A Two Level Model of the Steady State Response of the Atmosphere to Midlatitude Heating with Various Zonal Structures

By Andrew J. Weaver

Departments of Mathematics and Oceanography, University of British Columbia
Vancouver, British Columbia, Canada, V6T 1Y4

(Manuscript received 19 February 1987, in revised form 12 June 1987)

Abstract

The effect of the horizontal structure of midlatitude oceanic heating on the stationary atmospheric response is examined by means of two simple two level quasigeostrophic β-plane channel models. Solutions are obtained for three non-periodic zonal heating structures (line source, segmented cosine, and segmented sine), and these are compared with the results for a continuously stratified model. Little difference is observed between the solutions for these two different models.

There are two cases which emerge in obtaining analytic solutions. In case 1, for large meridional wavenumbers, there exists a large local response and a constant downstream response. In case 2, for small meridional wavenumbers, the far field response is now sinusoidal. A critical wavenumber separating these two cases is obtained.

The effect of oceanic heating on the atmosphere over the Kuroshio region is examined in an attempt to explain the large correlations observed between winter Kuroshio oceanic heat flux anomalies, and the winter atmospheric surface pressure and 500 & 700 mb geopotential heights, both upstream and downstream of the heating region. It is found that the model response is consistent with the observed correlations. When western North Pacific heating and eastern North Pacific cooling are introduced into the model, a large low pressure response is observed over the central North Pacific. This feature is in excellent agreement with the observed correlations.

1. Introduction

In recent years many investigators have attempted to explain the observed correlations between sea surface temperature (SST) in the North Pacific Ocean, and the overlying atmospheric circulation in the troposphere (see Frankignoul, 1985a for a review). Since the pioneering works of Namias (1959) and Uda (1962), where this correlation was established, research has been carried out to resolve the question of cause and effect. Davis (1976), on the basis of lag-correlations between monthly SST anomalies and sea level pressure, concluded that the atmosphere was indeed driving the ocean, although, in a later paper using seasonal statistics, Davis (1978) showed that autumn and winter sea level pressure anomalies could be reasonably predicted from summer and autumn SST anomalies respectively. However, numerical atmospheric general circulation model (AGCM) investigations (e.g., Huang, 1978) have shown that the atmosphere is relatively insensitive to midlatitude SST anomalies unless they are unreasonably large (Chervin et al., 1980). The linear analysis of Webster (1981, 1982) attempted to resolve this apparent paradox. He argued that a significant atmospheric response to midlatitude SST anomalies could only arise when the basic flow was small (i.e., during summer).

Since the surface heat flux is a more accurate way of representing the oceanic thermal influence on the atmosphere than the SST used by the above authors, researchers have begun to investigate the relationship between the oceanic heat flux Q and the overlying atmospheric circu-
lation. \( Q \) is comprised of the latent, sensible and backward radiative heat fluxes. Zhao and McBean (1987b) have recently examined the possible relationship between monthly mean surface heat flux anomalies and the overlying atmospheric circulation patterns. They found that surface heat flux anomalies over the Kuroshio region are significantly correlated with downstream atmospheric conditions in following months. As a continuation of their study, Weaver and Mysak (1986b) reworked their analysis using seasonal statistics. Once again, large correlations were observed between Kuroshio heat flux anomalies and atmospheric conditions downstream. The most striking correlations were observed between winter Kuroshio heat flux anomalies and the winter 500 & 700 mb geopotential heights and surface pressure over the North Pacific, eastern Asia and Canada.

Most linear wave theories of large-scale thermally-driven atmospheric motions have assumed a forcing \( Q_\infty \cos kx \) in order to model the differential heating associated with land and sea thermal contrasts (e.g., Frankignoul 1985a, who uses a two level \( \beta \)-plane channel model; Pedlosky 1979, who employs a continuously stratified \( \beta \)-plane model). One purpose of this paper is to introduce more realistic zonal structures of the oceanic diabatic heating anomalies and hence to determine the effects of these distributions on the atmospheric stationary wave response. To accomplish this we use a simple two level linear quasigeostrophic model.

Weaver, Mysak and Bennett (1987), hereafter called WMB, examined the effects of localized heating on the steady state atmospheric response using a continuously stratified quasigeostrophic model, linearized about a constant zonal flow \( U_0 \). They found that for parameters applicable to North Pacific wintertime conditions, there was a strong local response and a constant far field response. A second purpose of this paper is therefore to examine the effect of vertical shear on the results of WMB.

Zhao and McBean (1987a) also examined the principal patterns (EOF) of variability of the anomalies of total heat transfer from the ocean to the atmosphere over the North Pacific Ocean. In winter they found that the first EOF consisted of a bipolar pattern, with anomalous heating over the western North Pacific and anomalous cooling over the eastern North Pacific of nearly equal magnitude (see for example Fig. 8a below). They found that this first principal component accounted for about 21% of the total variance. In this paper we employ one particular zonal heating structure to model this phenomenon and hence to seek a simple explanation of the aforementioned correlations of Zhao and McBean (1987b) and Weaver and Mysak (1986b).

The outline of this paper is as follows. In Section 2 the model and the assumed heating distribution are introduced. In Section 3 a line source zonal heating structure is examined. The solution obtained from this line source heating is used as a Green's function in Section 4 to obtain solutions for segmented cosine and segmented sine distributions. In Section 5 the solutions obtained in Sections 3 and 4 are analyzed and compared with the results of WMB. Finally, in Section 6 we examine the effect of sine forcing over one wavelength in order to model the atmospheric response to heating with the structure of the first EOF of Zhao and McBean (1987a).

2. Description of the model

The two level quasigeostrophic model in the \( \beta \)-plane approximation is used. In pressure coordinates the linearized vorticity and thermodynamic equations for a small stationary disturbance are (Holton, 1979, Frankignoul, 1985a)

\[
\bar{U} \frac{\partial}{\partial x} \phi + \bar{\beta} \frac{\partial}{\partial x} \phi - f_o \frac{\partial}{\partial p} \omega = 0, \tag{1}
\]

\[
\bar{U} \frac{\partial}{\partial x} \frac{\partial \phi}{\partial p} - \frac{\partial \phi}{\partial x} \frac{\partial U}{\partial p} + \frac{\partial}{f_o} \omega = - \frac{R q'}{C_p f_o p}, \tag{2}
\]

where \( \phi \) is the geostrophic streamfunction, \( \bar{U}(p) \) is the zonal wind, and \( \omega \) is the vertical velocity. The mean static stability parameter

\[
\bar{\sigma} = \frac{R}{p} \left\{ \frac{RT}{C_p p} - \frac{\partial T}{\partial p} \right\}
\]

is assumed to be constant and the diabatic heating \( q' \) is assumed to have the form

\[
q' = Q \cos ky \delta(x - p). \tag{4}
\]
Two Level Model

\[ \begin{align*}
\omega_0 = 0 \quad & \quad p_0 = 200 \text{ mb} \\
U_1, \ \phi_1 & \quad - & \quad - & \quad - & \quad - & \quad p_1 \\
U_2, \ \phi_2 & \quad - & \quad - & \quad - & \quad - & \quad p_2 \\
U_3, \ \phi_3 & \quad - & \quad - & \quad - & \quad - & \quad p_3 \\
\omega_0 = 0 \quad & \quad p_0 = 1000 \text{ mb}
\end{align*} \]

Fig. 1 Schematic diagram of the two level model indicating the five pressure levels. In Model 1, \( p_1 = 400 \text{ mb}, p_2 = 600 \text{ mb}, p_3 = 800 \text{ mb} \); in Model 2, \( p_1 = 500 \text{ mb}, p_2 = 800 \text{ mb}, p_3 = 900 \text{ mb} \).

The solution for the line source heating (4) will be used as a Green's function in Section 4 to obtain the atmospheric response to heating \( q' \) with other zonal structures.

In the two level approximation the vorticity is calculated at levels 1 and 3 (Fig. 1) and the thermodynamic equation (2) is applied at level 2. The boundary condition \( \omega = 0 \) is enforced at the surface, \( p_4 = 1000 \text{ mb} \) and at the tropopause, \( p_4 = 200 \text{ mb} \). If \( U_1 \) and \( U_3 \) are the constant zonal flows at levels 1 and 3 respectively, the resulting system of equations is

\[ U_1 \frac{\partial}{\partial x} \varphi_1 + \frac{\partial}{\partial x} \phi_1 + \frac{f_0}{\Delta p_1} \omega_2 = 0, \]  

(5)

\[ \Delta p_i = p_{i+1} - p_i, \quad \Delta p_2 = p_3 - p_1, \quad \Delta p_3 = p_4 - p_2 \]

where \( q' \) is given by (4). Here we have used the interpolated values \( \frac{p_2 - p_1}{\Delta p_2} U_2 + \frac{p_3 - p_2}{\Delta p_3} U_3 \) and \( \phi_2 = \frac{p_2 - p_1}{\Delta p_2} \phi_2 + \frac{p_3 - p_2}{\Delta p_3} \phi_1 \) for \( U \) and \( \phi \) at level 2.

3. Solution of the equations

The \( \beta \)-plane channel is assumed to be infinite in extent so that we may Fourier transform (5) – (7). Let

\[ (\phi_1, \phi_2, \omega_2) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\xi x} (\phi_1, \phi_2, \bar{\omega}_2) dx \]

(8)

where

\[ (\phi_1, \phi_2, \omega_2) = (\phi_1, \phi_2, \bar{\omega}_2) \cos y. \]

(9)

The system of differential equations (5) – (7) is then transformed into a system of algebraic equations which yield

\[ \omega_2 = -\frac{Q_0 f_1(\xi)f_2(\xi)e^{-i\xi x}}{\sqrt{2\pi} \sigma C_p \rho \xi (f_1(\xi)f_2(\xi) - \lambda^2 \alpha_i f_1(\xi) + \alpha_5 f_3(\xi))}, \]

(10)

\[ \phi_1 = -\frac{Q_0 f_1(\xi) f_3(\xi) e^{-i\xi x}}{\sqrt{2\pi} \Delta p_1 \sigma C_p \rho \xi (f_1(\xi)f_3(\xi) - \lambda^2 \alpha_i f_1(\xi) + \alpha_5 f_3(\xi))}, \]

(11)

\[ \phi_2 = -\frac{Q_0 f_2(\xi) f_1(\xi) e^{-i\xi x}}{\sqrt{2\pi} \Delta p_2 \sigma C_p \rho \xi (f_2(\xi)f_1(\xi) - \lambda^2 \alpha_i f_1(\xi) + \alpha_5 f_3(\xi))}, \]

(12)

where

\[ \alpha_i = \frac{2U_1 \Delta p_1}{\Delta p_1 + \Delta p_3}, \quad \alpha_5 = \frac{2U_3 \Delta p_3}{\Delta p_1 + \Delta p_3}, \quad f_1(\xi) = \beta - U_1 (l^2 + \xi^2), \quad f_3(\xi) = \beta - U_3 (l^2 + \xi^2). \]

(13)

The deformation radius \( \lambda^{-1} \) is defined such that

\[ \lambda^2 = \frac{f_2(\Delta p_1 + \Delta p_3)}{2 \sigma \Delta p_1 \Delta p_2 \Delta p_3}, \]

(14)

The expression \( f_1(\xi) f_3(\xi) - \lambda^2 \alpha_i f_1(\xi) + \alpha_5 f_3(\xi) \) in (10) – (12) has four zeros which occur at

\[ \xi = \pm \{ -l^2 - \gamma \pm |\gamma^2 + \delta|^{1/2} \}^{1/2}, \]

(15)

where

\[ \gamma = \frac{\lambda^2 (\alpha_i U_1 + \alpha_5 U_3) - \beta (U_1 + U_3)}{2U_1 U_3} \]

(16)
are both independent of the meridional wave-number $l$. Before taking the inverse Fourier transform of (10) - (12) we must find the physical location of the zeros defined by (15) and the pole at $\xi = 0$ in the complex plane. To do this we include Rayleigh friction and Newtonian cooling in the problem (see Appendix I). For simplicity the Rayleigh friction and Newtonian cooling coefficients are assumed to be equal ($\tau_0$).

We let the $\beta$-plane channel be centered at $40^\circ$N, so that $f_0 = 9.4 \times 10^{-5} \text{ s}^{-1}$ and $\beta = 1.8 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$. We also take $U_1 = 18 \text{ ms}^{-1}$, $U_3 = 6 \text{ ms}^{-1}$ and $\sigma = 3 \times 10^{-6} \text{ m}^4 \text{s}^2 \text{kg}^{-2}$, typical of wintertime conditions (Frankignoul, 1985a). There are two models which we consider. The first is for $p_1 = 400 \text{ mb}$, $p_2 = 600 \text{ mb}$, and $p_3 = 800 \text{ mb}$, so that $\Delta p_1 = \Delta p_2 = \Delta p_3 = 400 \text{ mb}$, $\alpha_1 = U_1$, $\alpha_3 = U_3$, and hence the levels are equally spaced. In this model the deformation radius is $\lambda^{-1} = 7.4 \times 10^5 \text{ m}$, and from (16) and (17), $\gamma = 1.1 \times 10^{-12} > 0$ and $\delta = 4.4 \times 10^{-24} > 0$. To focus more on near surface heating we also consider a second model for which $p_1 = 500 \text{ mb}$, $p_2 = 800 \text{ mb}$, and $p_3 = 900 \text{ mb}$, so that $\Delta p_1 = 600 \text{ mb}$, $\Delta p_2 = 400 \text{ mb}$, $\Delta p_3 = 200 \text{ mb}$, $\alpha_1 = \frac{3}{2} U_1$, $\alpha_3 = \frac{1}{2} U_3$. For this model the deformation radius is $\lambda^{-1} = 6.4 \times 10^5 \text{ m}$, and from (16) and (17), $\gamma = 4.1 \times 10^{-12} > 0$ and $\delta = 1.2 \times 10^{-23} > 0$.

There are two cases which need to be considered for both models:

$$\omega_x = \frac{\chi \cos ly}{2(\eta_1^2 - \eta_2^2)} \left[ \frac{f_1(i \eta_1) f_3(i \eta_3)}{\eta_1} e^{-\eta_1 |x-p|} - \frac{f_1(i \eta_2) f_3(i \eta_3)}{\eta_2} e^{-\eta_2 |x-p|} \right] \text{sgn}(x-p),$$

where $i \eta_1 = 1, i \eta_2 = \pm 2$, and

$$\chi = \frac{Q_R R f_1(i \eta_1) f_3(i \eta_3)}{\delta C_p p_3 U_3 U_1},$$

$$\text{sgn}(x-p) = \begin{cases} 1, & \text{if } x > p; \\ -1, & \text{if } x < p. \end{cases}$$

Thus the vertical velocity consists of two components decaying exponentially away from the heating source. There is a discontinuity at $x = p$.

**case 1:** $|\eta^2 + \delta|^{1/2} - \eta^2 > 0$. In this case there are two conjugate pairs of pure imaginary roots. Here we write

$$f_1(\xi) f_3(\xi) - \lambda^3 \left[ U_1 f_1(\xi) + U_3 f_3(\xi) \right] = U_1 U_3 (\xi^2 - \xi_1^2) (\xi^2 - \xi_2^2),$$

with

$$\xi_1 = i \left( |\eta^2 + \delta|^{1/2} - \eta^2 \right),$$

and

$$\xi_2 = i \left( |\eta^2 + \delta|^{1/2} + \eta^2 \right),$$

so

$$\omega_x = \frac{Q_R R f_1(i \xi_1) f_3(i \xi_2)}{\sqrt{2\pi} \delta C_p p_3 U_1 U_3 (\xi_1^2 - \xi_1^2) (\xi_2^2 - \xi_2^2)}.$$

In taking the inverse Fourier transform of (21) we close the contour of integration for $x > p$ in the upper half plane (UHP) and for $x < p$ in the lower half plane (LHP) (see Fig. 2). Thus

$$\omega_x = \frac{1}{\sqrt{2\pi}} \int_{C_R} e^{i \xi \xi} \omega d\xi,$$

$$= \left\{ \begin{array}{ll} \sqrt{2\pi} i \Sigma \text{Res at } \xi_1, \xi_2, & \text{if } x > p; \\ \sqrt{2\pi} i \Sigma \text{Res at } -\xi_1, -\xi_2, & \text{if } x < p. \end{array} \right.$$
and
\[
\psi_3 = -\frac{\chi f(x)}{\Delta p_3} \left\{ \frac{f_1(i\eta_1)}{2\eta_1^2(\eta_1^2-\eta_2^2)} e^{-\eta_1|x-p|} \right\} + \frac{f_3(i\eta_3)}{2\eta_3^2(\eta_3^2-\eta_2^2)} e^{-\eta_2|x-p|},
\]
where we have used (9) once more and \( H(x-p) \) is the Heaviside step function defined by
\[
H(x-p) = \begin{cases} 
1, & \text{if } x > p; \\
0, & \text{if } x < p.
\end{cases}
\] (27)

The streamfunction at both the upper and lower levels consists of a component exponentially decaying away from the source and a component constant for all \( x \). At \( x = p \) both \( \psi_1 \) and \( \psi_3 \) have a step discontinuity.

\textbf{case 2:} \( \eta^2 < (\eta^2 + \delta)^{1/2} \).

In this case (15) has two complex conjugate pure imaginary roots and two real roots of opposite sign. When Rayleigh friction and Newtonian cooling are included in the problem (Appendix I), the roots \( \xi = \pm \sqrt{-\eta - \gamma + \eta^2 + \delta} \) both move into the UHP (Fig. 3).

As in case 1, we close the contour of integration of the inverse Fourier transform in the UHP for \( x > p \) and in the LHP for \( x < p \), apply Jordan's Lemma to the integrals over the semi-circular regions of Fig. 3, and obtain
\[
\tilde{\omega}_n = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\xi x} \tilde{\omega} d\xi
\]
\[
= \begin{cases} 
\sqrt{2\pi} i\Sigma \operatorname{Res} \left. \pm \xi_1, \xi_2 \right|_{x > p}; \\
\sqrt{2\pi} i\Sigma \operatorname{Res} \left. \mp \xi_2 \right|_{x < p}.
\end{cases}
\] (28)

Evaluating the residues at \( \pm \xi_1 \) and \( \pm \xi_2 \) and substituting the result into (28) using (9) gives
\[
\omega_n = \frac{\chi f(x)}{2(\xi_1^2+\eta_2^2)} \left\{ \frac{2f_1(\xi_2)f_3(\xi_1)}{\xi_1} \sin \xi_1(x-p) H(x-p) + \frac{f_1(i\eta_1)f_3(i\eta_3)}{\eta_2} e^{-\eta_2|x-p|} \operatorname{sgn} (x-p) \right\}.
\] (29)
The solution (29) consists of the exponentially decaying component which arises from the poles at \( \xi = \pm \xi_2 \) (cf., 23), and a sinusoidal component only for \( x > p \). Once more a sign discontinuity is present at \( \xi = p \).

To obtain \( \psi_1 \) and \( \psi_3 \) we notice that the pole at \( \xi = 0 \) bifurcates into two poles with the inclusion of Rayleigh friction and Newtonian cooling, one in the UHP and one in the LHP (Appendix I). After some residue calculus similar to the above, one obtains

\[
\phi_2 = \frac{Xf_0 \cos \frac{y}{2}}{Dp_1} \left\{ \frac{f_1(0)}{\xi_2^2} \cos \xi_2^2(x-p)H(x-p) - \frac{f_1(i\eta_2)}{2\xi_2^2(\xi_2^2+i\eta_2)} e^{-\eta_2^2|x-p|} \right\},
\]

and

\[
\phi_1 = -\frac{Xf_0 \cos \frac{y}{2}}{Dp_3} \left\{ \frac{f_1(0)}{\xi_1^2} \cos \xi_1^2(x-p)H(x-p) - \frac{f_1(i\eta_1)}{2\xi_1^2(\xi_1^2+i\eta_1)} e^{-\eta_1^2|x-p|} \right\}.
\]

Both \( \phi_1 \) and \( \phi_2 \) retain the contribution from the poles at \( \xi = \pm \xi_2 \) which decays exponentially away from the heat source. The constant term is now found for both \( x > p \) and \( x < p \). For \( x > p \) there is a sinusoidal term due to the contribution from the poles at \( \xi = \pm \xi_1 \). As in case 1 the streamfunction solutions are discontinuous at \( x = p \).

### 4. Solutions for other heating structures

In this section we consider the atmospheric response to the zonal heating distributions

\[
q_c = \frac{\pi Q_0}{4x_0} \cos \frac{y}{2x_0} \cos \left( \frac{\pi x}{2x_0} \right),
\]

and

\[
q_s = -\frac{\pi Q_0}{4x_0} \cos \frac{y}{2x_0} \sin \left( \frac{\pi x}{2x_0} \right),
\]

where

\[
\int_{-\infty}^{\infty} q_c(x, y) dx = \int_{-\infty}^{\infty} q_s(x, y) dx,
\]

with \( q' \) defined by (4). Here \( \Pi \left( \frac{x}{2x_0} \right) \) is the rectangular function, defined by

\[
\omega_0 = \frac{\pi^2 \chi \cos \frac{y}{2}}{2(\xi_1^2-\xi_2^2)} \left\{ f_1(i\eta_1) - f_2(i\eta_2) \right\} e^{-\eta_1^2|x|} \sin \frac{\pi x}{2x_0} - e^{-\eta_2^2|x|} \sinh \eta_1 x, \quad \text{if} \quad |x| \geq x_0;
\]

\[
-\frac{f_1(i\eta_2) - f_2(i\eta_2)}{\eta_2(\pi^2+4\eta_2^2x^2)} e^{-\eta_2^2|x|} \cosh \eta_2 x, \quad \text{if} \quad |x| \leq x_0.
\]

The heating is thus confined to the positive part of one period of the cosine function in (32) and to one period of the inverted sine function in (33). Motivation for these zonal structures will be given in Section 6.

The solutions to (5)–(7) with diabatic heating of the form (32) and (33) are obtained by using the solutions (23), (25), (26) and (29), (30), (31) as Green's functions. Hence

\[
\left\{ \begin{array}{l}
\end{array} \right.
\]

The heating is thus confined to the positive part of one period of the cosine function in (32) and to one period of the inverted sine function in (33). Motivation for these zonal structures will be given in Section 6.

The solutions to (5)–(7) with diabatic heating of the form (32) and (33) are obtained by using the solutions (23), (25), (26) and (29), (30), (31) as Green's functions. Hence

\[
\end{array} \right.
\]

The heating is thus confined to the positive part of one period of the cosine function in (32) and to one period of the inverted sine function in (33). Motivation for these zonal structures will be given in Section 6.

The solutions to (5)–(7) with diabatic heating of the form (32) and (33) are obtained by using the solutions (23), (25), (26) and (29), (30), (31) as Green's functions. Hence

\[
\end{array} \right.
\]
\[
\phi_1 = - \frac{\chi f_0 \cos y}{\Delta p_1} \left\{ \frac{f_3(0)}{\eta_1^3 \eta_2^3} \begin{cases} 1, & \text{if } x \geq x_0; \\ \frac{1}{2} \left( 1 + \sin \frac{\pi x}{2x_0} \right), & \text{if } |x| \leq x_0; \\ 0, & \text{if } x \leq -x_0; \end{cases} \right\},
\]

\[
\frac{\pi^2 f_3(i\eta_1)}{2\eta_1^3(\eta_1^2 - \eta_2^2)(\pi^2 + 4\eta_1^2 x_0^2)} \begin{cases} e^{-\eta_1 x_0} \cosh \eta_1 x_0, & \text{if } |x| \geq x_0; \\ e^{-\eta_1 x_0} \cosh \eta_1 x + \frac{2x_0 \eta_1}{\pi} \cos \frac{\pi x}{2x_0}, & \text{if } |x| \leq x_0; \end{cases}
\]

\[
\frac{\pi^2 f_3(i\eta_2)}{2\eta_2^3(\eta_1^2 - \eta_2^2)(\pi^2 + 4\eta_2^2 x_0^2)} \begin{cases} e^{-\eta_2 x_0} \cosh \eta_2 x_0, & \text{if } |x| \geq x_0; \\ e^{-\eta_2 x_0} \cosh \eta_2 x + \frac{2x_0 \eta_2}{\pi} \cos \frac{\pi x}{2x_0}, & \text{if } |x| \leq x_0; \end{cases}
\] (38)

and

\[
\phi_2 = \frac{\chi f_0 \cos y}{\Delta p_3} \left\{ \frac{f_1(0)}{\eta_1^3 \eta_2^3} \begin{cases} 1, & \text{if } x \geq x_0; \\ \frac{1}{2} \left( 1 + \sin \frac{\pi x}{2x_0} \right), & \text{if } |x| \leq x_0; \\ 0, & \text{if } x \leq -x_0; \end{cases} \right\},
\]

\[
\frac{\pi^2 f_1(i\eta_1)}{2\eta_1^3(\eta_1^2 - \eta_2^2)(\pi^2 + 4\eta_1^2 x_0^2)} \begin{cases} e^{-\eta_1 x_0} \cosh \eta_1 x_0, & \text{if } |x| \geq x_0; \\ e^{-\eta_1 x_0} \cosh \eta_1 x + \frac{2x_0 \eta_1}{\pi} \cos \frac{\pi x}{2x_0}, & \text{if } |x| \leq x_0; \end{cases}
\]

\[
\frac{\pi^2 f_1(i\eta_2)}{2\eta_2^3(\eta_1^2 - \eta_2^2)(\pi^2 + 4\eta_2^2 x_0^2)} \begin{cases} e^{-\eta_2 x_0} \cosh \eta_2 x_0, & \text{if } |x| \geq x_0; \\ e^{-\eta_2 x_0} \cosh \eta_2 x + \frac{2x_0 \eta_2}{\pi} \cos \frac{\pi x}{2x_0}, & \text{if } |x| \leq x_0; \end{cases}
\] (39)

**case 2:** \( l^2 < (\gamma^2 + \delta)^{1/2} - \gamma \).

\[
\omega_2 = \frac{\pi^2 \chi \cos y}{2(\xi_1^2 + \eta_2^2)} \left\{ \frac{2f_1(\xi_1)f_3(\xi_1)}{\xi_1(\pi^2 - 4\xi_1^2 x_0^2)} \begin{cases} \cos \xi_1 x_0 \sin \xi_1 x, & \text{if } x \geq x_0; \\ \frac{1}{2} \sin \xi_1 (x + x_0) - \frac{x_0 \xi_1}{\pi} \cos \frac{\pi x}{2x_0}, & \text{if } |x| \leq x_0; \\ 0, & \text{if } x \leq -x_0; \end{cases} \right\},
\]

\[
+ \frac{f_1(i\eta_2)f_3(i\eta_2)}{\eta_2^2(\pi^2 + 4\eta_2^2 x_0^2)} \begin{cases} e^{-\eta_2 x_0} \cosh \eta_2 x_0 \sgn x, & \text{if } |x| > x_0; \\ \sin \frac{\pi x}{2x_0} - e^{-\eta_2 x_0} \sinh \eta_2 x, & \text{if } |x| < x_0. \end{cases}
\] (40)

\[
\phi_3 = \frac{\chi f_0 \cos y}{\Delta p_3} \left\{ \frac{f_3(0)}{\xi_1^3 \eta_2^3} \begin{cases} 1, & \text{if } x \geq x_0; \\ \frac{1}{2} \left( \cos \xi_1 (x + x_0) + \sin \frac{\pi x}{2x_0} \right), & \text{if } |x| \leq x_0; \\ 0, & \text{if } x \leq -x_0; \end{cases} \right\},
\]

\[
- \frac{\pi^2 f_1(\xi_1)}{\xi_1^3(\xi_1^2 + \eta_2^2)(\pi^2 - 4\xi_1^2 x_0^2)} \begin{cases} \cos \xi_1 x_0 \cos \xi_1 x, & \text{if } x \geq x_0; \\ \frac{1}{2} \left( \cos \xi_1 (x + x_0) + \sin \frac{\pi x}{2x_0} \right), & \text{if } |x| \leq x_0; \\ 0, & \text{if } x \leq -x_0. \end{cases}
\]

\[
- \frac{\pi^2 f_3(i\eta_2)}{2\eta_2^3(\xi_1^2 + \eta_2^2)(\pi^2 + 4\eta_2^2 x_0^2)} \begin{cases} e^{-\eta_2 x_0} \cosh \eta_2 x_0, & \text{if } |x| \geq x_0; \\ e^{-\eta_2 x_0} \cosh \eta_2 x + \frac{2x_0 \eta_2}{\pi} \cos \frac{\pi x}{2x_0}, & \text{if } |x| \leq x_0. \end{cases}
\] (41)

and
The solutions (37)–(42) approach the delta heating solutions (23), (25), (26), (29), (30), (31) in the limit as \( x_0 \to 0 \). It is also interesting to note that \( \phi_2 \) and \( \phi_3 \) are all continuous at \( x = \pm x_0 \).

The substitution of \( f_s \) and \( G(x, y; \rho) \) into (36) yields:

\[
\begin{align*}
\omega_2 &= \frac{\pi^3 x \cos y}{4(\eta_1^2 - \eta_2^2)} \left\{ \frac{f_1(i \eta_1) f_2(i \eta_1)}{\eta_1(\pi^2 + \eta_1^2 x_0^2)} \right\} e^{-\frac{\eta_1}{\eta_2} |x|} \sinh \eta_2 x_0, \\
\phi_1 &= -\frac{x \cos y}{4 \Delta p_1} \left\{ \frac{f_1(0)}{\eta_1^2 \eta_2^2} \right\} 0, \\
\phi_3 &= \frac{x \cos y}{4 \Delta p_1} \left\{ \frac{f_1(0)}{\eta_1^2 \eta_2^2} \right\} 0, \\
\end{align*}
\]

and

\[
\begin{align*}
\omega_2 &= \frac{\pi^3 x \cos y}{4(\eta_1^2 - \eta_2^2)} \left\{ \frac{f_1(i \eta_1) f_2(i \eta_1)}{\eta_1(\pi^2 + \eta_1^2 x_0^2)} \right\} e^{-\frac{\eta_1}{\eta_2} |x|} \sinh \eta_2 x_0 \text{ sgn} \ x, \\
\phi_1 &= -\frac{x \cos y}{4 \Delta p_1} \left\{ \frac{f_1(0)}{\eta_1^2 \eta_2^2} \right\} 0, \\
\phi_3 &= \frac{x \cos y}{4 \Delta p_1} \left\{ \frac{f_1(0)}{\eta_1^2 \eta_2^2} \right\} 0, \\
\end{align*}
\]

\( \text{case 2: } l^2 < (\eta^2 + \delta)^{1/2} - \gamma. \)
The solutions for the streamfunctions $\psi_1$ and $\psi_3$ and the vertical velocity $\omega_z$ are once more continuous at $x = \pm x_0$.

5. Discussion

The solutions for $\omega_z$, $\psi_1$, and $\psi_3$, presented in the previous two sections, have been plotted for the equally spaced level model (Model 1) in Figs. 4a, b, c along the line $y = 0$, for a wave-number $l = 1.4 \times 10^{-6} \text{m}^{-1}$, typical of case 1. Here we use $Q_0 = 0.01 \text{ Wkg}^{-1}$, $C_p = 1005 \text{ Jkg}^{-1} \text{K}^{-1}$, $R = 290 \text{ Jkg}^{-1} \text{K}^{-1}$, and the parameters of Section 3. As we have normalized the heating structures (32) and (33) to satisfy (34), we must multiply the solutions given in Section 4 by a factor of $10^6$. For the line source heating solutions (solid line), there is a strong local response. In the streamfunction plots (Figs. 4b, c) this local response decays rapidly to a constant far field response. When heat forcing is spread over a broader region (dashed line; $x_0 = 2.0 \times 10^6 \text{m}$), the local response diminishes in magnitude. The solutions $\omega_z$, $\psi_1$, $\psi_3$ for the unequally spaced level model (Model 2), where the thermodynamic equation is applied at the 800 mb (as opposed to the 600 mb) level, are plotted in Figs. 5a, b, c. As expected the zonal structure of Figs. 5a–c is
Fig. 4 Model 1 solutions for case $I > \{y^2 + \delta\}^{1/2} - \gamma$ and the parameters of Section 3. The solid line is for line source heat forcing (4), the dashed line is for segmented cosine forcing (32), and the dotted line is for segmented sine forcing (33). a) - Vertical velocity $\omega$. b) - Streamfunction at the upper level, $\phi$. c) - Streamfunction at the lower level, $\phi$. The vertical axis in a) is in $10^{-4}$ m s$^{-1}$, and in b) and c) is in $10^{6}$ m$^2$s$^{-1}$. In all three plots the $x$-axis is in $10^{7}$ m.

Fig. 5 Model 2 solutions for case $I > \{y^2 + \delta\}^{1/2} - \gamma$, and the parameters of Section 3. The solid line is for line source heat forcing (4), the dashed line is for segmented cosine forcing (32), and the dotted line is for segmented sine forcing (33). a) - Vertical velocity $\omega$. b) - Streamfunction at the upper level, $\phi$. c) - Streamfunction at the lower level, $\phi$. The vertical axis in a) is in $10^{-4}$ m s$^{-1}$, and in b) and c) is in $10^{6}$ m$^2$s$^{-1}$. In all three plots the $x$-axis is in $10^{7}$ m.
Table 1  Real and imaginary parts of the zeroes of (A3) for Model 1 (equally spaced levels). Rayleigh friction and Newtonian cooling coefficients of \( r_0 = 1.0 \times 10^{-14} \text{s}^{-1} \) and \( r_0 = 1.0 \times 10^{-18} \text{s}^{-1} \) were used. Two wavenumbers, \( l = 1.4 \times 10^{-6} \text{m}^{-1} \) and \( l = 0.8 \times 10^{-6} \text{m}^{-1} \), corresponding to case 1 and case 2, were also selected.

<table>
<thead>
<tr>
<th>wavenumber ( \times 10^{-6} )</th>
<th>pole ( \xi )</th>
<th>( r_0 = 1.0 \times 10^{-14} )</th>
<th>( r_0 = 1.0 \times 10^{-18} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = 0.8 )</td>
<td>( \xi_a )</td>
<td>( 5.97 \times 10^{-41}, -6.66 \times 10^{-16} )</td>
<td>( 5.97 \times 10^{-45}, -6.66 \times 10^{-20} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_b )</td>
<td>( -9.86 \times 10^{-32}, 1.49 \times 10^{-16} )</td>
<td>( -1.20 \times 10^{-35}, 1.49 \times 10^{-19} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_c )</td>
<td>( -7.99 \times 10^{-7}, 6.16 \times 10^{-16} )</td>
<td>( -7.99 \times 10^{-7}, 6.16 \times 10^{-20} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_d )</td>
<td>( 7.99 \times 10^{-7}, 6.16 \times 10^{-16} )</td>
<td>( 7.99 \times 10^{-7}, 6.16 \times 10^{-20} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_e )</td>
<td>( 1.32 \times 10^{-23}, 2.01 \times 10^{-6} )</td>
<td>( -6.26 \times 10^{-23}, 2.01 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_f )</td>
<td>( 1.32 \times 10^{-23}, -2.01 \times 10^{-6} )</td>
<td>( -6.26 \times 10^{-23}, -2.01 \times 10^{-6} )</td>
</tr>
<tr>
<td>( l = 1.4 )</td>
<td>( \xi_a )</td>
<td>( 3.96 \times 10^{-16}, 1.62 \times 10^{-16} )</td>
<td>( 3.96 \times 10^{-19}, 1.62 \times 10^{-19} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_b )</td>
<td>( -9.86 \times 10^{-16}, 1.62 \times 10^{-16} )</td>
<td>( -3.96 \times 10^{-19}, 1.62 \times 10^{-19} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_c )</td>
<td>( -9.87 \times 10^{-16}, -8.26 \times 10^{-7} )</td>
<td>( -8.27 \times 10^{-16}, -8.26 \times 10^{-7} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_d )</td>
<td>( 8.27 \times 10^{-16}, -8.26 \times 10^{-7} )</td>
<td>( 8.27 \times 10^{-16}, -8.26 \times 10^{-7} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_e )</td>
<td>( -2.95 \times 10^{-23}, 2.32 \times 10^{-6} )</td>
<td>( -2.95 \times 10^{-23}, 2.32 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_f )</td>
<td>( 2.95 \times 10^{-23}, -2.32 \times 10^{-6} )</td>
<td>( 2.95 \times 10^{-23}, -2.32 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

Table 2  Real and imaginary parts of the zeroes of (A3) for Model 2 (unequally spaced levels). Rayleigh friction and Newtonian cooling coefficients of \( r_0 = 1.0 \times 10^{-14} \text{s}^{-1} \) and \( r_0 = 1.0 \times 10^{-18} \text{s}^{-1} \) were used. Two wavenumbers, \( l = 1.4 \times 10^{-6} \text{m}^{-1} \) and \( l = 0.8 \times 10^{-6} \text{m}^{-1} \), corresponding to case 1 and case 2, were also selected.

<table>
<thead>
<tr>
<th>wavenumber ( \times 10^{-6} )</th>
<th>pole ( \xi )</th>
<th>( r_0 = 1.0 \times 10^{-14} )</th>
<th>( r_0 = 1.0 \times 10^{-18} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l = 0.8 )</td>
<td>( \xi_a )</td>
<td>( -5.02 \times 10^{-41}, 9.83 \times 10^{-16} )</td>
<td>( -5.21 \times 10^{-41}, 9.83 \times 10^{-29} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_b )</td>
<td>( -4.93 \times 10^{-32}, -8.17 \times 10^{-16} )</td>
<td>( -1.20 \times 10^{-35}, -8.17 \times 10^{-28} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_c )</td>
<td>( -6.68 \times 10^{-7}, 6.55 \times 10^{-16} )</td>
<td>( -6.68 \times 10^{-7}, 6.55 \times 10^{-29} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_d )</td>
<td>( 6.88 \times 10^{-7}, 6.55 \times 10^{-16} )</td>
<td>( 6.88 \times 10^{-7}, 6.55 \times 10^{-29} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_e )</td>
<td>( -1.42 \times 10^{-23}, 3.03 \times 10^{-6} )</td>
<td>( -7.55 \times 10^{-24}, 3.03 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_f )</td>
<td>( -1.23 \times 10^{-23}, -3.03 \times 10^{-6} )</td>
<td>( -5.69 \times 10^{-24}, -3.03 \times 10^{-6} )</td>
</tr>
<tr>
<td>( l = 1.4 )</td>
<td>( \xi_a )</td>
<td>( 3.25 \times 10^{-16}, 1.12 \times 10^{-16} )</td>
<td>( 3.25 \times 10^{-19}, 1.12 \times 10^{-19} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_b )</td>
<td>( -3.25 \times 10^{-16}, 1.12 \times 10^{-16} )</td>
<td>( -3.25 \times 10^{-19}, 1.12 \times 10^{-19} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_c )</td>
<td>( 8.49 \times 10^{-16}, -9.35 \times 10^{-7} )</td>
<td>( 8.49 \times 10^{-16}, -9.35 \times 10^{-7} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_d )</td>
<td>( -8.49 \times 10^{-16}, 9.35 \times 10^{-7} )</td>
<td>( -8.49 \times 10^{-16}, 9.35 \times 10^{-7} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_e )</td>
<td>( 2.45 \times 10^{-23}, 3.24 \times 10^{-6} )</td>
<td>( 2.45 \times 10^{-23}, 3.24 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>( \xi_f )</td>
<td>( -2.45 \times 10^{-23}, -3.24 \times 10^{-6} )</td>
<td>( -2.45 \times 10^{-23}, -3.24 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

similar to that of Figs. 4a–c. The magnitude of the streamfunction at the upper level is significantly smaller in Fig. 5b than in Fig. 4b, while at the lower level the magnitude of \( \psi_3 \) is only slightly smaller in Fig. 5c. Similarly the magnitudes of \( \omega_2 \) differ only slightly for the two models (Fig. 5a, cf., Fig. 4a). Since \( \eta_1 \) and \( \eta_2 \) are larger for Model 2 than Model 1 (see Tables 1 and 2) the atmospheric response in Model 2 is more locally trapped.

Figures 4b, c and 5b, c are very similar to those obtained by WMB and indicate that wind shear would have little effect on their results for a continuously stratified model. Also we note that the results obtained above are very different from those that one would obtain if periodic heat forcing were used. If the system (5)–(7) were forced with periodic heating of wavenumber \( k \), the solutions for \( \omega_2, \phi_3, \phi_5 \) would also be periodic with wavenumber \( k \).
Fig. 6 Model 1 solutions for case 2 $P<\{T^2+\delta\}^{1/2}-\gamma$, and the parameters of Section 3. The solid line is for line source heat forcing (4), the dashed line is for segmented cosine forcing (32), and the dotted line is for segmented sine forcing (33). a) – Vertical velocity $\omega_v$. b) – Streamfunction at the upper level, $\psi_1$. c) – Streamfunction at the lower level, $\psi_3$. The vertical axis in a) is in $10^{-4}\text{mbs}^{-1}$, and in b) and c) is in $10^{6}\text{m}^2\text{s}^{-1}$. In all three plots the x-axis is in $10^7\text{m}$.

A discussion of the response for the segmented sine heating distribution will be delayed until the next section. It is interesting however to note that since, $\int_{-\infty}^{\infty} q_1(x, y)dx=0$ (see (33)), the far field response is zero.

For small wavenumbers (case 2 – a case not examined in detail by WMB) the far field solutions are very different from those described above. This is illustrated for Model 1 in Figs. 6a, b, c and for Model 2 in Figs. 7a, b, c. Here $l=0.8 \times 10^{-6}\text{m}^{-1}$ while the other parameters remain the same. Once more a strong local response exists for the line source heating solutions (solid line in Figs. 6a–c). This diminishes in magnitude as the heating is spread over a broader region (dashed line in Figs. 6a–c). Unlike the solution one would obtain for forcing at a given wavenumber $k$, a natural wavenumber $\xi_2$ (given by 19) for the downstream disturbance, is forced. This corresponds to a wavelength of about
Fig. 7 Model 2 solutions for case 2 $\ell^2 < \{4^2+\delta\}^{1/2} - \gamma$, and the parameters of Section 3. The solid line is for line source heat forcing (4), the dashed line is for segmented cosine forcing (32), and the dotted line is for segmented sine forcing (33). a) Vertical velocity $\omega_0$, b) Streamfunction at the upper level, $\phi_1$, c) Streamfunction at the lower level, $\phi_3$. The vertical axis in a) is in $10^9 \text{mbs}^{-1}$, and in b) and c) is in $10^4 \text{m}^2 \text{s}^{-1}$. In all three plots the x-axis is in $10^7 \text{m}$.

800 km if the parameters mentioned above are used. When the heating is distributed over a broader expanse, the magnitude of this far field stationary wave pattern decreases, although there is no effect on the wavelength or phase.

For Model 2 (Figs. 7a–c) the response once more has a similar zonal structure to that of Model 1, although the wavelength of the downstream stationary wave pattern has increased slightly due to a slightly smaller $\xi_1$. The magnitudes of $\omega_0$, $\phi_1$, $\phi_3$ are somewhat smaller especially for the upper level streamfunction $\phi_3$.

The contribution from the first term in the RHS of (30) and (31) now forces the far field mean response of $\phi_1$ (Figs. 6b, and 7b) to be greater than zero and of $\phi_3$ to be less than zero (Figs. 6c, and 7c).

The segmented sine solutions (dotted line in Figs. 6a–c, 7a–c; $x_0 = 4.55 \times 10^8$) differ from those of case 1 in a similar manner to that described above.

In many of Figures 4–7 $\lim_{x \to -\infty} (\omega_0, \phi_1, \phi_3) \neq \lim_{x \to +\infty} (\omega_0, \phi_1, \phi_3)$. When we try to apply these results to the sphere, which is periodic in $x$, this mismatching needs some interpretation. For simplicity we examine Model 1 for case 2 where the levels are equally spaced. The forced wavenumber for the downstream stationary wave pattern is $\xi_1 = 8 \times 10^{-7} \text{m}^{-1}$ (see Table 1), which corresponds to wavenumber 4 on the sphere. Suppose now that we are on the sphere. The time dependent unforced versions of (5)–(7) allow free wave solutions $\propto e^{i(kx+\ell y-ct)}$ which have phase speed in the $x$-direction (Holton, 1979 p. 218)

$$c_\ell = U_m - \frac{\dot{\bar{\delta}}(K^2+\lambda^2)}{K^2(K^2+2\lambda^2)} \pm \delta^{1/2},$$

where

$$\dot{\bar{\delta}} = \frac{\beta^2 \lambda^4}{K^4(K^2+2\lambda^2)^2} \left( \frac{U_T^2(2\lambda^2-K^2)}{(K^2+2\lambda^2)} \right),$$

and

$$U_m = \frac{U_1+U_3}{2}, \quad U_T = \frac{U_1-U_3}{2},$$

$$K^2 = k^2 + \ell^2.$$

Now we let $k = \xi_1$ and we use the parameters of Section 3 to get two stable modes for which $c_{\ell 1} = 6.3 \text{ms}^{-1}$ and $c_{\ell 2} = 0.015 \text{ms}^{-1}$. These Rossby modes would propagate around the globe (at 40°N) in $t_1 = 57$ days and $t_2 = 66$ years(!), respectively. A typical time scale for dissipative damping is about $t_d = 6$ days (Frankignoul,
Since \( t_2 \ll t_1 \) and \( t_2 \), an eastward propagating wave would be damped rather quickly and would not likely propagate around the globe. Thus the assumption \(-\infty < x < \infty\) is indeed justified. The response is mainly downstream of the heating with a small exponentially decaying response upstream.

In order to compare the model results with work by others we focus our attention on the segmented cosine heat forcing solutions (dashed line in Figs. 4–7). Egger (1977), using a two level, linear hemispheric model with friction, examined the stationary atmospheric response to a rectangular heat source. His heat source had a width of 1700 km, analogous to \( l = 1.8 \times 10^{-6} \) (case 1) in this model, and was centered at 45°N with a 2000 km horizontal extent. The atmospheric response near the surface consisted of a strong low downstream of the heating region with a weak high upstream. The dashed lines of Figs. 4c and 5c are in excellent agreement with his observations. If friction were included in the model herein, the response downstream would diminish as discussed above. The resulting solutions would be almost identical to Egger's Figs. 2 and 3.

Hoskins and Karoly (1981) also examined the linear wave response of the atmosphere to mid-latitude heating using a five level primitive equation model. They considered heating over a broad meridional extent and hence fall into case 2 in the model of this paper. In their model, Hoskins and Karoly observed weak ascending motion ahead of the heating and weak descending motion behind, with very small extrema. This is in excellent agreement with Fig. 7a (dashed line). Weak ascending motion is clearly visible ahead of the heating and weak descending motion behind. Similarly they observed a strong lower level trough and an upper level high downstream of the heating region. These results are also apparent in Figs. 7b and 7c.

As mentioned above, the downstream stationary wave pattern for case 2 corresponds to wavenumber 4 for Model 1 and to wavenumber 3 for Model 2, on the sphere. It is interesting to note that in the GCM studies of Chervin et al. (1980) and Frankignoul (1985b), the atmospheric response to SST anomaly heating was dominated by zonal wavenumber 3–4 perturbations, as predicted by the model herein.

6. Application to the Kuroshio region

The horizontal distribution of oceanic heat flux anomalies varies greatly from year to year (Zhao and McBean 1986, Weaver and Mysak 1986a). This is illustrated in Figs. 8a, b for winter (December, January, February), where anomalies between ten and thirty percent of the seasonal long term mean are observed in the proximity of and eastward of the Kuroshio region. The segmented cosine heating source (32) was therefore introduced to model heating over a broad expanse (Fig. 8a), whereas the line source heat forcing (4) was used as a model for localized heating (Fig. 8b).

On the basis of a simultaneous monthly cor-

---

**Fig. 8** Oceanic heat flux anomaly maps over the North Pacific a) – Winter 1963; b) – Winter 1978 (from Weaver and Mysak, 1986a). The oceanic heat flux has units Watts m\(^{-2}\) and is comprised of the latent, sensible and back radiative heat fluxes and has been described in an appendix (written by N.E. Clark) to Barnett (1981).
relation analysis, Zhao and McBean (1987b) proposed that these heat flux anomalies have a profound effect on the overlying atmospheric circulation. They averaged the oceanic heat flux data (obtained from the N.E. Clark data set at Scripps Institute of Oceanography), over an area termed the Kuroshio region (25–30 N, 130–150 E and 30–35 N 130–160 E; see hatched area in Fig. 9a), and correlated this with the atmospheric surface pressure and 700 mb geopotential height, over the northern hemisphere. Figures 9a–c illustrate results analogous to those of Zhao and McBean using seasonal statistics as opposed to monthly mean data. Here winter is defined as December, January, February.

The oceanic heat flux in winter has been averaged over the Kuroshio region and correlated with the winter surface pressure (Fig. 9a) and 700 & 500 mb geopotential heights (Figs. 9b, c, respectively), over the northern hemisphere. Since the surface pressure data was available for the period 1950–1979, the 700 mb data for the period 1963–1979, and the 500 mb data for the period 1956–1979, the correlation coefficients \( r = 0.36, r = 0.47, r = 0.40 \) correspond to the 0.05 confidence level in Figs. 9a, b, c, respectively.

There are three distinct regions of significant correlation: a)– Over eastern Asia, Kuroshio heat flux anomalies are significantly correlated with the surface pressure and the 500 & 700 mb geopotential height data, b)– Over the central North Pacific there exists an extremely strong negative correlation, especially with the surface pressure data (Fig. 9a); c)– Over northern Canada there is another region of significant positive correlation, but only at the 700 and 500 mb heights (Figs. 9b, c, cf. Fig. 9a).

Zhao and McBean (1987b) argued that the higher the pressure over eastern Asia, the colder and dryer the air is over Siberia and the higher the wind velocities are over the Kuroshio region. Hence, the heat flux over the Kuroshio region would be larger. The resulting heat flux anomaly, they further argued, would create lower than normal pressures over the Kuroshio region. Since the influence of the Kuroshio region extends further eastward than the hatched area of Fig. 9a (Zhao and McBean 1986), they concluded that

Fig. 9 Correlations between winter oceanic heat flux anomalies over the Kuroshio region (the hatched area in the western Pacific) and a) – surface pressure; b) – 700 mb geopotential height; c) – 500 mb geopotential height; (from Weaver and Mysak, 1986b). A correlation coefficient of \( r = 0.36, r = 0.47, r = 0.40 \), corresponds to the 0.05 confidence level in a), b), and c) respectively.
the lower than normal pressures would extend further eastward and hence cause the Aleutian low to shift southwestward during winter.

In order to investigate the possible effect of the Kuroshio heating anomalies on the downstream atmospheric conditions, and hence to theoretically test the hypothesis of Zhao and McBean (1987b), we use the segmented cosine heat forcing model with $x_0 = 2.0 \times 10^6$ m corresponding to heating over an area of $47^\circ$ extent. Here we assume that the oceanic heat flux anomalies manifest themselves as diabatic heating anomalies at the 800 mb level (Model 2). The origin $(x, y) = (0, 0)$ is taken near the east coast of Asia at $120^\circ$E, $40^\circ$N. We have chosen Model 2 instead of Model 1 since over the Kuroshio the oceanic heat flux leads to shallow heating, especially in winter (Nitta and So, 1980, Hoskins and Karoly, 1981, Masuda, 1983).

Finally, in an attempt to model the atmospheric response to heating with the structure of the first EOF of Zhao and McBean (1987a) (see Section 1), we consider heating of the form (33) with $x_0 = 4.55 \times 10^6$ m. This corresponds to positive anomalous heating over the western half of the North Pacific and negative anomalous heating over the eastern half of the North Pacific.

A meridional wavenumber $l = 1.4 \times 10^6$ m$^{-1}$ (ie., a wavelength of $4.5 \times 10^6$ m), is chosen. This corresponds to positive heating over a region of $20^\circ$ extent, centered at $40^\circ$N (see Figs. 8a, b), and falls into case 1 as described earlier. The streamfunctions at the upper and lower levels has been plotted in Figs. 5b and 5c respectively, for both the segmented cosine model (dashed line) and the segmented sine model (dotted line).

When Kuroshio heating is included alone, the local response at both levels consists of a gradual transition from the region $x < -x_0$, to the far field region $x > x_0$. The negative response is consistent with the observed large negative correlations (Figs. 9a–c) downstream of the heating region, and hence is consistent with the argument proposed by Zhao and McBean (1987b). In the region $x < -x_0$ there is a small positive response which is apparent in the correlation plots of Figs. 9a–c. As observed in WMB, the streamfunction at the upper level is of insignificant magnitude when compared to the response at the lower level. Thus the atmospheric response is trapped near to the surface.

Although the above results are consistent with the observations, a more satisfactory agreement is obtained when cooling in the eastern Pacific is included. Once more Figs. 5b, c portray the streamfunction at the upper and lower level, respectively. The positive response upstream of the Kuroshio region is once more barely visible in the lower level plot (Fig. 5c). Over the area corresponding to the central North Pacific there is a large negative response in Fig. 5c. This feature is in excellent agreement with the correlations of Figs. 9a–c. Once more the results are similar to those of WMB in that the upper level response is of insignificant magnitude when compared to the lower level.

In the spatial correlation plots (Figs. 9a–c) the observed results show significant correlations at all heights, whereas the response of Model 2 is weak at the upper level. The most significant correlations however occur in the SLP plot (Fig. 9a), where the response of Model 2 is largest. At the upper level the response of the model is small (Fig. 5b) and hence would not be expected to contribute to the observed correlations. It appears therefore that the correlations in the upper level plots (Figs. 9b, c) simply arise from the spatial correlations of the geopotential field. This ridge/trough structure in turn forces Kuroshio heating anomalies (and eastern Pacific cooling anomalies). The heating/cooling anomalies then “feedback” and intensify the low in the lower troposphere over the central North Pacific (Fig. 9a). This mechanism is in agreement with the argument of Zhao and McBean given earlier in this section.

Acknowledgments

The author is indebted to Drs. G.A. McBean and Y.P. Zhao for helpful discussions on this research and for making available their data and statistical results prior to publication. Drs. L.A. Mysak, A.F. Bennett and K. Hamilton are also gratefully acknowledged for their helpful comments. This research was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) in the form of a 1967 Science and Engineering Scholarship and
by a Teaching Assistantship from the Department of Mathematics, UBC. Financial support by NSERC Strategic Grant G–1485, awarded to L.A. Mysak, K. Hamilton and C. Groot, for the study of the Meteorological and Oceanographic Influences on Sockeye Tracks (MOIST), is also acknowledged.

References


Appendix I

Inclusion of Rayleigh Friction and Newtonian Cooling

We include Rayleigh friction and Newtonian cooling in the problem to examine the physical location in the complex plane of the poles at \( *=0, \pm \xi, \pm \xi \). To accomplish this we replace in the governing equations (5)–(7), where \( r_0 > 0 \) is the Rayleigh friction and Newtonian cooling coefficient. If one carries through the analysis
of Section 3, the resulting expression for $\omega_8$ is (cf., (10)).

$$\omega_8 = -\frac{XU_jU_3e^{-i\xi_1}R_1(\xi)R_3(\xi)}{\sqrt{2\pi}} \left\{ R_1(\xi)R_3(\xi) - i\xi\left[ \left( \frac{r_0\alpha_1}{U_1} + i\alpha_3\right)R_1(\xi) + \left( \frac{r_0\alpha_3}{U_3} + i\alpha_3\right)R_3(\xi) \right] \right\},$$  \hfill (A1)

where

$$R_1, s(\xi) = i\xi f_1, s(\xi) - r_0(l^2 + \xi^2),$$  \hfill (A2)

and $f_1(\xi)$ and $f_3(\xi)$ are given by (13). Similar expressions may be obtained for $\psi_1$ and $\psi_3$.

We must therefore find the location of the zeroes of

$$R_1(\xi)R_3(\xi) - i\xi\left[ \left( \frac{r_0\alpha_1}{U_1} + i\alpha_3\right)R_1(\xi) + \left( \frac{r_0\alpha_3}{U_3} + i\alpha_3\right)R_3(\xi) \right],$$  \hfill (A3)

which is sixth order in $\xi$.

The inclusion of Rayleigh friction and Newtonian cooling introduces two more zeros in the denominator of (A1) near $\xi = 0$. As $r_0 \to 0$ the numerator and denominator of (A1) both vanish at $\xi = 0$ and we retrieve (10).

The zeros of (A3) were obtained numerically using the parameters of Section 3 for both $r_0 = 1.0 \times 10^{-14} \text{s}^{-1}$ and $r_0 = 1.0 \times 10^{-18} \text{s}^{-1}$, and have been compiled in Table 1 for Model 1 and in Table 2 for Model 2. Two wavenumbers, $l = 1.4 \times 10^{-6} \text{m}^{-1}$ and $l = 0.8 \times 10^{-6} \text{m}^{-1}$, corresponding to case 1 and case 2, were selected. The results indicate that for case 1 ($l^2 > |\gamma^2 + \delta|^{1/2} - \gamma$) the pole at $\xi = 0$ goes into the UHP ($\xi_a, \xi_b$ in Tables 1, 2). The poles at $i\eta_1, i\eta_2$ are both located in the UHP ($\xi_{d}, \xi_{e}$ in Tables 1, 2) and the poles at $-i\eta_1, -i\eta_2$ are both located in the LHP ($\xi_{c}, \xi_{f}$ in Table 1, 2).

For case 2 ($l^2 < |\gamma^2 + \delta|^{1/2} - \gamma$) the pole at $\xi = 0$ splits and goes into both the UHP and the LHP, whereas the real poles at $\pm \xi_1$ both move into the UHP ($\xi_a, \xi_e$ in Tables 1, 2) and hence give the sinusoidal contribution of (29), (30), and (31) for $x > p$ only. Once more the poles at $i\eta_2$ and $-i\eta_2$ remain in the UHP and LHP respectively.