = 0 \]  

Integrating equation (5) from the top of the Ekman layer \( D_e \) to the top of the disturbance \( H \) with respect to \( z \), we obtain

\[ \frac{d\zeta_{gr}}{dt} = -\frac{f + \zeta_{gr}}{H - D_e} W_E \]  

where \( W_E \) is the work done by the secondary circulation against the horizontal pressure forces.
Here, $\zeta_{gr}$ is assumed to be independent of height and $W=0$ at $z=H$. Substituting from (4) into (6) we obtain an equation which is easily integrated, and we see that the spin-down time may be written as

$$\frac{1}{\tau} = 2\pi \cdot D e \cdot \frac{f + \zeta_{gr}}{H - D e}$$

(7)

If we replace $(f + \zeta_{gr})$ in equation (7) with $f$, we obtain the spin-down time of a synoptic scale disturbance in the free atmosphere (Holton 1977). If $D e \approx 500$ m, $H \approx 1$ km and $\zeta_{gr} \approx 10^{-4}$ sec$^{-1}$, $\tau$ for a small scale cyclonic vortex will be of the order of ten hours, which is about one-tenth of that for synoptic scale disturbances.

3. An analysis of the secondary circulation in a cyclonic vortex

A cyclonic circulation is formed by night in the planetary boundary layer of the Kanto plains when the surface pressure gradient is weak. In the lowest layer, surface friction produces an inflow, which requires upward motion and a slow compensating outward flow by continuity. These flows constitute a secondary circulation superposed on the primary one. It has been shown by Harada (1981) that this secondary circulation seems to exhibit the action of Ekman pumping. An analysis of the secondary circulation will be shown.

The study utilizes wind data collected during a special experimental observation period of the South Kanto Atmospheric Environmental Experiment Project. Twenty-four observing stations take hourly pibal observation by the method of single theodolite tracking. Randomly spaced wind data are objectively interpolated on a grid spaced 6.3 km horizontally and 100 m vertically by the correction method (Harada 1981). An example of wind distribution on the grid is shown in Fig. 1. In the northwestern part of the Kanto plains a cyclonic vortex with horizontal dimension of 100 km is seen.

In Fig. 2 the mean radial cross-sections of the azimuthal velocity of the vortex are shown. The origin of radial distance is placed at the center of the cyclonic circulation in the bottom layer, but may shift a little from the center in the upper layer when the axis of the vortex is slanted. We notice that the primary circulation of the vortex has its wind peak at a height of 200-700 m where the mean azimuthal wind is as great as 5 m sec$^{-1}$ at 0200 JST.

The mean radial cross-sections of resultant wind vectors composed of radial and vertical velocity components are shown in Fig. 3. The vertical velocity is calculated from the continuity equation integrated with respect to height. The kinematic method of computing vertical motions is open to cumulative bias errors. The presence of the errors necessitates adjustment techniques. O'Brien's (1970) method, which is recommended by Smith (1971), is used in the present computation. Assuming the value of the vertical velocity at the top of the column the errors are distributed to all levels. In the present analysis the vertical velocity is assumed to vanish at 2000 m. The figures indicate a marked characteristic of the secondary circulation associated with a circular vortex: a strong inflow in the lowest layer and a compensating outflow above it with vertical motion at the central part of the vortex. The absolute values of radial velocities range from 0 to 1.5 m sec$^{-1}$ and the upward motion is as great as 2.5 cm sec$^{-1}$ when the secondary circulation is active in pumping action. This value of the vertical velocity is compatible with that estimated in the previous section for a small scale disturbance. There exists a possibility that an assumption of axial symmetry is improper to describe the real vortex which does not always seem to be axial symmetry. This assumption seems, however, to be applicable to the present analysis, since calculated velocity distributions...
Fig. 2 Mean radial cross-sections of the azimuthal velocity (m sec\(^{-1}\)) of the vortex, August 4 to 5, 1975.

Fig. 3 Mean radial cross-sections of resultant wind vectors composed of radial and vertical velocity components, August 4 to 5, 1975. Broken lines indicate the level where the wind is parallel to the gradient wind. In the vector representation, grid intervals in the vertical and radial directions correspond to 2.5 cm sec\(^{-1}\) and 1.5 m sec\(^{-1}\), respectively.

It is clearly shown in Fig. 3 that there exists a level at which a radial component of the wind velocity of the primary circulation vanishes in the mean radial cross-sections. Early in the evening the level stays at around 500 m, and gradually comes down until it reaches to 200-300 m at dawn. Since the wind approaches its gradient-wind value at the level, it can be regarded as the top of the Ekman layer. Therefore, we see that the depth of the Ekman layer is of the
order of several hundred meters and gradually decreases with time.

Fig. 3 shows that the wind continues to flow inward in the lowest layer. This will contribute to intensification of the cyclonic circulation in that layer. On the other hand, surface friction tends to attenuate the circulation in the layer. In addition to these actions a mechanism of generating or breaking down the cyclonic vortex might be operating. In order to see the relative effect of these actions the vertical time section of the angular momentum is shown in Fig. 4. The shape of the vortex is assumed to be a right cylinder with a depth of 1500 m which is divided into disks of 100 m in depth. The ratio of the angular momentum of the cylinder on that of the disks is shown in percentages.

The level of the maximum ratio, which is situated at 400 m in the evening, comes down to 200 m at dawn. The ratio slightly increases in the layer of the lowest few hundred meters with time and decreases in the layer just above the level of the maximum ratio. This is indicative of a very interesting fact. Although surface friction tends to decrease the circulatory motion of the bottom layer, it is not the motion of that layer, but that above the Ekman layer that is attenuated in advance relatively in terms of vertical distribution. In the lower layer the circulatory motion tends to be intensified by the cross-isobaric flow to lower pressure against the dissipation by surface friction. Above the Ekman layer the air receives negative work done by the cross-isobaric flow to higher pressure. Therefore, although whole angular momentum of the vortex increases until 2200 JST and then decreases, the primary circulation tends to be dissipated earlier in the upper layer than in the Ekman layer.

As for the spin-down time of a small cyclonic vortex with the circular isobars, we estimated it at the order of ten hours. The disturbance studied in this report has the life span of about ten hours. It has been shown (Harada 1981) that the life span of the cyclonic vortices formed in the lower layer of the Kanto plains by night range from a few hours to ten. The observed life span of the vortices is of the same order of magnitude expected from the theory. However, small scale disturbances in the real atmosphere are not only dissipated by the action of the Ekman pumping, but also are subject to external forces which might tend to break them down. Therefore, the observed life span of the cyclonic circulation will not necessarily be a measure of the spin-down time of the circulation.

4. Summary

It has been analytically shown that a secondary circulation superposed on a primary one is formed in a cyclonic vortex with a horizontal dimension of about 100 km. This circulation is composed of a radial flow toward the center of the vortex in the lowest layer, a return flow above it and an upward flow in the vortex. When the secondary circulation is active in motion, an upward motion maximum of 2.5 cm sec$^{-1}$ together with a radial motion maximum of 1.5 m sec$^{-1}$ is observed.

The order of magnitude of the vertical velocity at the top of the Ekman layer associated with a small scale disturbance is theoretically estimated at a few centimeters per second. This value is ten times as large as that of a synoptic scale disturbance and is quite compatible with that of upward motion derived by the present analysis. Also, the spin-down time of a small scale cyclonic vortex is estimated at the order of ten hours.

Dissipation of the angular momentum of the cyclonic circulation found in the Ekman layer and in the layer just above it is investigated. Although surface friction will cause a decrease in angular momentum in the friction layer, the relative distribution of angular momentum shows an increase in the lowest layer and a decrease above the Ekman layer. This will be a result of the work done by the cross-isobaric flows due to the secondary circulation, and will be an indication of the action of Ekman pumping.

Fig. 4 Vertical time section of the ratio (percentage) of the angular momentum of the vortex on that of disks of 100 m in depth, August 4 to 5, 1975.
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