NOTES AND CORRESPONDENCE

A Supplementary Study of Simulations of Mountain Wave — Critical Level Interactions**By Kikuro Tomine***Institute of Geosciences and Astronomy, The National Defense Academy, Hashirimizu, Yokosuka 239, Japan**(Manuscript received 12 February 1986 in revised form 19 December 1986)***Abstract**

An extraordinarily deep local depression formed on the lee side of a mountain range is investigated under a nonhydrostatic condition and is compared with the results of a hydrostatic model (Tomine, 1984). It is found that nonhydrostatic effects on the large amplitude mountain wave seem insignificant within a framework of the present model.

1. Introduction

Large amplitude mountain wave affected by a critical level has been investigated by some authors. Among them, Peltier and Clark (1979) observed strong downslope windstorms at Boulder, Colorado, and stressed that both nonlinear and nonhydrostatic effects could be important for the windstorms associated with lee waves. It was described in their paper that strong reflection of gravity waves by the wavebreaking level and quasi — resonant amplification of the waves were also important. In their subsequent paper (Clark and Peltier, 1984), an amplified resonant lee wave was simulated by changing the height of the critical layer.

Using their nonlinear hydrostatic model, Klemp and Lilly (1978) simulated mountain waves over which a critical level exists. They recognized the large magnitude of momentum flux and reflections of the wave from the critical layer in the model, which had been not simulated in their linear hydrostatic model. But they wondered whether the reflection from the regions of wave breakdown (or from the critical layer) was realistic even in any other numerical models.

Of course, any overturning instability and wave breakdown cannot be so realistically reproduced in the hydrostatic model in the isentropic coordinate system by Klemp and Lilly (1978). But it seems very important for us that their results showed the large amplitude mountain wave owing to nonlinearity.

Tomine (1984) simulated a large amplitude mountain wave in a shear flow including a critical level in his nonlinear hydrostatic model. The overturning instability was clearly seen in the vicinity of the critical level, showing that wave breaking took place there. However, the large amplitude mountain wave in Tomine (1984) could not be explained as any resonant mountain wave shown as by Clark and Peltier (1984), because the ratio of the height of the critical level to the vertical hydrostatic wavelength in his work was 0.31. This is too small to show any resonance. The smallest ratio needed for resonance to take place was 0.75 in Clark and Peltier (1984). In case of Klemp and Lilly (1978), it was 0.45 when the large magnitude of momentum flux occurred. This large amplitude mountain wave must be essentially the same as that obtained by Tomine (1984) in which the ratio was smaller than 0.75.

Here we are much interested in the large amp-

litude mountain wave shown by Klemp and Lilly (1978) and Tomine (1984) which shows the importance of nonlinearity. But before investigating in detail characteristics of the mountain wave, we need make sure nonhydrostatic effects on it. Since reflection from the wave breaking level, as described by Klemp and Lilly (1978) and Peltier and Clark (1979), might be modified by hydrostatic assumption. We will simulate mountain waves using both hydrostatic and nonhydrostatic models and compare the results by the both models.

2. Governing equations and numerical model

Two - dimensional disturbances are considered as in the previous paper (Tomine, 1984). They obey the Boussinesq approximation. The governing equations for a nonhydrostatic model are as follows:

$$\frac{d\zeta}{dt} - \frac{g}{\Theta} \frac{\partial \theta}{\partial x} = D_1 + D_2, \quad (1)$$

$$\frac{d\theta}{dt} = C_1, \quad (2)$$

$$\nabla^2 \psi = -\zeta, \quad (3)$$

$$D_1 = K_1 \nabla^2 \zeta, \quad D_2 = -K_2 \zeta,$$

$$C_1 = K_1 \nabla^2 \theta,$$

where ζ , χ and θ are the vorticity, the stream function and the deviation of potential temperature from a constant value Θ (300 K), respectively. g the acceleration of gravity and K_1 the eddy diffusion coefficient for momentum and heat assumed to be constant in the whole region of the numerical mode ($30 \text{ m}^2 \text{ sec}^{-1}$). D_2 the Rayleigh damping term for the disturbances

(Tomine, 1984). In a hydrostatic model $-\frac{\partial u}{\partial z}$

is substituted for vorticity ($\zeta = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}$)

in equation (1). We represent the governing equations by the variables deviated from the initial state as follows:

$$\left. \begin{aligned} \zeta &= \zeta_i + \zeta' \\ \psi &= \psi_i + \psi' \\ \theta &= \theta_i + \theta' \end{aligned} \right\}, \quad (5)$$

where the suffix i means the initial state and

dashed variables are the deviations. Substituting (5) into (1), (2), (3) and (4), the perturbation equations are

$$\left. \begin{aligned} \frac{\partial \zeta'}{\partial t} + u_i \frac{\partial \zeta'}{\partial x} + J(\psi', \zeta') - \frac{g}{\Theta} \frac{\partial \theta'}{\partial x} &= D_1 + D_2, \\ \frac{\partial \theta'}{\partial t} + u_i \frac{\partial \theta'}{\partial x} + w \frac{d\theta_i}{dz} + J(\psi', \theta') &= C_1, \\ \nabla^2 \chi' = -\zeta', \quad u_i &= -\frac{d\chi_i}{dz}, \quad w = \frac{\partial \chi'}{\partial x}. \end{aligned} \right\} \quad (6)$$

The next step of the numerical calculation is to expand these perturbation quantities to Fourier series of the horizontal direction following Tanaka (1975).

$$\begin{pmatrix} \zeta' \\ \psi' \\ \theta' \end{pmatrix} = \int_0^K \left\{ \begin{pmatrix} \zeta_{ck} \\ \psi_{ck} \\ \theta_{ck} \end{pmatrix} \cos \frac{kx}{z_0} - \begin{pmatrix} \zeta_{sk} \\ \psi_{sk} \\ \theta_{sk} \end{pmatrix} \sin \frac{kx}{z_0} \right\} dk,$$

where z_0 ($= 1 \text{ km}$) is the thickness of the wind shear layer in the model, all these Fourier coefficients are functions of height and time following Tanaka (1975). We assume the maximum wave number K to be $2/3 \text{ (km}^{-1}\text{)}$ without missing so much energy.

(a) Initial state

Initial condition of the flow are given by

$$u_i(z) = -U \tanh(z - z_c)/z_0,$$

$$\theta_i(z) = \gamma(z - z_c),$$

$$U = 20 \text{ m sec}^{-1},$$

$$\gamma = 5 \text{ K km}^{-1},$$

$$z_0 = 1 \text{ km},$$

$$z_c = 3 \text{ km}.$$

The initial wind profile is illustrated in Fig. 1. The broken line in the figure represents the initial position of a critical level. Characteristic Richardson number is

$$R_i = -\frac{g}{\Theta} \frac{r z_i}{U^2} \cong 0.41$$

(b) Geometry of the computational domain, the mountain profile and boundary conditions

The computational domain is 9 km high and the vertical grid size Δz is 30 m. The interval of

Fig. 1. Vertical profile of general wind with a critical level. The broken line in the figure represents the original critical level.

the horizontal wave number Δk and time increment Δt are $2/300 \text{ km}^{-1}$ and 4 sec, respectively. The profile of the mountain range $f(x)$ is

$$f(x) = h \exp(-x^2/\lambda^2)$$
$$= \frac{h\lambda}{\sqrt{\pi}} \int_0^\infty \exp\left(-\frac{\lambda^2 k^2}{4}\right) \cos(kx) dk,$$

where h and λ , the height and the width of the mountain, are 500 m and 15 km.

Rayleigh damping term D_2 in eq. (1) is applied only to the upper layer with 3 km thickness as in the previous paper (Tomine, 1984). The rigid lid and slipping boundary conditions are adopted to the top of the model. The lower boundary conditions are assumed at the bottom ($z = 0$),

$$\theta_b = \gamma(-f(x) - z_c),$$
$$\phi_b = z_0 U \ln \{ \cosh(-z_c/z_0) \}$$
$$- f(x) U \tanh(-z_c/z_0),$$
$$\zeta_b = 0,$$

where $f(x)$ represents the profile of the mountain range.

3. Comparisons of results in the hydrostatic and nonhydrostatic models

Distributions of the stream function after 180 minutes from the initial state are shown in Figs. 2(a) and 2(b) for the hydrostatic and nonhydrostatic models, respectively. The both flow patterns are not so different from each other. A large rotor is observed just below the original critical level and a strong wind region on the lee side of the mountain range as in the previous

a

b

Fig. 2. Distributions of stream function ($\times 2 \times 10^4 \text{ m}^2 \text{ sec}^{-1}$) after 180 minutes from the initial state in the (a) hydrostatic model and the (b) nonhydrostatic model.

Fig.3. Distributions of horizontal velocity deviations ΔU ($\times 20 \text{ m sec}^{-1}$) at 180 minutes in the nonhydrostatic model.

paper (Tomine, 1984). Distributions of wind velocity deviations (ΔU) from the initial state in the nonhydrostatic model at 180 minutes are

Fig. 4. Evolutions of vertical distributions of vertical momentum flux ($\times 4 \times 10^{-2} \text{ m}^2 \text{ sec}^{-2}$) in the (a) hydrostatic model and the (b) non-hydrostatic model.

shown in Fig. 3. A strong wind region on the lee side and a weak wind region above there corresponding to the large rotor are observed clearly. Inclination of the positive ΔU region to the windward side from the vertical becomes large in the vicinity of the critical level. It means that gravity waves of large vertical wave number (m) become predominant there (cf. Bretherton, 1966). In the hydrostatic model the same aspects of ΔU are observed.

Evolutions of vertical distributions of the vertical momentum flux in the both models are shown in Fig. 4. The zero-flux line is clearly shifted from the initial critical level. The increase of the momentum flux suggests the existence of

growing mountain waves.

4. Conclusions

The weight of hydrostatic assumption on the large amplitude mountain waves is investigated by numerical simulations using the hydrostatic and nonhydrostatic models. In the both, the simulated flow fields are almost the same. Therefore the nonhydrostatic effects on the large amplitude mountain waves seem to be insignificant at least in the present case.

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山岳波とクリティカルレベルの相互作用の
数値シミュレーションによる研究の補稿

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