Evolution of Raindrops in an Axisymmetric Cumulus Model

Part I. Comparison of the Parameterized with Non-Parameterized Microphysics

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Abstract

A numerical simulation study on the evolution of raindrops in a maritime warm cumulus is presented by taking interaction of dynamical and microphysical processes into consideration. The model is two-dimensional with axisymmetric feature of deep moist convection. Microphysical processes considered in the model are condensation of water vapour on condensation nuclei, diffusional growth of droplets, stochastic coalescence, evaporation, sedimentation, and drop breakup.

In Part I, the effect of the way of treatment of water drops on the aspect of precipitation as well as on the overall behavior of a model precipitating convective cloud with detailed microphysical processes is investigated. Characteristic features of the parameterized microphysics model used Berry's and Kessler's parameterizations are also examined with emphasis on the comparison with the detailed microphysics model.

The result shows that the way of treatment of water drops has a significant effect on the evolutionary process of raindrops as well as on the dynamical aspect of the cloud. A smaller number of size-groups of water drops considerably distorts physically meaningful features of various aspects of clouds, leading to an exceedingly earlier onset of rainfall and a spuriously high rainfall efficiency through the life time. This is due to a numerical spreading of the droplet spectrum.

The parameterized microphysics model in combination with the Marshall-Palmer distribution for raindrop sizes also fairly distorts the result and gives an exceedingly high rainfall efficiency as well as a spuriously earlier onset of precipitation. The effect is caused by overestimation of mean terminal velocities of raindrops, particularly in the developing stage of the cloud.

The present results are compared with those by other researchers.

1. Introduction

Numerical modeling studies of a precipitating convective cloud present many useful informations on the mechanism of precipitation as well as the dynamical behavior of the cloud. Ever since the role of condensed water in the life cycle of a convective cloud was clarified by Das (1964), Takeda (1966), and Srivastava (1967), the importance of interaction of dynamical and microphysical processes in the evolution of clouds has been definitely recognized through many studies (Liu and Orville, 1969; Ogura and Takahashi, 1971; Takeda, 1971; Wilhelmson and Ogura, 1972; Murray and Koenig, 1972; Soong

and Ogura, 1973; Lopetz, 1973; Shiino, 1978; etc.) although microphysical processes are in most cases are parameterized in these models. The formation of water drops in a cloud is primarily dominated by the aspect of cloud dynamics, which in turn is controlled by the drag force of water drops together with their evaporation and the overall effects of microphysical processes.

In formulating both dynamical and microphysical processes in detail in one model, researchers always encounter various kinds of difficult problems such as highly complicated microphysical processes including nuclei, mutual interaction between cloud and its environmental atmosphere,

numerical technique, computational burden, verification of the validity of the result, and so on. Because of these difficulties, microphysical processes are always treated rather simply or in parameterized forms if the main purpose of their researches is to investigate any dynamical aspect of the cloud. However, if their major concerns are the effect on the dynamics of microphysical processes, or the mechanism of precipitation in clouds, microphysical processes are treated in detail with discrete size-distributions of water drops (Danielsen *et al.*, 1972; Ogura and Takahashi, 1973; Takahashi, 1973, 1974, 1975, 1976, 1981; Silverman and Glass, 1973; Soong, 1974; Takeda, 1975; Yau, 1980; Hall, 1980).

In the models with detailed microphysics, there are two major objectives. One is to clarify the role of physical processes such as mixing, turbulence, advection as well as the effect of the nucleus spectrum or updraft velocity on the broadening of the droplet size-distribution during the stage in which coalescence does not yet take place, since the condensation process tends to produce droplets of nearly uniform sizes after some tens of seconds of growth (Howell, 1949; Warner, 1969b). Another is to investigate how the size-distribution of cloud droplets with a narrow spectrum changes with time by stochastic coalescence toward a broader spectrum. This subject is always confronted with a difficult problem because of an extremely wide range of sizes of water drops from micron meters to near centimeter. This difficulty, however, was partly broken through by Berry (1967). He introduced a logarithmic spacing for the wide range of droplet mass, which made it possible to accomplish a necessary stretching of radius categories and to simplify numerical procedures. Nelson (1971) is the first to have attempted to use Berry's scheme in a numerical study on the initiation of warm rain although the dynamical framework of his cloud model is extremely simple. He stressed in the study the importance of the stochastic coalescence and the vertical water transport on the initiation of warm rain. Since then, stimulated by Nelson's successful result, Berry's algorithm was widely used, particularly in one-dimensional modeling of warm clouds (Ogura and Takahashi, 1973; Takahashi, 1973; Silverman and Glass, 1973; etc.). The attempts in multi-dimensional models have also been performed by Takahashi (1974, 1975, 1981) and Soong (1974). Takahashi focussed his attention

on a shallow warm cumulus and succeeded in a realistic simulation of the notable characteristics of tropical showers (heavy, short-lived, and localized) and of the wind gusts and wind change just before showers by slab- and axisymmetric as well as three-dimensional models with use of the Berry's scheme. Takahashi used the same size-groups of droplets as in Nelson (1971).

In a model in which stochastic process is treated, whether the model realizes physically reasonable spreading of droplet spectra with time or not essentially determines the reliability of the model. This reliability is strongly dependent upon the number of size-groups of water drops included in the model. The larger the number over a fixed radius range of droplets, is the result more realistic. In such a treatment, however, it needs a tremendous amount of computer time as well as its memories. On the other hand, in a model with a smaller number of size groups, a numerical spreading of the droplet spectrum will considerably distorts physically meaningful behavior of the model cloud. This effect has not been well investigated so far in any realistic cloud model in spite of its importance. The research by Silverman and Glass (1973) with one-dimensional model may be the only one about this subject. However, in one-dimensional model with no double cylinders, some important factors such as compensating downward motion and evaporation at cloud boundaries, and the mutual interaction between a cloud and its modified ambient atmosphere can not be incorporated. These factors often exert some crucial effects on the evolution and decay of the cloud (Asai and Kasahara, 1967; Soong, 1974) and should not be disregarded.

The main purpose of the present study is to improve understanding on the effect of the way of treatment of water drops on the aspect of precipitation as well as on the overall behavior of a model precipitating convective cloud with detailed microphysical processes. Characteristic features of the parameterized microphysics model used Berry's and Kessler's parameterizations are also examined with emphasis on the comparison with the detailed microphysics model.

2. Governing equations of the model

2.1 *A schematic representation of the interaction between cloud dynamics and microphysics*

A schematic representation of the interaction

Fig. 1 A schematic representation of interaction between cloud dynamics and microphysics in the formation and growing process of precipitation particles in a warm cumulus cloud.

of cloud dynamics with microphysics included in the present model is given in Fig. 1. In the figure, the boundary layer and the free atmosphere are differentiated by temperature and moisture profiles. The initial perturbation due to small scale convergence in the atmospheric boundary layer is given by a prescribed form.

2.2 Microphysical processes

Microphysical processes taken into consideration in the model are the formation of cloud droplets by condensation around condensation nuclei, droplet growth by water vapour diffusion, stochastic coalescence, evaporation, sedimentation, and drop breakup. The physical treatment of these processes is basically due to Ogura and Takahashi (1973). Meanwhile the presentation of the parameterized model is given in section 4.6. The model does not include ice phase.

- 1) Formation of cloud droplets around condensation nuclei

First, cloud droplets are formed by instantaneous condensation of supersaturated water vapor on condensation nuclei, which are initially assumed to be filled uniformly in the atmosphere. The droplets thus produced are assumed to take a prescribed size-distribution, which is represented with mass x of a droplet as follows:

$$f_0(x) = \frac{1}{\bar{x}_{f_0}} \exp\left(-\frac{x}{\bar{x}_{f_0}}\right). \quad (1)$$

This spectrum was used by Golovin (1963) and Berry (1967). With use of (1) the actual number of cloud droplets with mass x formed per unit time by condensation is written as

$$\left(\frac{\partial f(x)}{\partial t}\right)_{\text{con}} = \xi(r, z, t) f_0(x) / \Delta t, \quad (2)$$

where $\xi(r, z, t)$ is the number concentration of condensation nuclei. $f(x)dx$ means the number of droplets with mass x to $x+dx$ in the unit volume of air.

2) Droplet growth by water vapour diffusion

Water drops grow in supersaturated regions by water vapour diffusion. This process, however, does not change the total number of droplets and gives only minor change in their original distribution (Howell, 1949). Based on the theory of Fickian type diffusion of water vapour, the rate of change of mass x of a droplet is given by Byers (1965) as

$$\frac{dx}{dt} = \frac{4\pi R \sigma c_v}{L^2 \frac{m_w}{m_d} \frac{1}{\kappa R_g T^2} + \frac{m_d}{m_w} \frac{R_g T}{D' e_0(T)}}, \quad (3)$$

where the meanings of the various symbols are listed in Appendix. In (3), the effects of surface tension of a droplet and the molecular weight of nucleus are neglected. The ventilation coefficient c_v is given by Kinzer and Gunn (1951) as

$$c_v = 1 + 0.23 R_e^{1/2}. \quad (4)$$

With use of (3), the rate of change of the number concentration of water drops is given as

$$\left(\frac{\partial f(x)}{\partial t} \right)_c = - \frac{\partial}{\partial x} \left[f(x) \frac{dx}{dt} \right]. \quad (5)$$

3) Stochastic coalescence

After cloud droplets have grown to such sizes as they collide effectively by water vapor diffusion, the stochastic coalescence process will primarily dominate the distribution of water drops, leading to a significant broadening of the droplet spectrum. Following the Berry's (1967) notations, the stochastic coalescence equation is written as follows:

$$\begin{aligned} \left(\frac{\partial f(x)}{\partial t} \right)_{\text{coll}} &= \int_{x_0}^{x/2} dx' f(x_c) K(x_c, x') f(x') \\ &- \int_{x_0}^{\infty} dx' f(x) K(x, x') f(x'), \end{aligned} \quad (6)$$

where $x_c = x - x'$ and x_0 the smallest mass of a droplet considered. The collection kernel $K(x, x')$ is given with use of linear collision efficiency as

$$K(x, x') = \pi R_x^2 Y_c^2(R_x, R_{x'}) |V(x) - V(x')|. \quad (7)$$

The first term in the right hand side of (6) is the generation term of droplets with mass x due to coalescence and the second the loss term.

4) Sedimentation of water drops

The terminal velocity of a droplet is formulated under a basis of the experimental data by Gunn and Kinzer (1949) over the radius range greater than 40μ as follows:

$$\begin{aligned} V(R) &= 0.08 R^{1.5}, & \text{for } 40\mu \leq R < 100\mu & \quad (8a) \\ &= 0.8 R, & \text{for } 100 \leq R < 610 & \quad (8b) \end{aligned}$$

$$= 14 \times (2R)^{0.5}, \quad \text{for } 610 \leq R < 2000 \quad (8c)$$

$$= 194 R^{0.2}, \quad \text{for } 2000 \leq R. \quad (8d)$$

For droplets with radius less than 40μ , the regime of Stokes flow is applied following Ogura and Takahashi (1973), by which the terminal velocity is given as

$$V(R) = 1.26 \times 10^{-2} R^2, \quad \text{for } R < 40\mu. \quad (8e)$$

5) Evaporation of water drops

The rate of change of mass x of a droplet due to evaporation is given exactly by the same form as (3) except for the sign. In the present model, any droplet whose radius became smaller by evaporation than the smallest radius considered in the model is again treated as condensation nucleus.

6) Drop breakup

Large raindrops begin to breakup because of their internal instability against gravitation. Following Srivastava (1971), the governing equation of this process is written as

$$\begin{aligned} \left(\frac{\partial f(x)}{\partial t} \right)_B &= -f(x) P_B(x) \\ &+ \int_x^{\infty} f(x') Q_B(x', x) P_B(x') dx'. \end{aligned} \quad (9)$$

The first term in the right hand side of (9) is the loss of water drops with mass x by breakup and the second the increase due to breakup of larger raindrops than x . $P_B(x)$ is the probability of a water drop with mass x disintegrated per unit time and $Q_B(x', x)$ the number of water drops with mass x to x plus unit mass interval generated by breakup of a water drop with mass x' . The mathematical formulations of $P_B(x)$ and $Q_B(x', x)$ are designed by Srivastava (1971) as a result of a consideration of the experimental data presented by Komabayashi *et al.* (1964), which is given as follows:

$$P_B(x) = 2.94 \times 10^{-7} \exp(34 R_x), \quad (10)$$

$$Q_B(x', x) = \frac{ab}{3x} \left(\frac{R_x}{R_{x'}} \right) \exp \left(-b \frac{R_x}{R_{x'}} \right), \quad (11)$$

where both a and b are constants with $a=62.3$ and $b=7$, which satisfy mass conservation of water drops due to breakup.

Taking above-mentioned microphysical processes together with horizontal and vertical advections into consideration, the overall equation which governs the drop size-distribution in a deep convective cloud is given as follows:

$$\frac{\partial f(x)}{\partial t} = -u \frac{\partial f(x)}{\partial r} - w \frac{\partial f(x)}{\partial z} + \left(\frac{\partial f(x)}{\partial t} \right)_{\text{con}}$$

$$+\left(\frac{\partial f(x)}{\partial t}\right)_{C,E}+\left(\frac{\partial f(x)}{\partial t}\right)_{\text{coll}}+\left(\frac{\partial f(x)}{\partial t}\right)_B \\ +\frac{1}{\bar{\rho}}\frac{\partial}{\partial z}[\bar{\rho}V(x)f(x)]+D_f, \quad (12)$$

where the fourth term in the right hand side of (12) represents the rate of change of the number concentration due to either water vapour diffusion or evaporation, seventh the relative sedimentation and the last the turbulent diffusion.

The governing equation of the concentration of condensation nuclei $\xi(r, z, t)$, whose size-distribution is not taken into consideration in the present model, is expressed as

$$\frac{\partial \xi}{\partial t} = -u \frac{\partial \xi}{\partial r} - w \frac{\partial \xi}{\partial z} - \left(\frac{\partial \xi}{\partial t}\right)_{\text{con}} + \left(\frac{\partial \xi}{\partial t}\right)_E, \quad (13)$$

where the third term in the right hand side of (13) represents the loss due to the formation of cloud droplets and the fourth the gain due to evaporation of droplets. Turbulent term is disregarded in (13).

2.3 Dynamical processes

The basic design of the equations which govern the dynamics of a precipitating deep convective cloud is essentially the same as that derived by Soong and Ogura (1973), which is an extension of the anelastic system of equation for deep moist convection established by Ogura and Phillips (1962). The framework of the model is axisymmetric. In the following equations, any barred variable represents value of the environmental base state, the variable with prime symbol is the deviation from the base state, and that not barred or primed means the sum of these values.

1) Continuity equation of the air

The continuity equation of the air for deep moist convection is

$$\frac{\partial(r\bar{\rho}u)}{\partial r} + \frac{\partial(r\bar{\rho}w)}{\partial z} = 0. \quad (14)$$

2) Horizontal and vertical equations of motion, and vorticity equation

Basic equations which govern the air motion are

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial r} - w \frac{\partial u}{\partial z} - c_p \bar{\theta} \frac{\partial \pi'}{\partial r} + F_r, \quad (15)$$

$$\frac{\partial w}{\partial t} = -u \frac{\partial w}{\partial r} - w \frac{\partial w}{\partial z} - c_p \bar{\theta} \frac{\partial \pi'}{\partial z} \\ + g \left(\frac{\theta'}{\bar{\theta}} + 0.61 Q'_v - Q_c - Q_r \right) + F_z, \quad (16)$$

where F_r and F_z represent horizontal and vertical frictional forces due to eddy viscosity and are given as follows:

$$F_r = \frac{1}{r} \frac{\partial}{\partial r} \left(K_m r \frac{\partial u}{\partial r} \right) + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left(K_m \bar{\rho} \frac{\partial u}{\partial z} \right) \\ - K_m \frac{u}{r^2}, \quad (17)$$

$$F_z = \frac{1}{r} \frac{\partial}{\partial r} \left(K_m r \frac{\partial w}{\partial r} \right) + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left(K_m \bar{\rho} \frac{\partial w}{\partial z} \right), \quad (18)$$

where the eddy diffusion coefficient, K_m , is formulated by Smagorinsky (1963) as a function of deformation of the field due to the air motion as

$$K_m = (c\Delta)^2 \left\{ \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right)^2 \right. \\ \left. + 2 \left[\left(\frac{\partial u}{\partial r} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 + \left(\frac{u}{r} \right)^2 \right] \right\}^{1/2}, \quad (19)$$

where $c=0.2$ and $\Delta=(\Delta r \cdot \Delta z)^{1/2}$. The definition of the nondimensional pressure is as $\pi=(p/p_0)^{R_g/c_p}$. The reference pressure, p_0 , is assumed to be 1000 mb.

Under the following definition of vorticity

$$\eta = \frac{\partial(\bar{\rho}w)}{\partial r} - \frac{\partial(\bar{\rho}u)}{\partial z}, \quad (20)$$

the vorticity equation is written as

$$\frac{\partial \eta}{\partial t} = -u \frac{\partial \eta}{\partial r} - w \frac{\partial \eta}{\partial z} \\ + \left(\frac{2w}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial z} + \frac{u}{r} \right) \left(\eta + u \frac{\partial \bar{\rho}}{\partial z} \right) - uw \frac{\partial^2 \bar{\rho}}{\partial z^2} \\ + g \bar{\rho} \frac{\partial}{\partial r} \left(\frac{\theta'}{\bar{\theta}} + 0.61 Q'_v - Q_c - Q_r \right) + D_\eta, \quad (21)$$

where D_η is the eddy diffusion of vorticity which is given by

$$D_\eta = \frac{\partial(\bar{\rho}F_z)}{\partial r} - \frac{\partial(\bar{\rho}F_r)}{\partial z}. \quad (22)$$

Introducing a stream function defined by

$$\frac{\partial \phi}{\partial z} = -r\bar{\rho}u, \quad \frac{\partial \phi}{\partial r} = r\bar{\rho}w, \quad (23)$$

and substituting these relations into (20), the following elliptic partial differential equation is obtained:

$$\frac{1}{r} \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \phi}{\partial r} \right) = \eta. \quad (24)$$

3) Thermodynamic and water vapor equations

The thermodynamic equation in terms of potential temperature and the water vapour equation are respectively given by

$$\frac{\partial \theta'}{\partial t} = -u \frac{\partial \theta}{\partial r} - w \frac{\partial \theta}{\partial z} + \frac{L}{c_p \pi} (P_1 - P_6 - P_7) + D_{\theta'}, \quad (25)$$

$$\frac{\partial Q'_v}{\partial t} = -u \frac{\partial Q_v}{\partial r} - w \frac{\partial Q_v}{\partial z} - P_1 + P_6 + P_7 + D_{Q'_v}, \quad (26)$$

where P_1 is the condensation rate of water vapour, and P_6 and P_7 are the rate of evaporation of cloud water and rain water, respectively. The diffusion terms $D_{\theta'}$ and $D_{Q'_v}$ are expressed as

$$D_{\theta'} = \frac{1}{r} \frac{\partial}{\partial r} \left(K_{\theta} r \frac{\partial \theta'}{\partial r} \right) + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left(K_{\theta} \bar{\rho} \frac{\partial \theta'}{\partial z} \right), \quad (27)$$

and

$$D_{Q'_v} = \frac{1}{r} \frac{\partial}{\partial r} \left(K_v r \frac{\partial Q'_v}{\partial r} \right) + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} \left(K_v \bar{\rho} \frac{\partial Q'_v}{\partial z} \right). \quad (28)$$

In these equations, K_{θ} and K_v are assumed to be $K_{\theta} = K_v = K_m$.

3. Computational procedures

3.1 Grid arrangement and boundary conditions

Equations described in the previous section are numerically integrated as an initial value problem using a finite difference method. In the present study, a staggered system as shown in Fig. 2 is adopted. According to Soong and Ogura (1973), this grid system in combination with the modified upstream scheme eliminates

serious problems at the central axis of the cloud inherent in the ordinary upstream scheme. The computational domain under consideration has 6.4 km in the vertical and 8 km in the horizontal.

Boundary conditions used are rigid with free slip at all boundaries, i.e.,

$$w=0, \quad \frac{\partial \bar{\rho} u}{\partial z} = 0, \quad \text{at } z=0 \text{ and } z_H, \quad (29)$$

$$u=0, \quad \frac{\partial w}{\partial r} = 0, \quad \text{at } r=0 \text{ and } r_m. \quad (30)$$

These conditions imply $\eta=0$ and $\phi=0$ at all boundaries. Although the boundary conditions for θ , Q_v , $f(x)$, and ξ are not specified because of a staggered grid arrangement, the vertical derivatives of these variables at the vertical boundaries as well as the horizontal derivatives at the lateral boundaries are set to zero to avoid spurious diffusion at the boundaries (Soong and Ogura, 1973).

3.2 Computational scheme

The time increment Δt is taken to be $\Delta t=20$ sec as a result of several preliminary test runs within a Courant Friedrichs Lewy's stability criterion with use of $\Delta t=5$, 10 and 20 sec. The test shows little significant difference among these cases. The smaller time increment is much better for microphysical processes. It is, however, found that an exorbitant amount of computer time is required for the computation with $\Delta t=5$ sec. The mesh sizes are taken to be $\Delta r=\Delta z=400$ m, which produces 21 grid points in the horizontal and 17 in the vertical.

The computational schemes are the forward-difference in time, the modified upstream for the advective terms except for vorticity, for which an ordinary upstream scheme, and the centered difference for other space derivatives. The exact formulation of the advective term by the modified upstream scheme is to be referred to Soong and Ogura (1973).

Supersaturation is physically to be determined by the combined effects of the dynamical and microphysical processes. In the present model, however, it is assumed that supersaturation is not permitted at the end of the calculation of one time step, so that the tentative value of supersaturation is determined only by the dynamical tendency.

3.3 Scheme of numerical integration of the stochastic equation

According to many preliminary test runs, the

Fig. 2 Grid arrangement and boundary conditions used in the present study.

Berry's (1967) scheme often yields a significantly erroneous result on the conservation of total mass depending on the number of size-groups of water drops, which consequently affects the dynamics of the cloud. This is due to a difficulty of interpolation of $f(x_c)$ in (6). In the present study, therefore, a quite different scheme from Berry's in solving the stochastic equation is designed, except for a logarithmic scaling in terms of a "computational parameter" $J(J=1, 2, \dots, J_{\max})$ of the mass range of water drops, which is given by

$$x(J) = x_0 \exp[3(J-1)/J_0], \quad (31)$$

where J_0 is an adjustable scale factor of a given size-groups of water drops. With use of J , the rate of increase of the mass of a droplet due to coalescence is represented by

$$\frac{dx(J)}{dt} = \sum_{J'=1}^{J-1} f(J')K(J, J')x(J'). \quad (32)$$

Consequently, the mass $x(J)$ of a water drop is increased during the time increment Δt to

$$x'_J = x(J) + \frac{dx(J)}{dt} \Delta t, \quad (33)$$

while the number of water drops of the group J' is decreased during Δt to

$$f'_J(J') = f(J') - f(J)K(J, J')f(J')\Delta t, \quad (J'=1, 2, \dots, J-1). \quad (34)$$

The number of water drops with mass x'_J is divided into two groups surrounding x'_J when x'_J is in a range of $x(J'') \leq x'_J < x(J''+1)$ as

$$f_J(J'') = f(J) \frac{x(J''+1) - x'_J}{x(J''+1) - x(J'')}, \quad \text{for class } J''. \quad (35a)$$

$$f_J(J''+1) = f(J) \frac{x'_J - x(J'')}{x(J''+1) - x(J'')}, \quad \text{for class } J''+1. \quad (35b)$$

This procedure is repeated to all groups of $J(J=2, \dots, J_{\max})$ using the newest value of $f'(J')$ for each calculation. Thus, the new number concentration of water drops $f^{t+1}(J'')$ can be obtained as the sum of the residue of (34) and the summation of (35) with respect to all groups of J , that is,

$$\begin{aligned} f^{t+1}(J'') &= f(J'') - \sum_{J=J''+1}^{J_{\max}} f(J)K(J, J'')f(J'')\Delta t \\ &+ \sum_{J=1}^{J''} f(J) \frac{x(J''+1) - x'_J}{x(J''+1) - x(J'')} \\ &\quad (\text{for } x(J'') \leq x'_J < x(J''+1) \\ &\quad \text{and } J''=1, \dots, J_{\max}). \end{aligned} \quad (36)$$

Fig. 3 Test of the present scheme with use of the Golovin's kernel, $K(x+y)$, where x and y are mass of droplets, in solving the stochastic coalescence equation. Golovin's asymptotic solution as well as the result by the Berry's scheme is also presented.

This scheme does not need such an interpolation procedure of $f(x_c)$ as in Berry's. According to a preliminary test (Fig. 3) with use of the Golovin's kernel (Golovin, 1963), $K(x, y)$, the present scheme reasonably simulates the coalescence process under very good conservation of total mass of droplets, 0.998 at 60 min, in the light of the Golovin's asymptotic solution.

As for collision efficiencies, the theoretical results obtained by Hocking (1959) for $R \leq 30\mu$ and Shafrir and Neiburger (1963) for $30\mu < R \leq 136\mu$ are used in combination with the assumption of a fixed value equivalent to that for 136μ for droplets beyond 136μ .

3.4 Initial conditions

1) Air temperature and surface pressure

Temperature at the surface is assumed to be 301°K . In the atmospheric boundary layer up to 800 m, air temperature decreases dry adiabatically, from 800 m to 4.8 km it decreases with a lapse rate of $\Gamma = 0.63^\circ\text{K} (100 \text{ m})^{-1}$ and above 4.8 km it is assumed to be constant. Atmospheric pressure at the ground surface is assumed to be 1000 mb.

2) Moisture

Relative humidity is assumed to be 80% near the surface, 85% at 400 m, 90% at 800 m and above 800 m it decreases linearly with a lapse rate of 0.5% per 100 m.

The profiles of the air temperature and relative humidity assumed here are not so unrealistic except for the upper layer in the light of observations over the tropical and subtropical re-

gions in which warm clouds are frequently observed.

3) Mean size and dispersion of droplets formed by condensation

According to Warner (1969b), the number concentration of cloud droplets and the size-distribution formed by condensation depend on the vertical air velocity as well as the nucleus spectrum. The effect of updraft velocity, however, is not taken into consideration in the present model since any detailed treatment of the condensation process is not the major concern in this study. However, if the importance of the effect of updraft velocity on the overall behavior of the cloud is recognized in future it would be necessary to take the effect as well as turbulence into consideration.

In the present study, the mean radius of droplets $\bar{R}_{f_0}$ formed by condensation is assumed to take 10μ irrespectively of the vertical air velocity. The initial size-distribution of cloud droplets, whose normalized form is depicted in Fig. 4, has radius dispersion of 0.364, which is very close to a typical value of radius dispersion for maritime clouds observed by Warner (1969a) and other researchers around the cloud base. The number concentration of cloud droplets is prescribed by condensation nuclei, whose concentration is assumed to be 58 cm^{-3} , taking Squires' (1958) observational results into consideration.

Fig. 4 Normalized initial size-distribution of cloud droplets formed by condensation of water vapour. f_0 represents the number density and g_0 the mass density functions. $\bar{R}_{f_0}$ is the mean radius of droplets. $f_0(\ln R)$ has a radius dispersion of 0.364 close to a typical value in maritime clouds.

4) Initial disturbance

As an initial disturbance, such a small and weak circulation as with a maximum vertical air velocity of 1 m sec^{-1} at the grid point of $r=200\text{ m}$ and $z=800\text{ m}$, whose mathematical formulation is expressed by the following equation, is put into a region surrounded by $0 \leq r \leq 1.6\text{ km}$ and $0 \leq z \leq 1.6\text{ km}$,

$$\phi(r, z) = \phi_0 \left[\cos \frac{\pi(r-r_0)}{4\Delta r} \right]^2 \left[\cos \frac{\pi(z-z_0)}{4\Delta z} \right]^2, \quad (37)$$

where $r_0=z_0=800\text{ m}$. Furthermore, the region surrounded by $0 \leq r \leq 1400\text{ m}$ and $600 \leq z \leq 1800\text{ m}$ is assumed to be initially saturated.

3.5 Actual sequence of numerical integrations

The actual sequence of numerical integrations to get values at $t=\tau+1$ are briefly described by giving quantities ($u, w, \eta, \theta, Q_v, Q_c, Q_r, f(J)(J=1, 2, \dots, J_{\max}), \xi$) at $t=\tau$ as follows:

- ① $\eta^{\tau+1}$ is calculated by (21).
- ② With use of $\eta^{\tau+1}$, Poisson equation (24) is solved by the dimension reduction method (Ogura, 1969) to yield $\phi^{\tau+1}$ and consequently $u^{\tau+1}$ and $w^{\tau+1}$ at $t=\tau+1$.
- ③ The dummy values, $\theta^*, Q_v^*, f^*(J)(J=1, 2, \dots, J_{\max})$ and ξ^* at $t=\tau+1$ are computed from (25), (26), (12), and (13) by only the dynamical terms into consideration.

④ The saturation mixing ratio Q_{vs}^* corresponding to θ^* is obtained by Tetens' formula.

⑤ If $Q_v^* > Q_{vs}^*$, condensation takes place. In this article, it is assumed that supersaturated water vapour first condenses on condensation nuclei during Δt to form new cloud droplets in proportion to the concentration of condensation nuclei present. Then, if the air is not still released from supersaturation, growth of existing water drops by diffusion takes place by the equation (3) and $x'(J) = x(J) + (dx/dt)_J \Delta t$, ($J=1, \dots, J_{\max}$). In the present model, supersaturation is not permitted at $t=\tau+1$. This assumption overestimates condensation growth of droplets. The effect is indirectly examined in Fig. 5 with use of different initial mean sizes of droplets and different supersaturation ratios which is time-independent. The result suggests that the assumption of no supersaturation considerably overestimates droplet growth in a situation with a large number of small droplets but is not serious when the liquid water consists of a small number of larger drops. The exact way of adjustment of saturation is to be referred to

Fig. 5 Time variation of the droplet spectrum with different initial mean sizes under different supersaturation ratios. Initial liquid water is taken as 1 gm m^{-3} in the individual case. The left and the right are to be compared.

Soong and Ogura (1973) and Ogura and Takahashi (1973).

⑥ If $Q_v^* < Q_{vs}^*$, evaporation of water drops takes place until exact saturation is realized.

⑦ The stochastic coalescence equation (6) is solved by the scheme presented in section 3.3.

⑧ After ⑤, ⑥, and ⑦, the rearranged size-distribution by drop breakup is obtained by the following equation:

$$\frac{\partial f(J)}{\partial t} = -f(J)P_B(J) + \sum_{J'=J+1}^{J_{\max}} f(J')Q_B(J', J)P_B(J'), \quad (38)$$

which gives the size-distribution $f^{r+1}(J)$, ($J=1, 2, \dots, J_{\max}$) of water drops at $t=\tau+1$.

⑨ Finally, the concentration of condensation nuclei ξ^{r+1} is obtained by taking ⑤ or ⑥ into consideration for ξ^* .

3.6 Cases

In the present study, five cases of S11 to S121 with size-groups of 11, 16, 31, 61, and 121, respectively, over the radius range of 4 to 4096μ are studied (Fig. 6). The ratio of mass for the adjacent two droplets in the individual case is 8, 4, 2, $2^{1/2}$, and $2^{1/4}$, respectively. Water drops with radius smaller than or equal to 40μ are

Fig. 6 Size arrangement of water drops in five cases of stochastic model over the radius range of 4 to 4096 μ . The boundary value between cloud droplets and raindrops is taken to be 40 μ .

called cloud droplets and others raindrops. This division is based on a preliminary consideration on the droplet growth by water vapour diffusion, coalescence, and drop breakup with use of $\bar{R}_{f0}$ of 10 μ and σ of 0.0001 as shown in Fig. 7. The result shows that the droplets are typically divided into smaller ones and larger ones with a boundary value of radius around 40 μ .

4. Results and discussions

4.1 A preliminary test of the scheme in solving the stochastic coalescence equation

The result of a preliminary test of the scheme in solving the stochastic coalescence equation for the individual case is presented in Fig. 8 by using essentially the same initial density function as (1) but with 1 gm m⁻³ of total mass of water drops under such a hypothetical condition as with constant liquid water and with no vertical velocity in the field. The initial mass density function is given by

$$g(\ln R) = 3 \left(\frac{x}{\bar{x}_{f0}} \right)^2 \exp \left(- \frac{x}{\bar{x}_{f0}} \right). \quad (39)$$

In the test run, the mean radius of droplets $\bar{R}_{f0}$ and the time increment are taken to be $\bar{R}_{f0} = 12 \mu$ and $\Delta t = 5$ sec, respectively, for efficient coalescence.

The figure shows that the conversion is completely satisfied for all cases within a given radius range of water drops, although raindrops continue growing to larger sizes beyond the boundary of 4096 μ since in the test runs the effect of breakup is not included. The result

Fig. 7 Time variation of the droplet spectrum by the combination of diffusional growth, coalescence and breakup. The initial mean radius is taken as 10 μ and the supersaturation ratio is fixed as 0.0001. It is to be noted that the droplets are typically divided into two groups through the time with a boundary value around 40 μ .

also shows that the way of broadening of the spectrum is significantly different among the cases. The behavior in Case SC61 is comparatively similar to that in Case SC121. However, Case SC16 and Case SC31 produce larger droplets within a shorter period than that in Case SC121, leading to a much faster broadening of the droplet spectrum. This is caused by a numerical effect artificially induced, and significantly distorts physically meaningful behavior of the spectrum. In Fig. 9, the similar test runs by

Fig. 8 Droplet growth due to stochastic coalescence in five cases under such a hypothetical system as with a constant total mass of 1 gm m^{-3} throughout the computation and with no vertical velocity in the field. Initial mass density function is given by (39) in the text. The size-distribution every 10 minutes is depicted. Conservation ratio of total mass shown in the individual figure gives that at 60 min.

Berry's scheme in the use of four-point Lagrangian interpolation are presented for the size-groups of 31, 61, and 121 in comparison with the present scheme. The figure shows that the reasonable result is highly dependent upon the interpolation scheme.

4.2 The dynamical behavior of the cloud at the central axis in Case S61

In this article, Case S61 is regarded as a representative one among the five cases. Fig. 10 shows the time-height crosssection of vertical velocity at the center of the cloud in Case S61. As soon as the initial disturbance is put into the

domain, a small cloud appears and begins to develop, being fed by the latent heat released by condensation. The updraft extends upward and increases with time, reaching the maximum of 6.8 m sec^{-1} at 21 min, which is located at the level of 4.4 km. Meanwhile, at around 20 min, downdraft starts developing at rather low level (1.6 km) and extends both up- and downwards. The maximum value attains to 9.2 m sec^{-1} at 24 min, which is realized at the 2 km level. Then, the field shows a typical gravitational oscillation. The excess potential temperature (Fig. 11) also increases with time in a region

Fig. 9 Time variation of the droplet spectrum by coalescence obtained by the Berry's scheme used Lagrangian four-point interpolation. The results for the size groups of 31, 61, and 121 are presented.

with strong updraft velocity. The maximum value is 3.2°K , which is observed near the portion of the maximum updraft. In the meantime, the lower portion of the cloud is soon occupied by the negative value caused primarily by vertical advection of the air with lower potential temperature. The maximum values of cloud water and rain water through the life time are 2.1 gm kg^{-1} and 3.9 gm kg^{-1} , respectively.

The exact life time is difficult to determine because of a typical gravitational oscillation in the fields of w and θ' although the major life time of the cloud is about 30 min or so as observed in Fig. 14 and Fig. 20. However, relatively weak rainfall continues up to about 50 min. This time is taken as life time for convenience in this article.

4.3 The two-dimensional features of the cloud

The vertical crosssections of w , u , θ' , Q_c , Q_r , and ξ in the r - z plane are shown in Fig. 12a and Fig. 12b to illustrate the two-dimensional features of the cloud.

At 12 min, most of the inside of the cloud is occupied by updraft and the broad but weak compensating downward motion is observed in the ambient atmosphere. In the upper half region of the cloud, fairly strong outflow is observed, while in the lower half inflow is dominant. The positive θ' caused by condensation of water vapour is limited in the upper region. The negative θ' in the uppermost layer of the cloud is due to evaporation of droplets through the mixing of the cloudy air with the dry air, while that in the lower half region is primarily due to the effect of vertical advection of the air with lower θ . At this time, most of the droplets are very small through the layer.

At 21 min, the characteristics of the cloud are significantly different from those at 12 min. The inner region of the upper half is still occupied by updraft, while in the outer region of the upper half and in the lower half layer, typical downdrafts are observed. The upper one is due to evaporation of droplets and the lower is caused by the combination of evaporation, water loading and vertical advection of the air with lower θ . The compensating downward motion in the ambient atmosphere moved upward and in the lower atmosphere newly developed upward motion is observed. In the major portion of the cloud, inflow is dominant and outflow is split into two and the typical one is concentrated near the uppermost as well as in the lowermost layer of the cloud, suggesting a circulation of the cloudy air. The positive θ' region is still present in the upper layer of the cloud. The larger portion, however, is occupied by negative θ' because of the intensified evaporation. Most of the cloud droplets have grown to large drops with radius greater than 40μ , so that the value of Q_c is very small through the layer in contrast to the larger value of Q_r .

Fig. 10 Time-height variation of vertical velocity at the central region of the cloud in Case S61. Isoplethes are every 1 m sec⁻¹.

Fig. 11 Time-height variation of excess potential temperature from the base state at the central region of the cloud in Case S61.

Fig. 13 shows the time variation of the vertical crosssection of downward flux of Q_r (precipitation). Precipitation first appears at 12 min in the boundary region of the middle layer of the cloud and moves inward with time preferentially. As soon as it reaches the center at 18 min in the lowermost level of the cloud, precipitation begins to enhance rapidly its area as well as its intensity. This series of events typically reflects nonlinear interaction between cloud dynamics and microphysics.

If we compare the present result with those by Soong (1974) and Takahashi (1975), although

their studies are limited to shallow clouds with cloud top around 3 km, the most significant difference is in the downdraft. The maximum values of up- and downdraft in their models are around 5 and 2 m sec⁻¹, respectively. The former is realized at the cloud center and the latter is observed near the cloud boundary. On the contrary, the corresponding values in the present model are about 7 and 9 m sec⁻¹, respectively, the latter being far larger than the former. Both are realized at the cloud center. One of the reasons of this difference is that their bubble like initial impulse is so large both

Fig. 12a Vertical crosssections of w , u , θ' , Q_c , Q_r , and ξ at 12 min in r - z plane. Cloudy region is shaded. Units are m sec^{-1} for w and u , $^{\circ}\text{K}$ for θ' , g Kg^{-1} for Q_c and Q_r , and cm^{-3} for ξ .

Fig. 12b As in Fig. 12a except for 21 min.

Fig. 13 Time variation of downward flux of Q_r in the r - z plane from 12 to 27 min. Positive regions are shaded. Unit is mm hr^{-1} .

in magnitude and spatial dimension in the light of the maximum depth of the cloud that the relatively strong updraft survives for a long time until the descending dry air outside the cloud approaches the ground and cut off the supply of moist air. In their model, the downward flux of precipitation water first reaches the cloud top at the center from the cloud boundary. Another is due to the coarse grid in the present model, twice as large as that in their model, which leads to an overestimation of vertical advection of the air with lower θ and causes significant reduction of net buoyancy in the updraft region. However, the time change of the downward flux of Q_r in the present model such as the first occurrence near the cloud base at the center as well as the gross behavior of the cloud is qualitatively consistent to the result in a three-dimensional model (Takahashi, 1981).

4.4 Effect of the number of size-groups

The effect of the number of size-groups on the aspect of rainfall of the model cloud is typically shown in Fig. 14. In Case S61, it begins to rain suddenly at about 24 min and the rainfall intensity increases rapidly with time to reach the maximum of 46.9 mm hr^{-1} within a minutes or so, then decreases rapidly at first and

then gradually. The time of occurrence of the peak value of rainfall intensity becomes earlier as the number of size-groups of water drops decreases, such that it is 26 min in Case S121, 25 in S61, 24 in S31, 23.5 in S16, and 23 in S11, respectively. The maximum value itself is not so affected. This is supposed to be primarily due to the thermal structure of the initial base state. The most noticeable feature in Fig. 14 is in the difference of the time of beginning of rainfall or of the duration time from the onset of rain to the maximum. The smaller the number of size-groups in the model, the sooner is the onset of rainfall and in such a case, the rate of increase of rainfall intensity is quite small. Thus, the onset times for Case S121 and Case S11 are 25 min and 14 min, and the duration times are 1 min and 9 min, respectively, indicating significant difference between the two cases. This characteristic feature is reflected in rainfall efficiency of the cloud (Fig. 15). Rainfall efficiency is defined as

Rainfall efficiency (t)

$$= \frac{\int_0^{r_m} \int_0^t R_I(r, t) dr dt}{\int_0^{r_m} \int_0^{z_H} \int_0^t P_1(r, z, t) dr dz dt}, \tag{40}$$

Fig. 14 Time variation of rainfall intensity at the central ground surface of the cloud for five cases. CB and CK are the results of the parameterized model used Berry's and Kessler's parameterizations in combination with the Marshall-Palmer raindrop size-distribution.

Fig. 15 Time variation of rainfall efficiency of the cloud for five cases. Results of the parameterized model, CB and CK, are also illustrated.

where R_I is the rainfall intensity. In Case S61, rainfall efficiency increases rapidly with time in accordance with the onset of rainfall and then gradually with time. At the end of the life time (51 min) it shows 19.7%. Meanwhile, in other cases, it is 18% (at 50 min for Case S121), 21.6 (at 52 for S31), 23.8 (at 53 for S16), and 27.7 (at 51 for S11), respectively, showing a higher efficiency in the case with a smaller number of size-groups. The difference between Case S121 and Case S11 is 9.7%, while that between Case S61 and Case S121 is 1.7%. The cloud characteristics obtained in the present study are summarized in Table 1. The result shows that the

Table 1 Cloud characteristics obtained in the present study.

Summary of Cloud Characteristics

Parameter	Stochastic Model (Non-parameterized model)					Continuous M. (Parameterized model)	
	Number of mass classes (G.N.)					Autoconversion for.	
	121	61	31	16	11	Kessler	Berry
Max. Updraft							
W _{max} (m/s)	6.9	6.8	7.3	7.6	7.9	7.8	8.1
Height (km)	4.4	4.4	4.4	4.4	4.4	4.4	4.4
Time (min)	2.1	2.1	2.1	2.1	1.7	1.9	1.8
Max. Cloud Wat.							
Q _{rmax} (g/kg)	2.5	2.1	1.7	1.1	1.2	2.6	1.8
Height (km)	3.0	2.6	2.6	2.2	2.6	3.8	5.0
Time (min)	1.3	1.2	1.1	9	1.0	1.6	1.9
Max. Precip. Wat.							
Q _{rmax} (g/kg)	4.2	3.9	3.5	3.6	4.1	2.9	3.0
Height (km)	3.8	3.8	4.2	4.6	4.6	4.2	4.6
Time (min)	2.0	1.9	2.0	2.2	1.9	2.2	2.2
Beginning of Rain (min)	25	24	22	17	14	15	10
Rain Duration (min)	25	27	30	36	37	35	39
End of Raining (min)	50	51	52	53	51	50	49
Max. Rainfall							
Rate (mm/hr)	41.8	46.9	44.0	37.0	46.1	44.1	45.7
Time (min)	2.6	2.5	2.4	2.4	2.3	2.4	2.3
Total Rain at Axis (mm)	3.4	3.4	3.5	4.0	5.5	6.1	7.8
Rainfall Efficiency (percent)	18.0	19.7	21.6	23.8	27.7	29.0	38.2
Conservation Ra. of Moisture	1.030	1.031	1.033	1.037	1.052	1.036	1.038

way of treatment of water drops has a profound effect on the behavior of the model cloud, particularly on the aspect of rainfall. The physically meaningful behavior of the model cloud without inclusion any artificial effect in the droplet spectrum is still unknown. However, Case S121 is supposed to be fairly close to a true solution and Case S61 is also a very good approximation to the true solution since Case S121 and Case S61 give a similar tendency as shown in Fig. 14, Fig. 15, and Table 1.

It is interesting to compare the present result with that in one-dimensional model by Silverman and Glass (1973). Their model cloud is a shallow one with a cloud top of about 3 km. One of the significant difference is in the precipitation efficiency. In their model cloud, any hydrometeor once thrown away from the cloud by outflow never comes back to the cloud and is regarded to be completely evaporated in the ambient atmosphere, although the modified effect of the air is not either taken into consideration. This causes an overestimated difference in the precipitation efficiency, that is, 39 to 21.7% between the size-groups of 16 and 67, in spite of the negligible difference in the maximum updraft and the total amount of water condensed. On the other hand, the variation of precipitation efficiency in our model is 23.7 to 18.0% within the size-groups of 16 to 121, although the values themselves may be a little bit underestimated because of a slightly intensified compensating downward motion of the air along the cloud boundary, which is inherent in the axisymmetric model with rigid boundary conditions. No significant difference in the updraft as well as in the liquid water in their cloud is primarily due to the shallow depth of their cloud capped by the inversion layer so that no significant nonlinear

interaction between updraft and water loading can occur during the developing stage of the cloud, while it is noticeable in the present model (Fig. 16) because of its larger depth. However, if their model is applied to deep clouds the results would be fairly different from those in

Fig. 16 Time variations of maximum updraft and maximum downdraft for Cases S61, S11, CB, and CK.

Fig. 17 Time-height variations of cloud water (left) and rain water (right) at the central region of the cloud for Case S61 and Case S11. Isoplethes are every 0.5 gm Kg^{-1} .

multi-dimensional model since the one-dimensional model is not so powerful as in shallow clouds for the simulation of deep clouds even though it is extended to multi-cylinder model (Takahashi, 1975).

4.5 Consideration on the droplet size-distribution

Fig. 17 shows the time-height variations of cloud water and rain water at the cloud center in Case S61 and Case S11. The maximum value of cloud water is considerably different between the two cases. The difference in rain water, specially in a lower region in the early stage is to be noted. These differences are more clearly shown in Fig. 18. In Case S11, the rate of growth of cloud droplets is fairly large compared with that in Case S61, which will relatively increase net buoyancy by the reduction of water loading by sedimentation, stimulating further development of the cloud (Fig. 16).

The time variation of the droplet size-distribution at the cloud center in Case S61 are presented in Fig. 19. At the 2200 m level, the spectrum is narrow up to the mature stage. This is suggestive of their growth being controlled by water vapour diffusion. The distribution near the top of the cloud shows more and more broader spectrum as the cloud top grows upward, showing bi- or trimodal distribution (e.g., points G and L). One of the noticeable features is that large drops are significantly formed around the midlevel (3800 m), where a large amount of water is accumulated, in the light of its maximum height, although they start developing in the upper layer during the growing stage of the cloud. Because of the typical bimodal distribution at the points H and I, it is apparently indicative of the dominant effect of stochastic coalescence. Larger drops whose terminal velocities overwhelm updraft, begin to fall rapidly.

Fig. 20 shows a comparison of the droplet size-distribution (right) every 3 minutes between Case S61 and Case S11 at the indicated levels in the time-height crosssection of liquid water (left) at the central region of the cloud. The distribution at the 2200 m level in Case S61 shows clearly one simple and comparatively narrow spectrum with a peak around 20μ up to 15 min, while in Case S11, the broadening of the spectrum is noticeable even at 9 min. At 15 min, a little amount of the droplets reaches near the largest sizes. This tendency is also recognized in the upper level and the typical

Fig. 18 Time variations of the maximum values of cloud water (upper), rain water (middle), and liquid water (lower) for Case S61 and Case S11. Results of the parameterized model are also shown.

bimodal distribution shown in Case S61 is significantly obscured in Case S11. This difference, however, is comparatively small in the decaying stage (after about 24 min) of the cloud.

Fig. 21a to Fig. 21c show the droplet size-distributions in a two-dimensional space of the cloud in Case S61 and Case S11 at 12 min (typical growing stage), 18 min (mature stage), and at 24 min (decaying stage), respectively. The numerical spreading of the droplet size-distribution mentioned above can be clearly observed in Case S11 through the domain of the cloud at 12 and 18 min. The distribution at 24 min in Case S11 shows a fairly similar tendency to Case S61. However, it should be noted that the distribution in Case S61 is that just before the most intense rainfall, while in Case S11 it is just after the rainfall as shown in Fig. 14.

Fig. 19 Time-height variation of liquid water at the central region of the cloud (upper left) and time variation of the droplet size-distribution depicted by water mass every three minutes at three levels, 2200 m, 3800 m, and 5400 m, at the central region. Any symbol in the size-distribution corresponds to that in the liquid water distribution.

Fig. 20 Time-height variation of liquid water at the central region of the cloud (left) and time variation of the droplet size-distribution at the two levels shown by dashed lines in the liquid water distribution (right). Case S61 and Case S11 are compared.

Fig. 21a Droplet size-distributions at all grid points in the $r-z$ plane of the cloud at $t=12$ min (developing stage). Each square corresponds to a grid point and its horizontal and vertical scalings are the same as those in Fig. 20. Case S61 and Case S11 are compared.

Fig. 21b As in Fig. 21a except for $t=18$ min (mature stage).

4.6 Features of the parameterized microphysics model in comparison with the stochastic model

In the parameterized model, liquid water condensed is first categorized as cloud water (Q_c). As soon as the cloud water exceeds a threshold value, autoconversion process takes place. Two kinds of autoconversion equations presented by Kessler (1965) and Berry (1968), which give the

rate of growth of rain water, can be expressed as

$$\begin{aligned} \left(\frac{dQ_r}{dt}\right)_{\text{auto}} &= K1 \left(Q_c - \frac{a_0}{\bar{\rho}}\right), \text{ for } Q_c > \frac{a_0}{\bar{\rho}}, \\ &= 0, \text{ for } Q_c \leq \frac{a_0}{\bar{\rho}}, \end{aligned} \tag{41a}$$

Fig. 21c As in Fig. 21a except for $t=24$ min (decaying stage).

$$\left(\frac{dQ_r}{dt}\right)_{\text{auto}} = \frac{(\bar{\rho}Q_c)^2}{60\left(2\bar{\rho} + \frac{0.0266}{Q_c} \frac{N_b}{D_b}\right)}, \quad (41b)$$

respectively, where $K1$ is the autoconversion coefficient and a_0 the threshold cloud water content wherein autoconversion begins. The typical values of $K1$ and a_0 are $K1=10^{-3} \text{ sec}^{-1}$ and $a_0=0.5 \text{ gm m}^{-3}$, respectively. Meanwhile, in (41b) N_b is the cloud droplet concentration at the cloud base and D_b the radius dispersion of initial cloud droplets which is defined as

$$D_b = \frac{\text{standard deviation of droplet radii}}{\text{mean droplet radius}}. \quad (42)$$

The typical values of N_b and D_b for maritime clouds are 50 cm^{-3} and 0.366 , respectively (Simpson and Wiggert, 1969). The size-distribution of raindrops is assumed to always follow a prescribed form, the Marshall-Palmer (1948) spectrum. Raindrops grow to larger sizes by collecting cloud water which is distributed uniformly in the space. The collection equation is given by

$$\left(\frac{dQ_r}{dt}\right)_{\text{coll}} = \frac{130\pi}{4} \Gamma\left(\frac{7}{2}\right) (\pi\rho_w)^{-0.875} n_0^{0.125} \times \bar{E}(\bar{D}/\bar{C}) Q_c (\bar{\rho}Q_r)^{0.875}, \quad (43)$$

where Γ is the gamma function. Equation (43) is presented by Kessler (1965). With use of (41) and (43), the governing equations of cloud water and rain water in the parameterized model are given as

$$\frac{\partial Q_c}{\partial t} = -u \frac{\partial Q_c}{\partial r} - w \frac{\partial Q_c}{\partial z} + P1 - P2 - P6 + D_{Qc}, \quad (44)$$

$$\frac{\partial Q_r}{\partial t} = -u \frac{\partial Q_r}{\partial r} - w \frac{\partial Q_r}{\partial z} + \frac{1}{\bar{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \bar{V}_T Q_r) + P2 - P7, \quad (45)$$

where $P2=(dQ_r/dt)_{\text{auto}}+(dQ_r/dt)_{\text{coll}}$. In the parameterized model, evaporation of cloud water is instantaneous so long as it is present, while rain water evaporates with a finite rate (see Shiino, 1978).

Result of the parameterized model is presented in Fig. 22, which illustrates the time-height cross-section of vertical velocity at the cloud center in Case CB. The general feature is considerably similar to Case S61 except for the maximum values of updraft and downdraft as well as the times of their occurrence. The dynamical feature of the cloud in Case CK (not shown here) is similar to that in Case CB. The main characteristics of the clouds in Case CB and Case CK are listed in Table 1.

However, the aspect of precipitation in Case CB is significantly different from Case S61. As shown in Fig. 14 and Table 1, it begins to rain at 10 min in Case CB, which is the earliest among the three, while in Case CK, it is 15 min and in Case S61 24 min. The rainfall intensity in Case CB, shows a gradual change after its peak value, being in contrast to Case S61. Case CK shows an intermediate tendency between Case CB and Case S61 up to the mature stage

Fig. 22 Time-height variation of vertical velocity at the central region of the cloud for Case CB.

of the cloud. In the decaying stage, however, it is similar to Case CB.

This difference in rainfall intensity is reflected in rainfall efficiency (Fig. 15). The value in Case CB is highest among the three and in particular it overwhelms even the magnitude of Case S11 in which a significant artificial effect is included. At the end of the life time, Case CB gives precipitation efficiency of 38.2%, which is about twice as much as that in Case S61, while in Case CK it is 29%, which is about 1.5 times of Case S61.

4.7 Physical consideration on the result of the parameterized model

To investigate of the physical mechanism which enables one to explain these different features, the result of Case CB is compared with that in Case S61.

Fig. 23 shows the time-height crosssections of cloud water and rain water for both cases. In the detailed microphysics model, supersaturated water vapour first condenses on condensation nuclei to form cloud droplets as long as condensation nuclei are present. In a region with no condensation nuclei, however, it condenses directly on the surface of water drops by diffusion to increase their mass. On the other hand, in the parameterized model condensed water is always first treated as cloud water. This different treatment bears a noticeable difference in the distribution of cloud water. However, the most significant feature in Case CB which is contrast to Case S61 is the time-height variation

Fig. 23 Time-height variations of cloud water (dotted line) and rain water (solid line) in Case S61 (upper) and Case CB (lower).

of rain water, particularly in a lower half region of the cloud up to the mature stage. In Case S61 the isoplethes show a highly concentrated pattern around the region with maximum rain water, while in Case CB, rain water reaches the ground surface at considerably earlier time. This suggests that in Case CB, the downward trans-

Fig. 24 Time-height variation of the downward flux of rain water at the central region of the cloud in Case S61 (upper) and Case CB (lower). Isoplethes are every 10 mm hr^{-1} .

port of rain water is taking place at an exceedingly early time in a lower half region of the cloud.

Fig. 24 shows the time-height variation of the downward flux of rain water at the central region of the cloud for both cases. This figure substantiates the physical consideration mentioned above. In Case S61, rain water is transported upward as a whole by updraft except for a thin layer near the cloud top during the growing stage. On the contrary, in case CB it is transported downward except for a region with strong updraft during the growing stage as soon as small amount of rain water is produced. In addition to this, the highly concentrated isoplethes in Case S61 are to be noted.

The significant difference shown in Fig. 24 is caused for the most part by the different treatment of terminal velocities of raindrops. In the stochastic model, the terminal velocity of a raindrop is represented by (8a) to (8d). The mean terminal velocity is given by

$$\bar{V}_T = \frac{\int_{x_{40}}^{x_{\max}} V(x)f(x)dx}{\int_{x_{40}}^{x_{\max}} f(x)dx}, \quad (46)$$

where the numerator means the downward flux of rain water and the denominator the rain water content. On the other hand, in the parameterized model the mean terminal velocity of raindrops, whose spectrum is assumed to follow the Marshall-Palmer distribution, is expressed by (see Shiino, 1978)

$$\bar{V}_T = \frac{1}{6} \pi^{-1/8} \Gamma\left(\frac{9}{2}\right) \rho_w^{-1/8} v_0 n_0^{-1/8} (\bar{\rho} Q_r)^{1/8}. \quad (47)$$

Results of the computation of the mean terminal velocity for Case S61 and Case CB are presented in Fig. 25. In Case S61, the mean terminal velocity is extremely small during the growing stage (up to about 17 min), which is far smaller than the updraft velocity through the depth of the cloud (see Fig. 10). On the contrary, in Case CB considerably large magnitudes, which fairly exceed the updraft, are estimated in a lower portion of the cloud (at about 1 km) as soon as a small amount of rain water is produced. The magnitude increases with time and extends both upwards and downwards. This is one of the important differences between Case S61 and Case CB. Second is in the way of increase of the mean terminal velocity. In Case

Fig. 25 Time-height variation of the mean terminal velocity of raindrops at the central region of the cloud in Case S61 and Case S11. Isoplethes are every 0.5 m sec^{-1} .

S61, the rapid increase is observed in the mature stage (about 17 min to 22 min), while in Case CB it is seen in the early period (up to about 12 min) of the growing stage.

At present, the author does not know any reliable data on the vertical profile of mean terminal velocity of raindrops in clouds. It is, however, definitely unquestioned that the time when the downward flux of rain water appears at cloud base is not so early as shown in Case CB in Fig. 24 in the light of the observations by Byers and Braham (1949), in which it is typically reported to be just before the mature stage. Therefore, the result of Case CB considerably overestimates the magnitude of mean terminal velocity of raindrops, particularly in the growing stage of the cloud, which has led to a spuriously high rainfall efficiency (Fig. 15) as well as an extremely earlier onset of rainfall at the ground surface (Fig. 14).

This is due to the assumption of the Marshall-Palmer distribution for raindrops. The results presented here show that the size-distribution of raindrops doesn't follow the Marshall-Palmer distribution particularly in the growing stage of the cloud. The assumption of the Marshall-Palmer distribution, therefore, results in a considerably erroneous behavior, particularly on the aspect of precipitation, which in turn affects the dynamical aspects of the cloud as shown in Table 1. This unfavorable effect in the parameterized model in combination with the Marshall-Palmer distribution should always be taken into consideration when it is used.

The similar unfavorable characteristics are also estimated in the stochastic model with a considerably low number of size-groups of water drops. In the stochastic model, however, it is not due to overestimation of the mean terminal velocity but primarily due to the spuriously early production of large raindrops through numerical spreading of the droplet spectrum.

5. Concluding remarks

A numerical study on the evolution of raindrops in a maritime warm cumulus is presented by taking interaction of dynamical and microphysical processes into consideration. The model is axisymmetric with a framework of deep moist convection. Microphysical processes considered in the model are condensation of water vapour on condensation nuclei, diffusional growth of droplets, stochastic coalescence, evaporation, re-

lative sedimentation, and drop breakup. The main purpose in Part I is to clarify the effect of the way of treatment of water drops on the behaviour of the model cloud with detailed microphysical processes. The results of five cases with size-groups of droplets of 11 (S11), 16 (S16), 31 (S31), 61 (S61), and 121 (S121) over the same radius range of 4 to 4096μ are compared. A new integration scheme to solve the stochastic coalescence equation is presented, which is different from Berry's (1967) but strictly conserves the total mass of water drops. In the parameterized model the characteristic features of Kessler's (1965) and Berry's (1968) parameterizations in combination with the Marshall-Palmer (1948) distribution for raindrops are examined with emphasis on the comparison with the detailed microphysics model.

The result shows that the number of size-groups of water drops in the detailed microphysics model has a profound effect on the evolutionary process of raindrops. The onset of rainfall as well as its intensity, its amount, and its precipitation efficiency is, therefore, significantly affected. The cloud model with only a small number of size-groups results in an exceedingly early onset of rainfall and a spuriously high rainfall efficiency at the end of the life time. This is typically caused by the effect of numerical spreading of the droplet size-distribution. The physically meaningful behavior of the model cloud in which no artificial effect due to size-groups is included is still unknown in the present study. However, the rationality of the idea that the behavior of the cloud in Case S121 (including size-groups of 121) is very close to a physically reasonable one without serious numerical effect due to grouping of water drops and Case S61 is a very good approximation of Case S121 can be substantially supported by the present study.

The existing parameterized microphysics model also gives an exceedingly early onset of rainfall and a spuriously high rainfall efficiency at the end of the life time. This is caused by the assumption of the Marshall-Palmer distribution for raindrops, with which the mean terminal velocity of raindrops is exceedingly overestimated as to overwhelm updraft velocity, particularly in the developing stage of the cloud. Thus, the existing parameterized microphysics model significantly distorts in general physically meaningful behavior of the cloud, particularly the

aspect of precipitation. This unfavorable consequence should always be taken into consideration when one interpretes any result used these parameterizations.

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Appendix List of symbols

- a_0 : threshold cloud water content in gm m^{-3}
 c : universal constant in (19)
 c_p : specific heat of dry air at constant pressure
 c_v : ventilation coefficient
 D : droplet diameter
 D_b : radius dispersion of initial cloud droplets
 D_f : eddy diffusion of water drops
 D_{ac} : eddy diffusion of cloud water
 $D_{q'v}$: eddy diffusion of excess water vapour
 D_η : eddy diffusion of vorticity
 $D_{\theta'}$: eddy diffusion of excess potential temperature
 D' : diffusivity of water vapour in the air ($0.21 \times (P_0/P) \text{ cm}^2 \text{ sec}^{-1}$)
 $\overline{E(D/C)}$: averaged collection efficiency of raindrops for cloud water in (43)
 $e_0(T)$: saturated water vapour pressure over the plane water surface with ambient temperature T
 F_r : horizontal frictional force due to eddy viscosity
 F_z : vertical frictional force due to eddy viscosity
 $f(x)dx$: number of water drops with mass x to $x+dx$ per unit volume of air
 g : gravitational acceleration
 J : computational parameter of the mass range of water drops
 J_0 : adjustable scale factor in (31)
 $J_{\max}$: maximum computational parameter of the mass range of water drops
 $K(x, x')$: collection kernel
 K_1 : autoconversion coefficient
 K_m : eddy diffusion coefficient to momentum
 K_v : eddy diffusion coefficient of water vapour
 K_θ : eddy diffusion coefficient of potential temperature
 L : latent heat of condensation
 m_d : molecular weight of dry air
 m_w : molecular weight of water vapour
 N_b : concentration of cloud droplets at cloud base
 n_0 : intercept value (constant) in the Marshall-Palmer distribution
 P_1 : condensation rate of water vapour
 P_6 : rate of evaporation of cloud water
 P_7 : rate of evaporation of rain water
 $P_B(x)$: probability of a water drop with mass x disintegrating per unit time
 p : atmospheric pressure
 p_0 : atmospheric reference pressure
 $Q_B(x', x)$: number of water drops with mass x to x plus unit mass interval generated by breakup of x'
 Q_c : cloud water mixing ratio
 Q_r : rain water mixing ratio
 Q_v : water vapour mixing ratio
 Q_{vs} : saturated mixing ratio of water vapour
 R : radius of a droplet
 R_e : Reynolds number
 R_g : gas constant of dry air per unit mass ($2.87 \times 10^6 \text{ erg g}^{-1} \text{ K}^{-1}$)
 R_x : radius of a droplet with mass x
 r : horizontal (radial) distance
 r_m : horizontal distance of the computational domain
 T : temperature of air
 t : time
 u : horizontal (radial) velocity
 $V(x)$: terminal velocity of a water drop with mass x
 w : vertical velocity
 x : mass of a water drop
 x_0 : smallest mass of a droplet considered in the model
 x_c : mass of a counterpart which should be collide with x' to form x

$\bar{x}_{f_0}$: mean droplet mass in $f_0(x)$
 $Y_c(R_x, R_x')$: linear collision efficiency
 z : vertical distance
 z_H : height of the computational domain
 Δ : averaging scale in (19)
 Δr : horizontal grid interval
 Δt : time interval in integration
 Δz : vertical grid interval
 η : tangential component of vorticity
 θ : potential temperature
 κ : thermal conductivity ($2.4 \times 10^5 \text{ erg m}^{-1} \text{ sec}^{-1} \text{ K}^{-1}$)
 ξ : concentration of condensation nuclei
 π : nondimensional pressure
 ρ : density of air
 ρ_w : density of liquid water
 σ : supersaturation ratio
 ϕ : stream function

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軸対称積雲モデルにおける雨滴の成長

第一部 微物理過程を詳しく取り扱ったモデルと パラメータ化したモデルとの比較

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力学過程と微物理学的過程との相互作用を考慮した深い湿潤対流の軸対称モデルを用いて、暖かい海洋性積雲における雨滴の成長に関する数値シミュレーションを行った。水滴の成長に関わる微物理学的過程としては、凝結核への水蒸気凝結による雲粒の生成、水滴の水蒸気拡散による成長、併合、蒸発、落下、分裂が考慮されている。

第一部では微物理過程を詳しく取り扱った雲モデルにおける水滴の扱い方が、結果、特に降水の側面と雲の全体的な振舞に及ぼす影響が調べられる。更に、微物理過程に関する Berry と Kessler のパラメタリゼーションの特徴が、水滴の成長過程を詳しく扱ったモデルとの比較において調べられている。

数値計算の結果、水滴の扱いは、雨滴の成長過程と雲の力学的振舞に、無視できない程の影響を与えることが示される。即ち、水滴のクラスの数が多いモデルでは、雲の様々な側面において物理的に意味のある本来の振舞がゆがめられ、このため降水開始時刻が著しく早くなり、物理的に無意味な高い降水能率を評価してしまうことになる。こうした結果は水滴の粒径分布に与える数値計算上の人為的影響によってもたらされる。

また、生成された雨水（雨滴）の粒径を常に Marshall-Palmer 分布に仮定する上記のパラメタリゼーションを用いたモデルも、降水の開始と降水能率に対し同様な結果をもたらす。但しこれは上の場合と異って、特に雲の発達過程における雨滴の平均落下速度を、上記の仮定のため著しく過大評価することから起こっている。

数値計算の結果はこれまでに得られているいくつかの研究結果と比較されている。