Note on the Stability of Baroclinic Flow under Radiation Condition

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Abstract

We examine the stability of a baroclinic layer between two unbounded barotropic layers in a model neglecting *-effect. Without using geostrophic approximation, calculations are made for an extended range of the Rossby number, and for the Richardson number =0.5 and 50. For the large Rossby number the solutions in the barotropic layer may be sinusoidal (gravity wave). Then, we invoke "radiation condition" to choose between the gravity wave propagating upwards and the one propagating downwards. For the comparison, same calculations are made under the condition that horizontal rigid walls exist at both boundaries of the baroclinic layer.

The results of the calculations show that the nature of the disturbances is not drastically modified by the alteration of the boundary condition, although values of growth rate and phase velocity of the disturbances are to some extent changed.

With an increase of the Rossby number, one of two critical points (i.e. a level at which $U(z) = c = f/k$ or $-f/k$) appears in the baroclinic layer. However, for either boundary condition it is found that for more increased value of the Rossby number, steering level of the disturbances approaches to a boundary of the baroclinic layer, so that both critical points can not appear at the same time in the baroclinic layer.

1. Introduction

Since Charney (1947) and Eady (1949)'s pioneering work, the problem of baroclinic instability has been investigated by many authors. The motivation of these authors is to explain cyclogenesis in the earth's troposphere. Accordingly, the stability problem is usually solved under the boundary condition that vertical velocity vanishes at both boundaries of the baroclinic layer (assumption of horizontal rigid walls). However, an application of theory of baroclinic instability is not limited to cyclogenesis in the earth's troposphere. Up to the present day, the applications to the Jovian atmosphere (Stone (1976), Gierasch et al. (1979) and Conrath et al. (1981)) and to the middle atmosphere on the earth (e.g. Simmons (1974, 1977)) have been attempted. Although in Stone's study (1967) rigid horizontal walls were assumed to exist at the boundaries, it was pointed out that the assumption of rigid boundary condition is not suitable for the Jovian atmosphere (Gierasch et al. (1979)).

Gierasch et al. (1979) have investigated the instability of a baroclinic layer above a lower layer with a quiescent basic state. When the static stability of the lower layer is near zero (i.e. adiabatic), the growth rate of unstable waves are found to be drastically reduced. Conrath et al. (1981) extend Gierasch et al.'s model; the baroclinic layer above an adiabatic layer is also bounded by a stratosphere with strong stratification. A constant zonal wind shear is assumed in the baroclinic layer, while a constant zonal wind is assumed in the outer two layers. On the other hand, for examining the baroclinic instability in the middle atmosphere on the earth, Simmons (1974) investigated the instability of two unbounded fluid layers which are separated by the stratopause. In this model, zonal wind shear is assumed to be constant throughout the two layers, but the static stabilities are assumed to be different. It should be noted that geostrophic approximation is used in all the studies cited above.

It is known that in the Venus upper atmos-
sphere fast zonal winds, called the four-day circulation, prevail. It is certain that dynamic balance in that layer is cyclostrophic because of the large velocity of the zonal winds and the slow rotation of Venus. With hydrostatic approximation, the cyclostrophic balance is equivalent to the balance in the meridional plane between the moment of buoyant force due to the latitudinal temperature difference and that of the centrifugal force of the zonal winds with vertical shear. The dynamical balance of this kind is referred to as “thermal wind balance of the Venus type” in Matsuda (1980). In this dynamical balance, like thermal wind balance in westerly zone on the earth, owing to latitudinal temperature difference there exists available potential energy which can be, at least potentially, converted to kinetic energy of disturbances. Hence, the flow in thermal wind balance of the Venus type may be also baroclinically unstable. We wish to investigate this instability problem for the following two reasons. First, if such instability can occur for the parameter range appropriate to the actual state of the Venus upper atmosphere, the disturbance produced by such instability may be important to the general circulation of the Venus upper atmosphere, particularly a mechanism maintaining the four-day circulation. Second, this instability problem is interesting in the context of the theory of baroclinic instability, because the baroclinic instability of the flow in the cyclostrophic balance can be regarded as an extension of that in the geostrophic balance.

For examining the baroclinic instability in cyclostrophic balance in the Venus atmosphere, we must take the following points into consideration. First, the fluid layer with fast zonal winds on Venus is separated from the surface of the planet, so that we cannot assume the boundary of the layer under consideration to be rigid. This situation is similar to the terrestrial middle atmosphere and the Jovian atmosphere. Second, a basic state of dynamic balance in the Venus upper atmosphere (i.e. cyclostrophic balance) is different from that in the terrestrial and Jovian atmosphere. Hence, the perturbation equation governing disturbances is also different. In the perturbation equation in cyclostrophic balance, Coriolis factor (coefficient of Coriolis force term) becomes to be a function of altitude. Furthermore, the advection term is always important, so that geostrophic approximation cannot be adopted at all. Thus, we must examine the baroclinic instability with the equation perturbed about the cyclostrophic balance and with a different kind of boundary condition from rigid wall.

As a first step to solve this problem, in the present article, we examine only effects of the boundary condition; instability of baroclinic flows in geostrophic balance is examined in the absence of rigid wall. So, the situation considered by the present study is similar to that of the middle atmosphere and the Jovian atmosphere investigated in the studies cited above. However, in contrast with these studies, we do not use geostrophic approximation, because in connection with the Venus upper atmosphere we are interested in the case that advection terms are important.

The basic state considered in our model is illustrated in Fig. 1: a baroclinic layer with a constant wind shear ($\Lambda$) is bounded by two infinite barotropic layers. Static stability is assumed to be constant throughout the three layers. This basic state is contrast with Simmons (1974). In his model a constant wind shear is assumed throughout the layers, which have different static stabilities. The present instability problem without rigid wall may be formulated as an eigenvalue problem in the following way. As will be seen below, we easily obtain two independent solutions representing disturbances in the barotropic layers; these two solutions are either of exponential type or of sinusoidal type (gravity

\[ U = \Lambda H/2 \]
\[ U = 0 \]
\[ U = -\Lambda H/2 \]

Fig. 1 The model distribution of zonal wind: a baroclinic layer with constant wind shear and two barotropic layers with constant wind velocity.
wave). The former type appears for the small Rossby number, while the latter type can appear only for the large Rossby number (see below). It should be noted that the latter type cannot appear in the models which use geostrophic approximation. When the solutions are of the former type, we should retain only the solution which decays outward. On the other hand, when the solutions are of the latter type (gravity wave), we must invoke “radiation condition”; we retain only the solution expressing the wave whose origin lies in the baroclinic layer and whose energy propagates outward. Next, we must connect the solutions in the barotropic layers and the one in the baroclinic layer at the boundaries by assuming continuity of vertical velocity and of pressure. These four continuity conditions constitute an eigenvalue problem concerning the growth rate and phase velocity of disturbances. For the comparison, in this study we calculate also growth rates and phase velocities of disturbances for the case with rigid walls existing at the boundaries of the baroclinic layer.

Although the main purpose of the present article is to examine effects of the boundary condition on the baroclinic instability without using geostrophic approximation, we are also interested in the nature of disturbances for the large Rossby number even for the case with rigid walls. Indeed, Tokioka (1970) has examined the baroclinic instability of the atmosphere of weak stratification for the parameter range including the Rossby number which is large enough to admit the existence of one critical point in the baroclinic layer (for the critical point, see below). However, for more increased Rossby number it is not clear whether or not two critical points can exist in the fluid layer at the same time. We examine also this problem in the present study not only for the case of weak stratification but also for the case of strong one.

2. Mathematical procedure

In this section, we describe the mathematical procedure to treat the instability problem formulated in the introduction. First, we assume Eady's model, i.e. Boussinesq fluid in $f$-plane (Eady (1949) and Tokioka (1970)). Then, using conventional notations, a linearized perturbation equation for vertical velocity ($w$) in the baroclinic layer can be written down as

\[
Y(Y^2-1) \frac{d^2w}{dY^2} + 2 \left(1 - i \frac{k}{k_0} Y \right) \frac{dw}{dY} + k^2 - \frac{l^2}{k^2} Y \left( R_i + 2i \frac{k}{(k^2 + l^2)Y} \right) w = 0 ,
\]

(2-1)

where $Y = (k/f)(U-c)$. This equation is the same as the one discussed by Tokioka (1970) except that the term resulting from non-hydrostatic effect is retained in Tokioka (1970).

The perturbation equation governing disturbances in the upper barotropic layer is

\[
(\frac{Ro-\hat{c}}{Ro} - 1) \frac{d^2w}{d\zeta^2} + \frac{k^2 + l^2}{k^2} \frac{dw}{d\zeta} = 0 .
\]

(2-2)

That in the lower barotropic layer is

\[
(\frac{Ro+\hat{c}}{Ro} - 1) \frac{d^2w}{d\zeta^2} + \frac{k^2 + l^2}{k^2} \frac{dw}{d\zeta} = 0 ,
\]

(2-3)

where $\zeta = (k/f) \Lambda z$, $Ro = (k/f) \Lambda H / 2$ and $\hat{c} = \hat{c} + \hat{c} i = (k/f) c$. We call $Ro$ as the Rossby number below. We can easily solve eq. (2-2);

\[
w = A_1 \exp \left( i(-n_1)\zeta \right) + B_1 \exp \left( i(+n_1)\zeta \right) ,
\]

(2-4)

where

\[
n_1 = \sqrt{\frac{R_i}{k^2 + l^2}} \left( (Ro-\hat{c})^2 - 1 \right) .
\]

(2-5)

and $A_1$ and $B_1$ are arbitrary constants. Noting the fact that $Re ((i+n_1)\zeta) < 0$ for $(Ro-c_\gamma) < 1$ and $A Re((i+n_1)(Ro-c_\gamma)) \geq 0$, $A \Im((i+n_1)(Ro-c_\gamma)) \geq 0$ for $(Ro-c_\gamma) > 1$, we put $A_1 = 0$ based on the argument in the introduction. Similarly, we can obtain the solution in the lower barotropic layer:

\[
w = B_2 \exp \left( i(-n_2)\zeta \right) ,
\]

(2-6)

where

\[
n_2 = \sqrt{\frac{R_i}{k^2 + l^2}} \left( (Ro+\hat{c})^2 - 1 \right) .
\]

(2-7)

Next, we obtain solutions of (2-1) for the range of $-Ro - \hat{c} < \zeta < Ro - \hat{c}$. It should be noted that $\zeta = +1$ and $\zeta = -1$ are a singular point of eq. (2-1), while $\zeta = 0$ is, as will be shown below, only an apparent singular point. The first term on the left hand side of (2-1) changes its sign at $\zeta = \pm 1$. For $Ro > 1$ or $Ro < -1$ the solution of (2-1) is of sinusoidal type (gravity wave type), while for $-1 < Ro < 1$ the solution is of exponential type. From Howard's result, $|\hat{c}| < Ro$ for a stratified parallel flow (Howard (1961)) and its extension to a geostrophic flow (Pedlosky (1979)), we can expect that $|\hat{c}| < Ro$ for the present problem also, so that for $Ro$ larger than unity, at least one of the two critical points ($\zeta = +1$ or $-1$) exists in the range of $\zeta$ under consideration (i.e., $-Ro - \hat{c} < \zeta < Ro - \hat{c}$). However,
whether or not the another critical point is also present in the baroclinic layer depends on the value of \( c \). This problem was not made clear in Tokioka's study (1970). In the present study, we are interested in this problem also. For this reason, we need to obtain the solutions of (2-1) in the range of \( Y \) including \( Y = \pm 1 \). Several methods are available for obtaining numerically the solutions of (2-1). * In this study we obtain the solutions in the form of power series around \( Y = 0, +1 \) and \( -1 \), respectively.

When \( l \) (wave number in latitudinal direction) \( = 0 \), the two independent solutions in the vicinity of \( Y = 0 \) are written down in the form of infinite power series as

\[
w_1 = \sum_{n=0}^{\infty} a_n Y^n, \tag{2-8a}
\]

\[
a_n = \frac{(n-2)(n-3)+R_i}{n(n-3)} a_{n-2} \tag{2-8b}
\]

and

\[
w_2 = Y^2 \sum_{n=0}^{\infty} a_n Y^n, \tag{2-9a}
\]

\[
a_n = \frac{(n+1)n+R_i}{(n+3)n} a_{n-2}. \tag{2-9b}
\]

Since both (2-8) and (2-9) are of the form of regular solution, the point \( Y = 0 \) is an apparent singular point. Two independent solutions near \( Y = 1 \) are

\[
w_1 = \sum_{n=0}^{\infty} a_n (Y-1)^n, \tag{2-10a}
\]

\[
a_1 = -\frac{1}{2} R_i a_0, \tag{2-10b}
\]

\[
2n^2 a_n + \{3(n-1)(n-2)+R_i\} a_{n-1} + \{(n-2)(n-3)+R_i\} a_{n-2} = 0, \quad (n = 2, 3, 4, \ldots)
\]

and

\[
w_2 = w_1 \log (Y-1) + \sum_{n=1}^{\infty} b_n (Y-1)^n; \tag{2-11a}
\]

\[
2b_1 + R_i b_0 = (2R_i+3)a_0, \tag{2-11b}
\]

\[
2(n+2)^2 b_{n+2} + \{3(n+1)n+R_i\} b_{n+1} + \{n(n-1)+R_i\} b_n = -\{(n+2)a_{n+2}+3(2n+1)a_{n+1}+(2n-1)a_n\} \quad (n = 0, 1, 2, \ldots)
\]

While (2-10) is a regular solution, (2-11) involves a logarithm. In the same way, we can obtain two independent solutions \( w_1^- \) and \( w_2^- \) near \( Y = -1 \). When \( l \neq 0 \), we can obtain solutions near \( Y = -1, 0, +1 \) in the form of infinite power series again. Here, we omit to write down these solutions. The circles of convergence of each infinite power series are illustrated in Fig. 2.

Next, in order to obtain two independent solutions which are valid throughout the baroclinic layer it is necessary to connect the solutions obtained above to each other. First, we express \( w_1 \) and \( w_2 \) by a linear combination of \( w_1^+ \) and \( w_2^+ \):

\[
w_1 = a w_1^+ + \beta w_2^+, \tag{2-12a}
\]

\[
w_2 = \gamma w_1^+ + \delta w_2^+. \tag{2-12b}
\]

We determine coefficients \( a, \beta, \gamma \) and \( \delta \) from the continuity condition that the both sides of these equations and its derivatives coincide to each other at \( Y = 1/2 \), respectively. As understood from Fig. 2, these equalities are valid in \( |Y| < 1 \). Similarly, we express \( w_1 \) and \( w_2 \) by a linear combination of \( w_1^- \) and \( w_2^- \). In this way, we obtain the two independent solutions \( \bar{w}_1 \) and \( \bar{w}_2 \) which coincide with \( w_1 \) and \( w_2 \) in \( |Y| < 1 \), respectively, and are the linear combination of \( w_1^+ \) and \( w_2^+ \) in \( |Y-1| < 2 \) and that of \( w_1^- \) and \( w_2^- \) in \( |Y+1| < 2 \). This continuation of the solution is made only numerically in this study, so that we do not write down coefficients of the continuation here. In this way, we can obtain solutions valid throughout the baroclinic layer. We have already obtained the two independent solutions valid in the barotropic layers above. Upon these solutions, we must impose the condition that pressure and vertical velocity should be continuous at the boundaries between the barotropic and baroclinic layers. Noting that

\* The exact solutions of (2-1) can be expressed in terms of hypergeometric functions. See Yamanaka and Tanaka (1984).
\( p \) is expressed by \( w \) as

\[
p = 2iR_0 \frac{d^2 w}{dY^2} + \left( Y + i \frac{l}{k} \right) w \frac{\rho_0 f}{k^2 H} \quad \text{(2-13a)}
\]

in the baroclinic layer and

\[
p = 2iR_0 \frac{d^2 w}{dZ^2} \frac{\rho_0 f}{Y \left( 1 + \frac{l^2}{k^2} \right) k^2 H} \quad \text{(2-13b)}
\]

in the barotropic layers, we can write continuity of \( w \) and \( p \) across \( z = H/2 \):

\[
\begin{align*}
A\bar{w}_1(Y_1) + B\bar{w}_2(Y_1) & = Cw_0(Y_1) \quad \text{(2-14a)} \\
A\bar{w}_1(Y_2) + B\bar{w}_2(Y_2) & = Dw_0(Y_2) \quad \text{(2-14b)}
\end{align*}
\]

Note that through the variable \( Y_1 \) and \( Y_2 \), (2-15) involves \( \bar{c} \) to be determined as an eigenvalue. As described in the introduction, for the comparison we examine the stability problem under the boundary condition with horizontal rigid walls also. In this case, since the left hand sides in (2-14a), (2-14c) should vanish, we obtain the equation to determine \( \bar{c} \) as:

\[
\begin{align*}
& \begin{pmatrix}
1 & \frac{l}{k} \\
\frac{1}{1-Y_1^2} & 1
\end{pmatrix} \begin{pmatrix}
\bar{w}_1(Y_1) + \bar{w}_1'(Y_1) \\
\bar{w}_2(Y_1) + \bar{w}_2'(Y_1)
\end{pmatrix} \\
& \begin{pmatrix}
1 & \frac{l}{k} \\
\frac{1}{1-Y_2^2} & 1
\end{pmatrix} \begin{pmatrix}
\bar{w}_1(Y_2) + \bar{w}_1'(Y_2) \\
\bar{w}_2(Y_2) + \bar{w}_2'(Y_2)
\end{pmatrix} = 0.
\end{align*}
\]

The procedure described above is numerically carried out in the following way. First, for a given value of \( Ri \) and \( l/k \), coefficients of the power series are numerically estimated. In numerical calculations, each infinite power series are truncated after one hundred terms. Next, we obtain coefficients connecting these solutions at \( Y = 1/2 \) and \( Y = -1/2 \). For a given value of \( Ro \), we obtain coefficients of (2-14) as power series of \( Ro - \bar{c} \) or \( -Ro - \bar{c} \). Then, the left hand side of (2-15) (or (2-16)) can be regard as a function of \( \bar{c} \) only. The zero point of this function is determined mainly by Newton-Rapson method. The numerical calculations are carried out in double precision, partly in quadruple precision.

Note that through the variable \( Y_1 \) and \( Y_2 \), (2-15) involves \( \bar{c} \) to be determined as an eigenvalue. As described in the introduction, for the comparison we examine the stability problem under the boundary condition with horizontal rigid walls also. In this case, since the left hand sides in (2-14a), (2-14c) should vanish, we obtain the equation to determine \( \bar{c} \) as:

\[
\begin{align*}
& \begin{pmatrix}
1 & \frac{l}{k} \\
\frac{1}{1-Y_1^2} & 1
\end{pmatrix} \begin{pmatrix}
\bar{w}_1(Y_1) + \bar{w}_1'(Y_1) \\
\bar{w}_2(Y_1) + \bar{w}_2'(Y_1)
\end{pmatrix} \\
& \begin{pmatrix}
1 & \frac{l}{k} \\
\frac{1}{1-Y_2^2} & 1
\end{pmatrix} \begin{pmatrix}
\bar{w}_1(Y_2) + \bar{w}_1'(Y_2) \\
\bar{w}_2(Y_2) + \bar{w}_2'(Y_2)
\end{pmatrix} = 0.
\end{align*}
\]

The external parameters governing the present system are \( l/k, Ri \) and \( Ro \). It needs enormous computing time to calculate eigenvalues for a wide range in this three-dimensional parameter space. Moreover, in this study we are mainly interested in the effect of the boundary condition on baroclinic instability, so that we restrict our calculations to a rather limited parameter range. For the Richardson number, we consider only two cases, i.e. \( Ri=50 \) and \( Ri=0.5 \), which correspond to strong and weak atmospheric stratification, respectively. For the value of \( l/k \), calculations are made in its extended range.

\section{Results}

The eigenvalues \( \{c_r, \bar{c} \} \) obtained in our calculations are depicted in Figs. 3–6. We should note that Eq. (2-1) and the boundary conditions are invariant to the replacement of \( Y \) and \( w \) by \(-Y^*\) and \( w^*\), respectively. Therefore, if \( \bar{c} = c_r + i\bar{c}_l \) is an eigenvalue, then \( \bar{c} = -c_r + i\bar{c}_l \) is also an eigenvalue. For simplicity, we consider eigenvalues with \( \bar{c}_r \geq 0 \) only below. These figures show
Fig. 3 The dependency of non-dimensionalized growth-rate $\hat{c}_i=(k/f)c_i$ (a) and non-dimensionalized phase velocity $\hat{c}_r=(k/f)c_r$ (b) of the disturbances on the Rossby number for $Ri=0.5$ and $l=0$. The results of the calculations in the case with rigid wall are indicated by $\times$ and those without rigid wall are $\triangle$. Note that the eigenvalues with $\hat{c}_r \geq 0$ only are shown.

Fig. 4 Same as Fig. 3 but for $Ri=50$ and $l=0$.

Fig. 5 Same as Fig. 3 but for $Ri=0.5$ and $l/k =1$.

that for each set of $Ri$ and $l/k$ Ro-dependency of the eigenvalues in the case without rigid wall is analogous to that with rigid walls, although the eigenvalues themselves are quantitatively different between the two cases. First, we examine common characteristics of the disturbances in the two cases.

The disturbances obtained can be classified into three groups: the first group is ordinary unstable baroclinic waves which appear for the small Rossby number ($\hat{c}_r=0, \hat{c}_i \neq 0$), the second is neutral waves ($\hat{c}_r \neq 0, \hat{c}_i=0$) and the third is unstable waves which include a critical point ($Y=+1$ or $Y=-1$) in the baroclinic layer.
(\(c_r \neq 0, \delta_\ell \neq 0\)). For ordinary baroclinic waves, our calculations agree with the fact that is already known: there exists a cut-off wave number, and the large Richardson number (strong atmospheric stratification) suppresses both the growth rate of the disturbances and the extent of the unstable range of \(Ro\). In the case of \(Ri=50\), unstable baroclinic wave can appear only for \(Ro<0.2\), while in the case of \(Ri=0.5\), unstable baroclinic wave can appear for \(Ro<0.8\); the latter results agree with Tokioka's study (1970) on stability of baroclinic flow with the small Richardson number. In the case of \(Ri=0.5\) we have obtained neutral waves, although \(Ro\)-range of the neutral waves is very limited. In the case of \(Ri=50\), we cannot distinguish between neutral waves and waves with a critical point (\(Y=1\) or \(-1\)) only from values of \(\delta_\ell\), because the effect of strong stratification makes \(\delta_\ell\) of the latter waves to be so close to zero.

As described above, when \(Ro\) increases, one of the critical points enters into the baroclinic layer: \(-H/2<z<H/2\). Then, a neutral wave is replaced by an unstable wave with critical point. For waves with critical point, Fig. 3–6 show that \(\delta_r\) increases linearly with \(Ro\) and the value of \(Ro-\delta_r\) remains a constant value which is much smaller than unity. This fact seems valid independently of the boundary conditions and of the value of \(l/k\) and \(Ri\). Moreover, it is certain that this linearity is valid beyond the \(Ro\)-range calculated here. Hence, a steering level where \(U(z) - c_r = 0\) remains in the vicinity of the boundary (\(z=H/2\)), so that the critical point (\(Ro-\delta_r = +1\)) can not appear. Thus, we can conclude that the two critical points can not exist at the same time in the fluid layer for any value of \(Ro\). Accordingly, in the case without rigid wall, we cannot find a situation such that gravity waves are radiating from the both sides of the baroclinic layer. Moreover, from Figs. 3–6 we see that the growth rate of unstable waves with critical point are smaller than that of ordinary baroclinic waves. This implies that it is difficult for the disturbance with critical point to grow in actual atmospheres. Hence, it is not reasonable to expect that the instability of baroclinic layer produces gravity waves in actual atmospheres.

In the last, we describe the difference in the results between the case with and without rigid wall. For \(Ri=50\), the growth rate of ordinary baroclinic unstable waves and its extent in \(Ro\)-domain are larger in the case with rigid wall than in the case without it. For example, for \(l=0\), a maximum value of \(\delta_\ell\) is 0.43 in the former case, while maximum \(\delta_\ell\) is 0.29 in the latter case. Further, a cut-off value of \(Ro\) of unstable waves exists near 0.19 in the former case, while a cut-off \(Ro\) is 0.1 in the latter case. Hence, it is found that a baroclinic instability in an atmosphere of strong stratification for the small Rossby number is considerably weakened by removing rigid walls.

For \(Ri=0.5\), there is also some difference in the value of \(\delta_r\) and \(\delta_\ell\) between the cases with and without rigid wall, while there is little difference in the value of the cut-off Rossby number of ordinary baroclinic waves and the position of the limited range of neutral waves in \(Ro\)-domain. Fig. 3 and 5 indicate that the value of \(\delta_\ell\) is somewhat larger in the case without rigid wall than in the case with it. When a constant wind shear is assumed throughout the layers with different static stability, then a strong stratification in the outer layer is favourable for instability. Hence, existence of rigid wall, which can be regarded as a limit of strong stratification, is always the most favourable condition for instability. (See Eady (1949).) However, our model
in this study is different from the model with a constant wind shear. Hence, in our model the existence of rigid wall does not necessarily mean the most favourable condition for instability. So, our results of \( \hat{c}_\ell \) for \( Ri=0.5 \) does not contradict the notion about the case with a constant wind shear. It should be noted that the difference of \( \hat{c}_\ell \) between the two cases for \( Ri=0.5 \) is not so large as that for \( Ri=50 \). For example, for \( Ri=0.5 \) maximum growth rate of \( l=0 \) differs only by 30% between the two cases.

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