Theoretical Interpretations of the Eliassen-Palm Diagnostics of Wave-Mean Flow Interaction

Part I: Effects of the Lower Boundary

By Yoshikazu Hayashi

*Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, P.O. Box 308
Princeton, New Jersey 08542 USA*

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Abstract

In order to theoretically interpret the Eliassen-Palm diagnostics of wave-mean flow interactions, necessary and sufficient conditions are derived for the Eliassen-Palm flux divergence to approximate the zonal acceleration in the absence of mean damping.

It is shown by a simple analytical model that a violation of these conditions due to the effect of the lower boundary condition may explain why some observational diagnostic studies indicate that transient eddies accelerate westerlies in the mid-latitude upper troposphere in the absence of mean damping, although the transient E-P flux divergence is easterly there.

It is also suggested on the basis of simple diagnostic relations that the steady state upper tropospheric westerlies can be either enhanced or reduced by transient eddies depending on mean damping and radiative heating.

I. Introduction

It is well known (Eliassen, 1952; Phillips, 1954; Kuo, 1956) that eddies can not only accelerate zonal mean flows but also can induce mean circulations to maintain geostrophic and hydrostatic balance (see Lorenz, 1967, 1969 for a review of general circulations). In the mid-latitude troposphere, observed eddy momentum and heat fluxes induce an indirect cell (Vernekar, 1967; Crawford and Sasamori, 1981; Salustri and Stone, 1983). The observed eddy momentum flux convergence dominates the counteracting induced mean Coriolis force and accelerates *westerlies* in the upper troposphere, while the two effects work in the same direction and accelerate westerlies in the lower troposphere (Wiin-Nielsen and Vernekar, 1967; Pfeffer, 1981). In the absence of dissipation, heating, critical levels or wave transience ("non-acceleration condition"), waves do not accelerate mean flows, although

a mean vertical motion is induced by the eddy heat flux (Charney and Drazin, 1961; Holton, 1974; Boyd, 1976; Andrews and McIntyre, 1976, 1978a; Nakamura, 1979).

In the stratosphere, however, the induced mean Coriolis acceleration plays an important role. For example, in the presence of an instability or wave dissipation, equatorial Rossby-gravity waves accelerate easterlies through their dominant induced mean Coriolis acceleration (Hayashi, 1970). These waves may be responsible for the easterly phase of the quasi-biennial oscillation (Holton and Lindzen, 1972). In the presence of a wave transience or a critical level, mid-latitude planetary waves can cause a sudden warming due to their heat flux convergence and accelerate easterlies due to their induced mean Coriolis force (Matsuno, 1971).

In order to describe the net effect of eddies on the zonal flow in a simple manner, Andrews and McIntyre (1976) transformed the Eulerian-

mean equations in terms of the Eliassen-Palm flux divergence (EPFD) discussed by Eliassen and Palm (1960) and the residual mean circulations ($\bar{\phi}^*$). This EPFD consists of a combination of eddy momentum and heat flux divergences, while $\bar{\phi}^*$ consists of the Eulerian-mean circulations and heat fluxes. According to their scale analysis, the residual mean circulations are negligible and the acceleration can be approximated by the EPFD on the "tall mean flow condition." This condition is met when the vertical-lateral scale ratio of the mean acceleration is much larger than fN^{-1} , where f and N are the Coriolis and Brunt-Väisälä frequencies, respectively. In the absence of a horizontal shear the vertical component of the EPFD is equivalent to the "effective vertical momentum flux convergence" of Bretherton (1969) and Lindzen (1970). Under quasi-geostrophic scaling the EPFD is equivalent to the quasi-geostrophic eddy potential vorticity flux discussed by Dickinson (1969).

The Eliassen-Palm flux is also a useful measure of wave propagation (Edmon, *et al.* 1980; Palmer, 1982; Butchart, *et al.* 1982) and has been modified to describe local wave propagation (Hoskins, *et al.* 1983; Plumb, 1985) and to include the effect of eddy forcing of condensational heating (Stone and Salustri, 1984). Also, it has been used to reformulate energy cycle (Plumb, 1983; Kanzawa, 1984a).

Eliassen-Palm diagnostics have been widely applied to observed and simulated planetary waves (e.g., Sato, 1980; Edmon, *et al.* 1980; Dunkerton, *et al.* 1981; Palmer, 1981a, b; Kanzawa, 1982, 1984b; O'Neill and Youngblut, 1982; Karoly, 1982; Andrews, *et al.* 1983; Geller *et al.* 1983; Mahlman and Umscheid, 1984; Hartmann *et al.* 1984). According to Edmon *et al.* (1980), the observed stationary or transient EPFD indicates easterly mean flow forcing in the midlatitude troposphere except near the surface [see also Wiin-Nielsen and Sela (1977), Pfeffer (1981), Andrews *et al.* (1983), Geller *et al.* (1983)]. This easterly forcing appears to be contrary to the net westerly acceleration calculated by Wiin-Nielsen and Vernekar (1967, Fig. 3) and Pfeffer (1981, Fig. 7). However, their calculation is not proper for topographically forced stationary

waves, since the lower boundary condition for the Eulerian-mean vertical velocity depends on the topographic height which is neglected. On the other hand, the recent calculation by Lau and Holopainen (1984, Fig. 9) indicates that observed transient eddies accelerate westerlies in the upper troposphere, being consistent with a theoretical study of mixed baroclinic-barotropic instability (Shepherd, 1983, Fig. 6a, Fig. 8). These eddies also accelerate westerlies in the lower troposphere and reduce the mean vertical shear and the latitudinal gradient of the mean temperature in the lower troposphere.

The present paper theoretically interprets the Eliassen-Palm diagnostics of wave-mean flow interaction. Section 2 gives the zonal mean equations, while Section 3 derives conditions for the Eliassen-Palm diagnostics. Section 4 gives a simple analytical model to study the effect the vertical-lateral scale ratio and the boundary conditions, while Section 5 discusses the effect of mean damping. Conclusions and implications are given in Section 6. Appendix A lists symbols, Appendix B gives the boundary conditions and Appendix C describes the Green's function.

2. Zonal-mean equations

In this section the conventional and transformed Eulerian-mean equations are summarized. The symbols are listed in Appendix A.

a. Zonal-mean equations

The quasi-geostrophic Eulerian-mean equations on an f -plane in the log-pressure coordinates (see Holton, 1975) are simplified by neglecting the nonlinear advection terms and omitting the friction and heating terms as

$$\bar{u}_t = -M_y + f\bar{v}, \quad (2.1)$$

$$\bar{T}_t = -B_y - R_H^{-1}N^2\bar{w}, \quad (2.2)$$

$$(\rho_0\bar{v})_y + (\rho_0\bar{w})_z = 0, \quad (2.3)$$

$$f\bar{u}_z + R_H\bar{T}_y = 0. \quad (2.4)$$

$$M = \overline{u'v'}, \quad (2.5)$$

$$B = \overline{T'v'}, \quad (2.6)$$

$$\rho_0 = \rho_s \exp(-z/H), \quad (2.7)$$

where the prime denotes a deviation from the zonal mean which is indicated by an overbar. f and N are the Coriolis and Brunt-Väisälä frequencies which are assumed to be constant for simplicity. H is the constant scale factor of the log-pressure coordinate. R_H is the gas constant R , divided by H .

b. Transformed Eulerian-mean equations

The above Eulerian-mean equations are transformed, following Andrews and McIntyre (1976) as

$$\bar{u}_t = D + f\bar{v}^*, \quad (2.8)$$

$$\bar{T}_t = -R_H^{-1}N^{-2}\bar{w}^*, \quad (2.9)$$

$$(\rho_0\bar{v}^*)_y + (\rho_0\bar{w}^*)_z = 0, \quad (2.10)$$

and

$$f\bar{u}_z + R_H\bar{T}_y = 0, \quad (2.11)$$

where D is the Eliassen-Palm flux divergence and $(\bar{v}^*, \bar{w}^*)$ are the residual mean circulations defined by

$$D = -M_y + fR_HN^{-2}\rho_0^{-1}(\rho_0B)_z, \quad (2.12)$$

$$\bar{v}^* = \bar{v} - R_HN^{-2}\rho_0^{-1}(\rho_0B)_z, \quad (2.13)$$

and

$$\bar{w}^* = \bar{w} + R_HN^{-2}B_y. \quad (2.14)$$

When $f\bar{v}^*$ in (2.8) is larger than D , the tendency is not always of the same sign as D . When $\bar{v}^*$ in (2.13) is negligible, the second term in (2.12) can be interpreted as approximating $f\bar{v}$ induced by B .

In the absence of mean vertical shear and wave heating, the D of steady small amplitude waves approximates the radiation stress divergence, while the residual circulations and the second terms in (2.13) and (2.14) approximate the Lagrangian-mean circulations and the Stokes drift, respectively (Andrews and McIntyre, 1978b; Matsuno and Nakamura, 1979). The radiation stress can be interpreted as a pseudo-momentum flux (Andrews and McIntyre, 1978c; McIntyre, 1981; Plumb and Bell, 1982). For reviews of Eulerian and Lagrangian-mean circulations, see Uryu (1980), Matsuno, (1980), McIntyre (1980), Dunkerton (1980) and Holton (1980).

3. Conditions for Eliassen-Palm diagnostics

In this section, the necessary and sufficient conditions for D to approximate the net eddy-induced acceleration are derived from the elliptic equations.

a. Elliptic equations

Eliminating variables from the above Eulerian mean equations results in the elliptic equations as

$$(\bar{u}_t)_{yy} + f^2N^{-2}(\bar{u}_t)_{zH} = D_{yy}, \quad (3.1)$$

$$(\bar{T}_t)_{yy} + f^2N^{-2}(\bar{T}_t)_{zH} = -fR_H^{-1}D_{yz}, \quad (3.2)$$

$$(\bar{\phi})_{yy} + f^2N^{-2}(\bar{\phi})_{zH} = -R_HN^{-2}B_{yy} - fN^{-2}M_{yz}, \quad (3.3)$$

$$(\bar{\phi}^*)_{yy} + f^2N^{-2}(\bar{\phi}^*)_{zH} = fN^{-2}D_z, \quad (3.4)$$

where $()_{zH}$ is defined as

$$()_{zH} = ()_{zz} - H^{-1}()_z \quad (3.5a)$$

$$= \rho_0^{-1}[\rho_0()_z]_z. \quad (3.5b)$$

In (3.3), $\rho_0\bar{\phi}$ denotes the mass stream function defined by

$$(\rho_0\bar{\phi})_z = -\rho_0\bar{v}, \quad (3.6)$$

and

$$(\rho_0\bar{\phi})_y = \rho_0\bar{w}, \quad (3.7)$$

b. Necessary and sufficient conditions

In the present paper the vertical-latitude scale ratio of $\bar{u}_t$ is defined by

$$r_u \equiv (\bar{u}_t)_{yy} / (\bar{u}_t)_{zH}. \quad (3.8)$$

It follows from (3.1) that if and only if

$$|r_u| \gg f^2N^{-2}, \quad (3.9)$$

then

$$(\bar{u}_t)_{yy} \approx D_{yy}. \quad (3.10)$$

Thus, (3.9) is a diagnostically necessary and sufficient condition¹ for $\bar{u}_t \approx D$.

The condition (3.9) is essentially the same as the "tall mean flow condition" discussed by Andrews and McIntyre (1976) based on the scale analysis of the transformed Eulerian

¹ Strictly speaking, (3.9) is a necessary and sufficient condition for (3.10).

mean equations of a Boussinesq fluid. The present condition is more appropriate for a compressible fluid, since the first z derivative $(\)_z$ in (3.5a) usually dominates the second z derivative $(\)_{zz}$ for stratospheric planetary waves. It should be noted that (3.8) is defined with respect to $\bar{u}_t$ rather than D . When (3.8) is defined with respect to D as in Andrews *et al.* (1983, p. 2782), (3.9) is only a necessary condition for $\bar{u}_t \approx D$, as discussed below.

It follows from (3.1), that if

$$(\bar{u}_t)_{yy} \approx D_{yy} \quad \text{and} \quad (\bar{u}_t)_{zH} \approx D_{zH}, \quad (3.11)$$

then

$$|r_D| \gg f^2 N^{-2}, \quad (3.12)$$

where

$$r_D = D_{yy}/D_{zH}. \quad (3.13)$$

Thus, (3.12) is a necessary condition² for $\bar{u}_t \approx D$.

It should be noted that if (3.12) holds everywhere, D approximates a particular solution to (3.1). However, D does not approximate $\bar{u}_t$ at low levels, since D does not usually satisfy the lower boundary condition.

The solution to (3.1) is also given by the use of the Green's function (see Appendix C). When the condition (3.12) holds, the Green's function behaves like a delta function in the vertical and $\bar{u}_t$ is approximated by D for a large z .

4. Analytical model

In this section a simple analytical model is given to examine the effect of the scale ratio and the lower boundary conditions on wave-mean interactions.

a. Analytical solutions

It is assumed that eddy fluxes due to long and ultralong waves are given by

$$\rho_0 B = B_0 R_H^{-1} N^2 \exp(-mz) \sin ly, \quad (4.1)$$

$$\rho_0 M = M_0 f \exp(-mz) \cos ly, \quad (4.2)$$

and

$$\rho_0 D = -(mB_0 - lM_0) f \exp(-mz) \sin ly, \quad (4.3)$$

² Strictly speaking, (3.12) is a necessary condition for (3.11).

where B_0 and M_0 are positive constants. It should be noted that M can increase with height, although $\rho_0 M$ decreases with height.

The vertical-lateral scale ratio of D is given by

$$r_D = \frac{l^2}{m(H^{-1} - m)}. \quad (4.4)$$

The lateral boundary condition is given by

$$\bar{v} = 0 \quad \text{at } ly = 0 \text{ and } \pi. \quad (4.5)$$

Physically reasonable upper and lower boundary conditions are that the Lagrangian-mean mass fluxes vanish. In the present solution, these conditions are met by only imposing the following conditions.

The upper boundary condition is given by

$$\rho_0 \bar{w} < \infty \quad \text{at } z = \infty. \quad (4.6)$$

The lower boundary condition in the absence of topography is given by

$$\bar{w} \approx 0 \quad \text{at } z = 0, \quad (4.7)$$

which is equivalent to

$$(\bar{u}_t)_z \approx f^{-1} R_H B_{yy} \quad \text{at } z = 0. \quad (4.8)$$

The lower boundary condition (4.7) which is used for observational diagnostics is not exact ($d\phi/dt \neq 0$). However, this approximation will not seriously affect the upper troposphere. This condition is exact under the non-acceleration condition (see Appendix B).

If the waves are topographically forced in the absence of heating and friction at the surface, the lower boundary condition (see Appendix B) is reduced to

$$\bar{w} \approx -R_H N^{-2} B_y \quad \text{at } z = 0, \quad (4.9)$$

which is equivalent to

$$(\bar{u}_t)_z \approx 0 \quad \text{at } z = 0. \quad (4.10)$$

The solution to (3.3) in the absence of topography is given by the use of the lower boundary conditions (4.7) and (4.8) as

$$\rho_0 \bar{\phi} = -c [\exp(-mz) - \exp(-\mu z)] \sin ly, \quad (4.11)$$

and

$$\bar{u}_t = \frac{r_D}{r_D + f^2 N^{-2}} D + c f \mu \exp[-(\mu - H^{-1})z] \sin ly, \quad (4.12)$$

while the topographically forced solution is given by the use of the lower boundary conditions (4.9) and (4.10) as

$$\rho_0 \bar{\psi} = [-c \exp(-mz) + (c - B_0) \cdot \exp(-\mu z)] \sin ly, \quad (4.13)$$

and

$$\ddot{u}_t = \frac{r_D}{r_D + f^2 N^{-2}} \{D - (H^{-1} - m)(H^{-1} - \mu)^{-1} \cdot D(o) \exp[-(\mu - H^{-1})]\}. \quad (4.14)$$

In the above solutions c and μ are defined by

$$c = \frac{B_0 l^2 + M_0 f^2 N^{-2} l (H^{-1} - m)}{l^2 + f^2 N^{-2} m (H^{-1} - m)}, \quad (4.15)$$

and

$$\mu = [f^{-2} N^2 l^2 + (2H)^{-2}]^{1/2} + (2H)^{-1}, \quad (4.16a)$$

$$= [h_R^{-2} + (2H)^{-2}]^{1/2} + (2H)^{-1}, \quad (4.16b)$$

$$= [-N^2 (gh)^{-1} + (2H)^{-2}]^{1/2} + (2H)^{-1}, \quad (4.16c)$$

where

$$h_R = f N^{-1} l^{-1}, \quad (4.17)$$

and

$$h = -g^{-1} f^2 l^{-2}. \quad (4.18)$$

Here h_R is the Rossby depth used in the geostrophic adjustment theory, while h is the equivalent depth used in tidal theory. In the present problem this equivalent depth does not depend on the frequency by virtue of the quasi-geostrophic approximation. The negative value of h is consistent with that of zonally symmetric annual oscillations over a sphere discussed by Sawada and Matsushima (1964).

For $m = H^{-1}$, B and M are constant with height and $|r_D|$ approaches infinity, while $\rho_0 B$ and $\rho_0 M$ decay with height. In this case, M does not induce $\bar{\psi}$.

For $0 < m < H^{-1}$, $\rho_0 B$ and $\rho_0 M$ decay with height, while B and M increase with height. In this case, both B and M induce an indirect cell ($\bar{\psi} < 0$) for $B_0 > 0$ and $M_0 > 0$.

In the present calculation, it is assumed that $m = 0.5H^{-1}$, since the observed M increases with height in the troposphere. It is also assumed that $mB_0 = 2lM_0$ in (4.3), since the observed B contributes to D at least twice as much as M does. The constants are given

below:

$$f = 10^{-4} \text{ s}$$

$$N = 10^{-2} \text{ s}^{-1},$$

$$\pi/l = 5000 \text{ km}.$$

$$H = 7 \text{ km},$$

$$m^{-1} = 14 \text{ km},$$

$$\mu^{-1} = 6 \text{ km},$$

$$h = -2.6 \text{ km},$$

and

$$h_R = 16 \text{ km}.$$

b. Results

The conventional and transformed Eulerian-mean zonal momentum equations are summarized as

$$\bar{u}_t = f\bar{v} - M_y \quad (4.19a)$$

$$= f\bar{v}^* + D, \quad (4.19b)$$

$$(\bar{u}_t)_{yy} + f^2 N^{-2} (\bar{u}_t)_z H = D_{yy}, \quad (4.20)$$

where

$$D = f R_H^{-2} \rho_0^{-1} (\rho_0 B)_z - M_y. \quad (4.21)$$

Fig. 1a shows the vertical distributions (0–10 km) of $\bar{u}_t$ and D with the non-topographical boundary condition when both B and M , only B , and only M are used as forcing functions. The effective value of N is assumed to be 10^{-2} s^{-1} which is a representative value in the troposphere when the observed lapse rate is taken into account. When both B and M are imposed, it turns out that D is negative (westward acceleration), while $\bar{u}_t$ is positive (eastward acceleration) in agreement with some observational diagnostics. The discrepancy between $\bar{u}_t$ and D becomes larger when only B is imposed, as compared to that when only M is imposed. When both B and M are imposed, the negative D is primarily due to the effect of B , while the positive $\bar{u}_t$ is primarily due to the effect of M . It should be noted that the effect of B on $\bar{u}_t$ is not twice that of M , although B contributes to D twice as much as M does. This is because $\bar{u}_t$ depends not only on D but also on the surface B through the lower boundary condi-

Fig. 1 Vertical distributions at levels 0–10 km (a) and 40–50 km (b) of u_t (solid) and D (dashed) for $N=10^{-2} \text{ s}^{-1}$. Forcing with both B and M (left), B only (middle), M only (right). $-M_y$ is set to 1 at 10 km.

tion (4.8) in the absence of topography.

Fig. 1b is the same as Fig. 1a except for the levels 40–50 km. At these levels, the effect of the lower boundary is negligible, since it decays with height in (4.12). It is seen that $\bar{u}_t$ and D have the same sign for the three cases of forcing and are negative when both B and M are imposed. However, D still overestimates $\bar{u}_t$ by a factor of 2.4. At an infinite height, this factor approaches 2.3, as is expected from (4.12).

It is of interest to examine the lateral-vertical scale ratios defined by (3.8) and (3.13). In this model, $|r_u| \gg f^2 N^{-2}$ is rigorously a

necessary and sufficient condition for $\bar{u}_t \approx D$, since $(\bar{u}_t)_{yy} = -l^2 \bar{u}_t$ and $D_{yy} = -l^2 D$.

For this model, the ratio between D and $\bar{u}_t$ is related to r_u as

$$\frac{D}{\bar{u}_t} = \frac{1 + r_u / (f^2 N^{-2})}{r_u / (f^2 N^{-2})}. \quad (4.22)$$

This ratio becomes 0 and ∞ for $r_u / (f^2 N^{-2}) = -1$ and 0, respectively.

Table 1 lists the vertical distribution of the scale ratio of $\bar{u}_t$ normalized by $f^2 N^{-2}$. The normalized scale ratio of D is 0.77 at all the levels for the three cases of forcing. Above 40 km, r_u is close to r_D . This is due

Table 1 Vertical distributions of $r_u/(f^2N^{-2})$ for $N=10^{-2}\text{ s}^{-1}$.

$Z(\text{km})$	B and M	B only	M only
50	0.69	0.75	0.81
40	0.57	0.71	0.87
30	0.32	0.61	1.05
20	-0.06	0.40	1.72
10	-0.46	0.45	16.85
9	-0.50	0.00	186.14
8	-0.53	-0.04	-20.85
7	-0.56	-0.08	-9.95
6	-0.59	-0.12	-6.58
5	-0.62	-0.16	-4.95
4	-0.65	-0.21	-3.99
3	-0.68	-0.25	-3.36
2	-0.70	-0.29	-2.91
1	-0.72	-0.33	-2.58
0	-0.74	-0.37	-2.33

to the fact that $\bar{u}_t$ and D are nearly proportional at a great height, as expected from (4.12). In the troposphere, r_u becomes negative due to the effect of the lower boundary condition. With both B and M and with only B , $|r_u|/(f^2N^{-2})$ is smaller than 1, being consistent with the fact (Fig. 1a) that $\bar{u}_t$ and D are quite different.

Fig. 2 is the same as Fig. 1 except that N is replaced by its representative value ($2 \times 10^{-2}\text{ s}^{-1}$) in the stratosphere. It is seen in Fig. 2a (0–10 km) that $\bar{u}_t$ is still positive, although it is somewhat reduced in the upper troposphere when both B and M are used as the forcing. At the 40–50 km levels (Fig. 2b), $\bar{u}_t$ and D are close, their ratio being about 1.3 when both B and M are imposed.

Fig. 2 As in Fig. 1 except for $N=2 \times 10^{-2}\text{ s}^{-1}$.

Table 2 As in Table 1 except for $N=2\times10^{-2}\text{ s}^{-1}$.

$Z(\text{km})$	B and M	B only	M only
50	3.03	3.07	3.10
40	2.84	2.98	3.13
30	2.19	2.65	3.26
20	0.86	1.70	3.92
10	-0.33	0.28	13.24
9	-0.41	0.15	22.17
8	-0.48	0.04	83.13
7	-0.54	-0.07	-42.25
6	-0.60	-0.17	-16.16
5	-0.65	-0.26	-9.76
4	-0.69	-0.35	-6.88
3	-0.72	-0.42	-5.27
2	-0.76	-0.49	-4.24
1	-0.79	-0.55	-3.53
0	-0.82	-0.61	-3.02

Table 2 is the same as Table 1 except for $N=2\times10^{-2}\text{ s}^{-1}$. For this value $r_D/(f^2N^{-2})$ is 3.1. Above 40 km, r_u is close to r_D and larger than 1, being consistent with the fact that $\bar{u}_t$ and D are similar at these levels. In the troposphere, r_u is negative. With both B and M and with B only, $|r_u|/(f^2N^{-2})$ is smaller than 1, being consistent with the fact (Fig. 2a) that $\bar{u}_t$ and D are quite different.

Fig. 3 is the same as Fig. 1 ($N=10^{-2}\text{ s}^{-1}$) except that the lower boundary condition is replaced by the lower boundary condition (4.9) or (4.10) for topographically forced waves. It is seen in Fig. 3a (0-10 km) that D has the same sign as $\bar{u}_t$ but underestimates it. At the 40-50 km levels (Fig. 3b), D has the same

Fig. 3 As in Fig. 1 except for the lower boundary condition for topographically forced waves.

Table 3 As in Table 1 except for the topographical boundary condition.

Z(km)	B and M	B only	M only
50	0.81	0.81	0.81
40	0.87	0.87	0.87
30	1.05	1.05	1.05
20	1.72	1.72	1.72
10	16.85	16.85	16.85
9	186.14	186.14	186.14
8	-20.85	-20.85	-20.85
7	-9.95	-9.95	-9.95
6	-6.58	-6.58	-6.58
5	-4.95	-4.95	-4.95
4	-3.99	-3.99	-3.99
3	-3.36	-3.36	-3.36
2	-2.91	-2.91	-2.91
1	-2.58	-2.58	-2.58
0	-2.33	-2.33	-2.33

sign as $\bar{u}_t$ but overestimates it. Fig. 3b is very similar to Fig. 1b, since the effect of the lower boundary is negligible. When N is replaced by its stratospheric value, the stratospheric acceleration (not illustrated) is very similar to Fig. 2b and $\bar{u}_t$ is close to D .

Table 3 is the same as Table 1 ($N=10^{-2}\text{s}^{-1}$) except for the adoption of the topographical boundary condition. Above 50 km, r_u is close to $r_D=0.77$, since the effect of lower boundary is negligible there. In the troposphere r_u becomes negative due to the lower boundary condition. However, $|r_u|/(f^2N^{-2})$ is larger than 1, being consistent with the fact (Fig. 3a) that $\bar{u}_t$ has the same sign as D .

5. Effects of mean damping

In the previous sections the effects of mean damping on wave-mean flow interaction were not considered. It is of importance to study these effects, since waves affect not only mean flows but also mean damping which, in turn, affects mean flows. In this section, simple diagnostic relations are derived to study the effects of mean damping.

When eddies are added to the steady state equilibrated without eddies, the deviation (Δ) of the zonal mean from the steady state is governed by

$$\frac{\partial \Delta \bar{u}}{\partial t} = -\Delta M_y + f \Delta \bar{v} - \varepsilon \Delta \bar{u}, \quad (5.1)$$

$$\frac{\partial \Delta \bar{T}}{\partial t} = -\Delta B_y - R_H^{-1} N^2 \Delta \bar{w} - \gamma \Delta \bar{T}, \quad (5.2)$$

$$(\rho_0 \Delta \bar{v})_y + (\rho_0 \Delta \bar{w})_z = 0, \quad (5.3)$$

and

$$f \Delta \bar{u}_z + R_H \Delta \bar{T}_y = 0, \quad (5.4)$$

where ε and γ are the Rayleigh and Newtonian damping coefficients, respectively. These mean dampings crudely represent diffusion and radiative damping.

In the above equations, the perturbed mean advection terms involving the deviation (Δ) are neglected. This neglect is justified in the mid-latitudes according to a quasi-geostrophic Rossby number truncation. The *unperturbed* mean advection terms drop out together with the other unperturbed terms, since the unperturbed terms balance each other. The *unperturbed* mean advection terms play an important role in the steady state tropical Hadley circulation (Schneider and Lindzen, 1977; Schneider, 1977), particularly when internal friction is infinitesimally small (Held and Hou, 1980).

The effect of surface friction and surface sensible heat flux can be crudely incorporated by varying ε and γ as

$$\varepsilon = \varepsilon_R \quad \text{for } z > z_b, \quad (5.5a)$$

$$= \varepsilon_b \quad \text{for } 0 < z < z_b, \quad (5.5b)$$

and

$$\gamma = \gamma_N \quad \text{for } z > z_b, \quad (5.6a)$$

$$= \gamma_N + \gamma_b \quad \text{for } 0 < z < z_b. \quad (5.6b)$$

where z_b is the depth of the surface layer and it is assumed that the temperature at the earth's surface is not affected by eddy heat flux.

By omitting the time derivatives and neglecting ε_R as in observational diagnostics, the following steady state diagnostic relations can be obtained from (5.1)-(5.4) as explained below :

$$\Delta \bar{v}(z) = f^{-1} \Delta M_y(z) \quad \text{for } z > z_b, \quad (5.7)$$

$$\Delta \bar{v}(0) \approx -z_b^{-1} \rho_0^{-1}(0) \int_{z_b}^{\infty} \rho_0 \Delta \bar{v} dz, \quad (5.8)$$

$$\Delta \bar{w}(z) = \rho_0^{-1} \int_z^{\infty} \rho_0 \Delta \bar{v}_y dz, \quad (5.9a)$$

$$= f^{-1} \rho_0^{-1} \int_z^\infty \rho_0 \Delta M_{yy} dz \quad \text{for } z > z_b, \quad (5.9b)$$

$$\Delta \bar{u}(0) = \varepsilon_b^{-1} \{-\Delta M_y(0) + f \Delta \bar{v}(0)\}, \quad (5.10a)$$

$$\approx -\varepsilon_b^{-1} z_b^{-1} \rho_0^{-1}(0) \int_0^\infty \rho_0 \Delta M_y dz, \quad (5.10b)$$

$$\Delta \bar{T}(0) = -(\gamma_N + \gamma_b)^{-1} \Delta B_y(0), \quad (5.11)$$

$$\Delta \bar{T}(z) = \gamma_N^{-1} [-\Delta B_y(z) - R_H^{-1} N^2 \Delta \bar{w}(z)], \quad (5.12a)$$

$$= \gamma_N^{-1} R_H^{-1} N^2 f^{-1} \rho_0^{-1} \int_z^\infty \rho_0 \Delta D_y dz \quad \text{for } z > z_b, \quad (5.12b)$$

and

$$\begin{aligned} \Delta \bar{u}(z) \approx & -f^{-1} R_H \int_{z_b}^z \Delta \bar{T}_y(z) dz \\ & - f^{-1} R_H \Delta \bar{T}_y(0) z_b + \Delta \bar{u}(0). \end{aligned} \quad (5.13)$$

Eq. (5.7) follows from the zonal momentum balance (5.1) for $\varepsilon_R=0$. Eq. (5.8) is given by integrating the continuity equation (2.3) from $z=0$ to z_b and from $z=z_b$ to ∞ with $\rho_0 \bar{w}=0$ at $z=0$ and ∞ . Eq. (5.9a) results by integrating the continuity equation (2.3) from z to ∞ with $\rho_0 \bar{w}=0$ at $z=\infty$, while (5.9b) is given by inserting (5.7) into (5.9a). Eq. (5.10b) is found by inserting (5.8) into the momentum balance (5.10a). Eq. (5.11) follows from the heat balance (5.2) with $\bar{w}=0$ at $z=0$. Eq. (5.12b) is given by inserting (5.9b) into the heat balance (5.12a). Eq. (5.13) results by integrating the thermal-wind balance (5.4) from 0 to z .

According to (5.7) the steady state $\Delta \bar{v}$ depends on ΔM only and not on ΔB in the absence of internal friction. However, the transient state $\Delta \bar{v}$ depends on both ΔM and ΔB and in turn affects $\Delta \bar{u}$. As a result, the steady state $\Delta \bar{u}$ depends on both ΔM and ΔB , being consistent with the thermal wind relation (5.13) with (5.12b).

According to (5.9b), the *steady* state indirect cell is associated with positive and negative ΔM_{yy} in the southern and northern latitudes, respectively, whereas the *transient* indirect cell due to ΔM occurs in the absence of mean damping when ΔM increases with height ($\Delta M_z > 0$) according to (3.3).

According to (5.13), $\Delta \bar{u}$ is approximately determined by $\Delta \bar{u}(0)$, $\Delta \bar{T}_y(0)$ at the surface and $\Delta \bar{T}_y(z)$ above the surface layer. In the case of baroclinic waves, $\Delta \bar{u}(0)$ is positive and acts to increase westerlies, while $-\Delta \bar{T}_y(0)$ and $-\Delta \bar{T}_y(z)$ are negative and act to decrease the vertical shear. If γ_b and γ_N are sufficiently small and ε_b is sufficiently large, the effect of $\Delta \bar{T}_y(0)$ and $\Delta \bar{T}_y(z)$ dominate the effect of $\Delta \bar{u}(0)$. Then, $\Delta \bar{u}$ can become easterly although $\bar{u}_t$ is westerly in the absence of mean damping. This possibility may explain why the steady state westerlies in the mid-latitude upper troposphere are reduced by including transient eddies according to Trenberth's (1973, Fig. 4, Fig. 8) numerical experiments.

The effect of eddy fluxes on $\Delta \bar{u}$ can also be interpreted in terms of acceleration as follows. The combined effect of B and M is to accelerate westerlies in the absence of mean damping. However, the transient state $\Delta \bar{u}$ and $\Delta \bar{T}$ affect $\bar{u}_t$ and $\bar{T}_t$ not only directly through $-\varepsilon \Delta \bar{u}$ and $-\gamma \Delta \bar{T}$ but also through $\Delta \bar{v}$ and $\Delta \bar{w}$ which are affected by the mean damping. In particular, the surface friction enhances an *indirect* cell due to the Ekman pumping, since the surface flow is westerly in the presence of M . On the other hand, the surface sensible heat flux enhances a *direct* cell, since this heat flux counterbalances the effect of B . Thus, for sufficiently strong surface friction and weak surface sensible heat flux, there is the possibility that the indirect cell becomes strong enough to overcome the effect of westerly acceleration by $-\Delta M_y$.

6. Conclusions and implications

In order to theoretically interpret the Eliassen-Palm diagnostics of wave-mean flow interaction, necessary and sufficient conditions are derived for the residual mean circulations to be negligible and the Eliassen-Palm flux divergence (EPFD) to approximate the zonal acceleration ($\bar{u}_t$) in the absence of mean damping. These conditions are summarized as follows:

- (1) A diagnostically necessary and sufficient condition is that the vertical-lateral scale ratio of $\bar{u}_t$ as defined by (3.8) is much larger than $f^2 N^{-2}$, where f and N are

the Coriolis and Brunt-Väisälä frequencies, respectively.

- (2) A necessary condition is that the vertical-lateral scale ratio of EPFD as defined by (3.12) is much larger than $f^2 N^{-2}$.
- (3) Even if Condition (2) holds everywhere, the approximation does not necessarily hold at low levels due to the lower boundary conditions.

Condition (1) is essentially the same as the "tall mean flow condition" (Andrews and McIntyre, 1976) based on a scale analysis of a Boussinesq fluid. The present definition of the scale ratio (3.8) is more appropriate for a compressible fluid.

Condition (2) explains the large residual-mean circulation associated with the sudden warming due to planetary waves with a small vertical scale of EPFD as studied theoretically by Matsuno and Nakamura (1979).

Condition (3) may explain why the heat flux of thermally forced stationary planetary waves results in larger residual circulations than the eddy induced circulations in the troposphere, whereas that of topographically forced waves results in much smaller residual circulations than the eddy-induced circulations as studied theoretically by Trenberth (1983, Fig. 14, Fig. 13). Under the topographical lower boundary condition, an eddy heat flux results in less residual circulations than under the non-topographical condition as demonstrated by the present analytical model.

It is shown by a simple analytical model that a violation of Condition (1) due to the effect of the lower boundary conditions may explain why some observational diagnostic studies indicate that transient eddies accelerate westerlies in the mid-latitude upper troposphere, although the transient EPFD is easterly there. For Eady and Charney baroclinic wave (see Appendix D), however, easterlies are accelerated in the upper layer, since the eddy momentum flux vanishes for these waves.

It is shown by simple diagnostic relations that the steady state upper tropospheric westerlies can be either enhanced or reduced in the presence of mean damping and mean radiative heating by adding transient waves, although these waves accelerate westerlies in

the absence of mean damping. The reduction can occur when waves accelerate the surface westerlies and, in turn, enhance frictionally induced indirect mean circulations which decelerate the upper westerlies due to the Coriolis effect. This may explain the result of the numerical experiments of Trenberth (1973) and Taylor (1980). On the other hand, a recent diagnostic model of Schneider (1984, p. 1107) indicates that the steady state upper tropospheric westerlies are enhanced by the combined effects of eddy momentum and heat fluxes. Since these results can be sensitive to the model resolutions, radiative heating, surface sensible heat flux and surface drag, the effects of transient waves on the zonal wind should be re-examined by the use of realistic general circulation models. According to the preliminary results of experiments with a current GFDL general circulation model, the winter tropospheric jet is shifted equatorward in the absence of transient waves being qualitatively consistent with the eddy momentum flux convergence and divergence, while the spring tropospheric jet is increased in situ being qualitatively consistent with the EPFD. These results will be reported in detail elsewhere.

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Appendix A

List of symbols

y	northward distance
z	vertical "log pressure" coordinate [$\equiv -H \ln(p/p_s)$]
t	time
u	eastward velocity
v	northward velocity
w	"vertical velocity" in log pressure coordinates [$\equiv dz/dt$]
$\rho_0 \psi$	mass stream function of meridional circulation
T	departure of temperature from T_0

$T_0(z)$	basic state temperature
Q	heating rate per unit mass
ϕ	departure of geopotential from basic state
p	pressure
$\rho_0(z)$	basic state density
M	northward eddy momentum flux
B	northward eddy heat flux
D	Eliassen-Palm flux divergence
H	constant scale height
h	equivalent depth
h_R	Rossby depth
R	gas constant for dry air [$=287 \text{ m}^2 \text{ s}^{-2} \text{ K}^{-1}$]
R_H	gas constant divided by H [$\equiv R/H$]
c_p	specific heat at constant pressure
g	gravitational acceleration [$=9.8 \text{ m s}^{-2}$]
f	constant Coriolis parameter
N	constant Brunt-Väisälä frequency
l	meridional wavenumber of eddy flux
m	vertical decaying constant of eddy flux
μ	vertical decaying constant of homogeneous solution
r_u	vertical-lateral scale ratio of mean flow acceleration
r_D	vertical-lateral scale ratio of D
ϵ	Rayleigh friction coefficient
γ	Newtonian cooling coefficient

Appendix B

Boundary conditions

The lower boundary condition for Eulerian-mean flow in the presence of topography and in the absence of mean Ekman pumping is given (Charney and Drazin, 1961; Andrews, 1980) by

$$\bar{w} \approx g R_H^{-1} T_0^{-1} (\bar{v}' h')_y, \quad (\text{B.1})$$

where h is the topographic height.

In the absence of heating and Ekman pumping in stationary waves, $\bar{v}' h'$ is reduced to

$$\bar{v}' h' = -R_H N^{-2} B. \quad (\text{B.2})$$

The condition (B.1) is more rigorously written in terms of the vertical velocity ($g^{-1}(d\phi/dt)$) instead of $\bar{w}$ as

$$\bar{\phi}_t + R_H T_0 \bar{w} = g (\bar{v}' h')_y. \quad (\text{B.3})$$

This condition is reduced to

$$f^2 \rho_0^{-1} (\rho_0 \bar{\phi})_z + R T_0 \bar{\phi}_{yy} = -f M_y + g (\bar{v}' h')_{yy} \quad \text{at } z=0. \quad (\text{B.4})$$

It should be noted that (B.3) is reduced to (B.1) when the time derivative is negligible.

Appendix C

The Green's function

The solution to (3.1) for imposed D with arbitrary vertical structure and sinusoidal lateral structure is given by

$$\bar{u}_t = \int_0^\infty G(z-z') D_{yy} dz' + C_0 \exp(-\mu z) \cdot \exp(z/H) \sin(l y) \quad (\text{C.1})$$

where G is the Green's function for an infinite vertical domain given by

$$G(z) = G(0) \exp(-\mu z) \exp(z/H), \quad z > 0 \quad (\text{C.2a})$$

$$= G(0) \exp(\mu z), \quad z < 0, \quad (\text{C.2b})$$

with μ defined by (4.16a).

C_0 in (C.1) is determined by the lower boundary condition (4.8) and depends on $D(z)$ and $B(0)$.

It should be noted that $\bar{u}_t$ depends on $D(z)$ and $B(0)$ but does not explicitly depend on $M(z)$, although $\bar{u}_t$ implicitly depends on $M(z)$ through $D(z)$. The Green's function $G(z)$ behaves like a delta function relative to the vertical profile of D for sufficiently large μ and sufficiently large vertical scale of D . In this case, $\bar{u}_t$ is approximated by D at a sufficiently large height where the second term in (C.1) is negligible, being consistent with the necessary condition (3.12).

Appendix D

Mean circulations induced by Eady and Charney baroclinic waves

Baroclinically unstable waves studied by Eady (1949) and those by Charney (1947) are associated with poleward heat fluxes and zero momentum fluxes. The essential differences between these two waves are the absence of the beta effect, compressibility and the presence of a lid in the Eady wave. The eddy heat flux and E-P flux are vertically constant for the Eady wave, while these fluxes decrease

Fig. 4a Schematic latitude-height sections of eddy heat flux (vectors) and Eulerian-mean circulations (contours) for an Eady wave (left) and a Charney wave (right) in a channel. Steering levels are marked sl .

Fig. 4b Schematic latitude-height sections of E-P fluxes (vectors) and residual circulations (contours) for an Eady wave (left) and a Charney wave (right) in a channel. Steering levels are marked sl . This picture is partly based on Edmon *et al.* (1980, Fig. 2) and Hoskins (1982, Fig. 7.3).

with height for the Charney wave. As illustrated schematically by Fig. 4a, the eddy heat fluxes induce indirect cells which in turn accelerate westerlies and easterlies in the lower and upper layers, respectively.

Fig. 4b shows schematically that the residual circulations for the Eady wave consist of direct and indirect circulations in the lower and upper layers (see Kanzawa, 1984a, p. 42 for the analytical solution). The vertical velocity (2.14) at the lower and upper boundaries is consistent with the poleward eddy

heat fluxes. The mean Coriolis force due to these residual circulations accelerates westerlies and easterlies in the lower and upper layers, while the E-P flux divergence vanishes everywhere. These residual circulations resemble the Lagrangian-mean circulations induced by the growing Eady wave as studied by Uryu (1979, Fig. 2c).

In the case of the Charney wave, the E-P flux divergence (D) forces easterlies, while the residual circulations force westerlies at all the levels. The residual circulations dominate D and forces westerlies in the lower layer, while D dominates the residual circulation and forces easterlies in the upper layer. The results based on the Charney wave are similar to those (Fig. 1a) of the present model with a non-topographical boundary condition in the absence of an eddy momentum flux.

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波動—平均流相互作用の Eliassen-Palm 解析についての理論的解釈

第1部：地表境界条件の効果

林 良 一

Geophysical Fluid Dynamics Laboratory/NOAA

波動—平均流相互作用の Eliassen-Palm 解析についての理論的解釈を行った。

Mean damping のない場合, Eliassen-Palm flux divergence (EPFD) が平均流加速の近似となるための必要十分条件を導いた。地表境界条件の影響のためにこの必要十分条件が成立しなくなる可能性を簡単な理論的モデルで示し, 実際の中緯度上部対流圏の観測では非定常波に伴う EPFD は東風加速の傾向を持つにもかかわらず, 非定常波は西風を加速する理由を示した。

Mean damping や radiative heating の大きさによって, 上部対流圏定常状態の西風は非定常波動により強められたり弱められたりする可能性がある事を簡単な関係式で示した。