One Hypothesis in Glaciation Cycles of the Earth: (II) A Cause of Appearance of Oscillatory Climate in the Pleistocene

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Abstract

With use of a simple climate model, we discuss analytically the cause of appearance of oscillatory climate state on the earth. Introducing a geographical parameter into the climate model, we investigate a relationship between emergence of oscillatory climate and continental drift. Our results can explain mathematically the observed fact, which gives records during last million years, that it grew colder with a gradual increase of amplitude of climatic oscillations, and can indicate that oscillatory climate had appeared on the earth when the geographical condition had allowed it to exist. Our tentative calculations reveal that the oscillatory climate might have emerged on the earth, when a clear cryosphere had been established on the earth owing to drift of such lands as the Antarctica or Greenland from warm low-latitudes to ice-possible high-latitude regions, and when the past earth's glacial productive capacity had come to exceed about 70% or so of the present capacity.

1. Introduction

In the geological history of the earth, in the Precambrian and the Cambrian, a few cold epochs are known to exist. After the Paleozoic, the warm climate continued steadily, especially 180 million to 30 million years ago. But, 30 million years ago the climate began to cool and a slow decline in temperature occurred until oscillatory climate emerged. According to contemporary paleogeographic data, ice cover appeared in the Neocene, i.e., starting about 30 million years ago, ice began to accumulate on the Antarctica and a very rapid growth of the ice sheets there commenced about 10 million years ago, and on Greenland about 3 million years ago. As seen from Fig. 1, the oscillatory climate appeared on the earth a few million years ago, in the Pliocene, but clear large climatic and glacial oscillations seem to have begun later in the Pleistocene. During the last million years it grew colder with a gradual increase of amplitude of climatic oscillations. As shown in Fig. 1, midlatitude temperatures have undergone a series of oscillations, whose maximum amplitude is about 10°C. Fig. 1 also shows that midlatitude temperatures more or less monotonically declined by about 10°C from 60 million years ago until one million years ago.

One may have an objection to that Fig. 1 would represent the past earth's climate. However, it seems reliable that a transition from one stable climate state to an oscillatory climate had occurred in the Pliocene, and so, that the earth had experienced a transition between small ice sheets climate and large ice sheets one. We feel a great interest in a cause for such a drastic change in the past earth's climate. Why has our earth rushed into such an oscillatory climate system in the last million years? In this paper, we will discuss and try to account for this problem analytically with use of a simple climate model. From our result, a possibility may raise that the earth would have a self-sustained oscillatory climate system. Moreover, it will be shown that the continental drift would have largely

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caused an appearance of the large-scale climatic oscillations on the earth.

Watts and Hayder (1983), using a Weertman's simple ice sheet model (1976), showed that the transition between small ice sheets and large ice sheets can occur as a mathematical bifurcation; that is, the response of ice sheets forced by a periodic change in the climate point may exhibit bifurcation when certain climatic parameters undergo very small changes, and this bifurcation may be responsible for the initiation of large glacial cycles in the Pleistocene. Sergin (1980) also pointed out an importance of the continental ice sheets formations on the earth for the appearance of large-scale climatic oscillations, using numerical experiments with a simplified thermodynamic model of the glacier-ocean-atmosphere global system. We will indicate the importance of establishment of a climate system composed of the glacier-ocean-atmosphere, upon the present problem by a different way from theirs, i.e., analytically, based upon the other climate model. Our previous paper (Moriyama, 1986) indicates that the present earth's glacial cycles would be governed by the most dominant free oscillation system with 100 kyr period in the present climate field.

2. Basic equations

As our basic model, we use the climate model with ice sheet dynamics developed first in Källén et al. (1979). The equations governing the climate model are

\[
C_T dT/dt = Q\left[1 - \gamma (\alpha_0 + \alpha L) \right] - \kappa (T - T_0), \quad T \geq 0
\]

\[
C_L dL/dt = F(T, L), \quad L \geq 0
\]

where the two dependent variables are globally averaged temperature \(T\) which is the mass average of ocean temperature and atmospheric temperature, and latitudinal extent of the land ice-cover \(L\). \(C_T\) and \(C_L\) denote the heat capacity of the atmosphere-ocean-cryosphere system and the relaxation time of the continental ice sheets, respectively. \(F\) is a nonlinear function to be described in eq. (4). The right-hand side of eq. (1) indicates the shortwave radiation absorbed by the system and the outgoing longwave radiation. The effect of sea ice is included in \(\alpha(T)\). The formation and melting of sea ice is taken to be a function of the model temperature as adopted by Källén et al., and it is assumed that they occur faster than the relaxation time scale of the atmosphere-hydrosphere system. Eqs. (1) and (2) include the ice-albedo feedback and the precipitation-temperature feedback. As the developed KCG climate model, the feedback between visco-elastic response of bedrock to the ice load and ice-mass balance is also introduced by Ghil and Le Treut (1981). However, for simplicity, we do not consider the third feedback in this discussion, because it gives only the secondary effect on an appearance of the KCG model's self-sustained oscillations (See, Ghil and Le Treut (1981)).

Introducing the nondimensional variable for the ice width \(l\) which is defined by

\[
l = L/L^*,
\]

we obtain from eq. (2)

\[
C_L dl/dt = l^{1/3} \{1 + \varepsilon(T)\} l_T(T, l) - l,
\]
Eq. (4) describes the evolution of an ice sheet owing to changes in the precipitation and melting budget. The detailed derivation of these equations is given by Källén et al. (1979). From now on we call the climate model by Källén et al. as KCG model.

The notation is as follows.

- \( Q \) solar radiation incident at the top of the atmosphere;
- \( \gamma \) ratio of the surface of the continents to the surface of the earth;
- \( \alpha_{oc} \) temperature-dependent albedo of the ocean, determined by the extent of sea ice;
- \( \kappa(T - T_o) \) outgoing infrared radiation;
- \( L^* \) scaling factor for the latitudinal extent \( L \) of the ice sheet;
- \( \varepsilon(T) \) temperature dependent ratio between the snow accumulation rate and the ablation rate;
- \( s \) slope of the 0°C-isotherm;
- \( \beta(T - T_{oo}) \) height of the 0°C-isotherm above the northern edge of the ice sheet;
- \( \alpha_0 + \alpha_1 L \) albedo of continents, with dependence on the ice sheet extent.

In our calculations, we take the numerical values for the parameters from Källén et al., and Ghil and Le Treut (1981).

The model has three equilibrium solutions, two of which are linearly unstable saddle points, while the other is a stable focus. For example, in Fig. 2, the points P and Q on the line c are saddle points, and the point C is a focus. When \( C_{L}/C_T \) exceeds the critical value, the stable focus will turn into a locally unstable focus. For some choices of parameters, the transition from a stable focus to an unstable focus leads to the onset of a limit cycle around the focus (see, Källén et al., 1979; Ghil and Le Treut, 1981).

Thus, the KCG model has a nonlinear climatic oscillator. Physically, the model's self-sustained oscillations are caused, when the development of the ice sheet is slow enough compared to the thermal relaxation time of the ocean-atmos-

![Fig. 2 Phase plane for a given set of parameter values. The dotted curve illustrates the steady state curve in eq. (13). The curves a-c are those expressed by eq. (14). \( T_{e,t} \) and \( T_{e,u} \) are the mean annual temperatures at which \( \varepsilon(T) \) starts to increase with \( T \), and it reaches its maximum value, respectively, which are proposed by Källén et al. (1979). See text for further explanation.](image-url)
phere-cryosphere, by the interaction between
the ice-albedo feedback affecting the radiation
balance and the precipitation-temperature feed-
back affecting the ice mass balance.

3. Introduction of a geographical variable

In eq. (1) the land albedo \( \alpha_{L} \) and \( \alpha_{s} \) depends on the extent of the land ice cover. Minimum value in \( \alpha_{o}+\alpha_{s}L \) is reached when \( L = 0 \), that is no land ice, while maximum one occurs when \( L = L_{max} \), that is widely ice-covered
continent. The effect of appearance of continental
ice sheets on the global climate can be tested
by setting up a special condition for the KCG
model. To do this, we introduce a technical
coefficient \( \gamma_{1} \) or \( \xi (=\gamma_{1}/\gamma) \) through the follow-
ing process.

The three matters are to be considered as a
result of the geographical restriction on the ice-
sheet extent. As the geographical restriction, we
here consider the long-term variation of area of
ice-possible land due to such a geological event
as continental drift. First, let us assume that
a latitudinal ice-sheet extent under the past
geographical condition \( L' \) is restricted by a frac-
tion \( \xi' \) to \( L \) at the present condition compul-
sorily, due to the geographical restriction, as
follow;

\[
L' = L\xi'.
\]  
(6)

This is natural, because the latitudinal extent of
land where ice-sheets are to lie is limited by the
geographical condition at the epoch under con-
sideration and, owing to this limitation condi-
tion, ice-sheets cannot extend toward the higher-latitudes. Eq. (6) assumes that a northern
part of an ice-sheet with \( L \) is always cut due to
the geographical restriction, and an ice-sheet
with \( L' \) is newly built up with a certain fraction
to \( L \) which is assumed to be independent of the
magnitude of \( L \). Of course, eq. (6) is a crude
assumption, but it would be admitted, when
considering that the magnitude of variation of
\( L \) is only 10\% \sim 20\% of the magnitude of \( L \) itself
at the most, and so an error due to using the
same equation (6) for the maximum and the
minimum \( L \) is not so much. Here, one should
notice that \( \xi' \) is not simply a ratio of ice-possible
land areas between \( L \) and \( L' \).

Second, the geographical restriction on the
ice-sheet extent also affects the land albedo. Using \( \xi_{a} \) \( (\leq 1) \), we express the land albedo for
the epoch under consideration, as follow;

\[
\alpha_{o} + \alpha_{s}L\xi_{a}.
\]

This can be modified with the use of \( L' \) and \( \xi
\) \( (=\xi_{a}/\xi') \) as

\[
\alpha_{o} + \alpha_{s}L\xi_{a} = \alpha_{o} + \alpha_{s}L'\xi.
\]  
(7)

By substituting \( L' \) for \( L \) in eqs. (1) and (4), a
new \( L' \) equation system can be set easily. Here,
for this new \( L' \) equation system, the same para-
meter values as for the \( L \) system can be used ex-
cept \( h_{0}(=-\beta(T-T_{0})) \) which is the height of the
0°C-isotherm at the northern edge of the ice
sheet, since changes in other physical and cli-
matical parameters are beside our question.
Moreover, fortunately, even if \( h_{0} \) were to be
fixed at the value for the \( L \) system throughout
our discussion, this would not lead a fatal error,
because the contribution of \( h_{0} \) to eq. (4) is
lower by one order, for appropriate \( L' \)'s, than the
other terms in eq. (4). Thus, although roughly,
we will continue the discussion here, for sim-
plicity, using the fixed parameter values, in-
cluding \( h_{0} \), under the same condition as for the
\( L \) equation system. In that case, we can leave
eq. (4) without any changes and can also replace
eq. (1) with eq. (8), using the \( L \) equation system
in place of the \( L' \) equation system:

\[
C_{T} \frac{dT}{dt} = Q\{ 1 - [\gamma(\alpha_{o} + \xi_{a}(L)
+ (1-\gamma)\alpha_{o}T] } \}-K(T-T_{0}).
\]  
(8)

When \( \xi = 0 \) or \( \gamma_{1} \) \( (=\xi_{T}) = 0 \) in eq. (8), no
land ice condition is set by compulsion. Since
land ice is, in general, produced in the high
latitudes, \( \gamma_{1} = 0 \) would mean in geographical
sense that there is no land in the high latitudes
where ice sheets are to get on, even if the \( L > 0 \)
condition is established. Thus, in this case the
land albedo remains \( \alpha_{o} \) \( (=0.25) \). On the other
hand, the condition that \( \gamma_{1} = \gamma \) \( (=0.3) \) or \( \xi = 1 \)
obviously indicates the present geographical
condition.

As \( \alpha_{o} \leq \alpha_{land} \leq \alpha_{o} + \xi_{a}L_{max} \), where \( \alpha_{land} \)
is albedo for continental area, the maximum
land albedo $\alpha'_{\text{max}}$ is equal to $\alpha_0 + \xi \alpha_1 L_{\text{max}}$. Thus,

$$L_{\text{max}} = \frac{(\alpha'_{\text{max}} - \alpha_0)}{\xi \alpha_1}. \quad (9)$$

For simplicity, if $L_{\text{max}}$ is assumed to be proportional to $\xi$ (see Appendix in this connection),

$$L_{\text{max}} = \xi L_{\text{max}}^*, \quad (10)$$

then we obtain from eq. (9),

$$\alpha'_{\text{max}} - \alpha_0 = \xi \frac{(\alpha_{\text{max}} - \alpha_0)}{\alpha_1}, \quad (11)$$

where, $\alpha_{\text{max}}$ and $L_{\text{max}}^*$ denote the maximum land albedo and maximum $L$ in the case of $\xi = 1$, respectively. Of course, $L$ must not be larger than $L_{\text{max}}^*$:

$$L \leq L_{\text{max}}^*. \quad (12)$$

Thus, eqs. (10) and (12) provide the third necessary conditions for our geographical restriction problem.

A set of eqs. (4) and (8) can be interpreted as follows; now, the geographical restriction affects only the land albedo through $\xi$; this seems natural, when considering that, although the case with the same magnitude of extent of the ice-sheet as that of the present one is treated mathematically (or assumptively) in eqs. (4) and (8), the ice-sheet extent under the actual condition is to be smaller than $L$ which is treated mathematically in the set of eqs. (4) and (8), owing to the restriction by the actual geographical condition; thus, the climatic difference between the both cases for the mathematical $L$ and the actual $L$ is reflected and expressed only in the change in the land albedo in eq. (8); that is, all the effect of the geographical restriction on the ice sheet extent is included, en bloc, into the change in the land albedo, or $\xi$, by the above technical treatment. Of course, $L$ must complete the third necessary condition for its maximum value, i.e., eqs. (10) and (12), which also reflects strongly the geographical restriction.

One should note that $\gamma_1$ (or $\xi$) is not the fraction of the earth covered by continents: For, $\gamma$ remains 0.3 in eq. (8) throughout this study. As described above, $\gamma_1$ can be interpreted to give one guide for the geographical restriction for land ice formation. Eq. (10) shows that the maximum extent of the ice cover must be limited by the geographical condition, that is $\xi$ or $\gamma_1$. (As $L$ gives a length, it is thought proper that the ice-covered land albedo is proportional to $\xi^2$ in eq. (11), because albedo due to land ice-covered would be crudely proportional to $L^2$.)

The location and character of critical points of a nonlinear autonomous system composed of eqs. (4) and (8) is particularly important in determining the nature of the solutions. From eq. (4) we can obtain for $dI/dt = 0$,

$$I = - \frac{g(h + h/E - 1) - \sqrt{g^2(h + h/E - 1)^2 - g^2(1/E^2 + 1 + 2/E)(h - 1/2)^2}}{g^2(1/E^2 + 1 + 2/E)} \quad (13)$$

with $g = L^* \xi^2$, $h = \beta(T - T_{00})s + 1/2$ and $E = 1 + \varepsilon(T)$.

For $dT/dt = 0$, eq. (8) gives,

$$I = - \frac{s - \kappa(T - T_0)/Q - 1 + (1 - \gamma)\alpha_{0c}(T)}{\gamma \alpha_0} / \gamma \xi \alpha_1 L^*. \quad (14)$$

In the $T/I$ phase plane, the curve expressed by eq. (13) is independent of $\xi$. This is indicated by the dotted curve in Fig. 2. This curve feature reflects mainly the piecewise linear dependence of $\varepsilon$ on temperature. Between $T_{c, l}$ and $T_{c, u}$, $\varepsilon$ increases linearly with $T$. As widely known, this dependence of $\varepsilon$ on temperature is one distinguished feature of the KCG climate model, instead of using a constant $\varepsilon$ as in Weertman (1976). Some observations seem to support this dependence (e.g., Lorius et al., 1979; Hollin and Barry, 1979; Ruddiman and McIntyre, 1979).

From eqs. (10) and (14), as $I \leq I_{\text{max}}$, $\xi$ must satisfy the condition,

$$\xi = \sqrt{\frac{1 - \kappa(T - T_0)/Q - 1 + (1 - \gamma)\alpha_{0c}(T)}{\gamma (\alpha_{\text{max}} - \alpha_0)}} \quad (15)$$

Elimination of $\xi$ from eqs. (14) and (15) gives a limitation condition for the extent of the land ice-cover,
We call the upper limit curve for eq. (16) in the $T$-$l$ phase plane as the $l_{\text{max}}$-curve.

In Fig. 2, the $l_{\text{max}}$-curve intersects the line of eq. (13) at the point B. So, B gives a steady state solution which satisfies first the limitation condition equation (16) for $l$. At the point B, the value of $T$ is 280.927K, $l$ is 0.945762, $\xi$ is 0.8401537 (= $\xi_0$), and $\gamma_1$ is 0.2502461. As B is a focus, it is expected that the oscillatory climate could emerge on the earth only when $\xi > 0.8401537 = \xi_0$. The curve of eq. (14) displaces from a to c in Fig. 2 with increase in $\xi$.

By the way, when eq. (14) intersects first the line of eq. (13) at the point A, the steady state solution which is a focus appears on the system. However, one can easily understand that the point A cannot satisfy the limitation condition of eq. (15) or eq. (16).

The inclination of eq. (14) $dl/dT$ is, since $T$ is constant (= 310.12K) when $l=0$,

$$dl/dT = -\xi/\gamma_1 L^* Q.$$

On the other hand, that of eq. (13) is

$$dl/dT = -\gamma_1 (1-2h-2gl-2gl/E)/2g\{(1/E^2+1+2/E)gl+ (h+h/E-1)\}.$$  

In the vicinity of the point A for $T>T_{c,u}$ (= 283K), the inclination expressed by eq. (17) is $-0.03787$, while that of eq. (18) is $-0.04094$. This means that there is only one steady state solution which is an unstable saddle point, for $T>T_{c,u}$. Thus, for varying $\xi$, a focus is located only within the range between $T=T_{c,l}$ (= 273K) and $T=T_{c,u}$ (= 283K), where $T_{c,l}$ and $T_{c,u}$ are the same as those proposed by Källén et al. (1979), i.e., the lower and the upper temperatures used in a piecewise linear function of temperature for $\varepsilon$.

Here, when $\xi < \xi_0$, let us suppose that $l$ is fixed by the $l_{\text{max}}$ condition expressed by the following equation,

$$l_{\text{max}} = L_{\text{max}}/L^* = \gamma_1 (\alpha_{l,\text{max}}-\alpha_0)/\alpha_1 L^*.$$  

(19)

Although the actualization of this case is possible only under an extreme condition, to treat this condition is useful to consider one restriction, i.e., the upper limit for glacial condition, to the past "stable" climatic condition before the present oscillatory climate. Especially, it may be useful to obtain an information about the just prior climate to the oscillatory one into which the present earth's climate system had rushed. For, the possibility to realize this extreme climatic condition would increase with decrease in temperature, because for lower temperature, larger glacier would possibly appear. When $\xi < \xi_0$, the possibility clearly becomes highest at the just prior climate to the oscillatory one, i.e., at $\xi \sim \xi_0$. If $l$ is fixed, it is easy to show that the solution of eq. (8) has one asymptotic solution $T_a$, when $t$ is enough long, i.e., for the order to $t > C_L$.

$$T_a = \left[ Q \left( 1-\gamma_0 \gamma_1 (\alpha_{l,\text{max}}-\alpha_0)/\gamma \right. \right.$$

$$\left. \left. - (1-\gamma_0 \alpha_0) + \kappa T_a \right] / \kappa \right].$$

(20)

Eq. (20) also gives an intersecting point between the $l_{\text{max}}$-curve in eq. (16) and the steady state curve for eq. (8). This means that if $l$ were to be fixed by the $l_{\text{max}}$ value during the epoch under consideration for $\xi < \xi_0$, the climatic condition always tends to the $T_a$ in due time, because a geographical variation is thought to have certainly much longer time scale than $C_L$ which would have the value of only $10^3 \sim 10^4$ years (Moriyama, 1986). In Fig. 2, B' indicates such a $T_a$ for a bound condition at $\xi = \xi_0$.

4. Discussion

We performed some numerical experimentations for the geographical condition $\xi \geq \xi_0$, and found a limit cycle around each focus appearing in displacement from the point B to C in Fig. 2, for a wide range of various parameter values. Some of these are shown in Fig. 3a-c, which are computed by a set of same parameter values except for $\gamma_1$ or $\xi$. These numerical computations show an important fact: that is, increasing amplitude of limit cycle with increase in $\xi$; temperature at a critical point decreases with $\xi$.\end{document}
Fig. 3 Time variation of temperature $T$ and non-dimensional ice extent $l$ calculated with the same parameter values except for $\gamma_1$. Computations are all started with the same initial condition of $T=276.7K$, $l=0.72$. (a) is the case for $\gamma_1 = 0.2520 (=\xi_c)$, (b) for $\gamma_1 = 0.28$, and (c) for $\gamma_1 = 0.30$.

Fig. 4 Temperature oscillations obtained from the results of Fig. 3 and some calculations with other values for $\gamma_1$, but $\xi > \xi_c$ condition. They qualitatively agree well with the data in the Pleistocene in Fig. 1. For $\xi < \xi_c$ condition, steady state temperature and ice extent for $l_{max}$-curve, i.e., eq. (16), are also illustrated by solid and dashed curves, respectively. The points B and B' correspond to those in Fig. 2.
dition had allowed it to exist, quantitatively, the condition \( \xi \geq \xi_c \) corresponds to this.

The gradual increases of amplitudes of climatic variables with increase in \( \xi \) are caused mainly by the hydrological feedback for the evolution of ice sheets in the KCG model. In order for the focus to be unstable, i.e., the condition for the real parts of the eigenvalues of matrix obtained from the autonomous system composed of eqs. (4) and (8) to be positive, \( C_L/C_T \) must satisfy the following condition (see also, eq. (25) in the paper of Källén et al. (1979)),

\[
C_L/C_T > \frac{\varepsilon(T_s) + 1}{l_s^3[Q(1-\gamma)(d\alpha_o/dT)]_{T=T_s}}. \tag{21}
\]

where, the suffix \( s \) denotes values at the steady state. When \( C_L/C_T \) exceeds the critical value given by eq. (21), the stable focus will turn into a locally unstable focus. Although this equation does not depend on \( \xi \) explicitly, the effect of varying \( \xi \) upon appearance of the unstable focus is included in \( \xi \). It is obvious from eq. (21) that, among them, a change in \( \varepsilon(T_s) \) due to varying \( \xi \) affects mostly eq. (21), because \( \varepsilon(T_s) \) varies linearly from 0.1 to 0.5 between \( T_{*,l} (= 273K) \) and \( T_{*,u} (= 283K) \) while the other terms change much less. Since \( T_s \) decreases as the focus moves from the point B toward C in Fig. 2, the value of the right-hand side of eq. (21) also decreases. This means that the critical value of \( C_L/C_T \) for the stable focus to turn into a locally unstable focus decreases with increase in the value of \( \xi \). Consequently, it is supposed that, for a fixed value of \( C_L/C_T \) as in Fig. 3, the amplitudes of the limit cycle increase with increase in \( \xi \). This idea can also be driven from the fact that the amplitudes of the limit cycle increase smoothly with increase in the value of \( \xi \). The physical meaning expressed by varying \( \xi \) could be seen, though crudely, from the following tentative discussion.

Change in albedo of the earth due to increase in ice-covered land \( \Delta A \) can be expressed as follow (e.g., Pollack, 1979):

\[
\Delta A = \Delta a \cdot f \cdot I. \tag{22}
\]

Where, \( \Delta a \) is difference in albedo between newly ice-covered land and bare land, \( f \) is the fraction of surface area of newly ice-covered land to the whole surface on the earth, and \( I \) is ratio of solar radiation incident to the newly ice-covered land to its mean quantity for the year incident to the entire earth. Under the present geographical condition, if, as an extreme assumption, \( \alpha_{l,\text{max}} \) condition were to be realized from the no land ice condition on the earth, as \( \Delta A = -\gamma (\alpha_{l,\text{max}} - \alpha_0) \) and \( \Delta a = -q (\alpha_{l,\text{max}} - \alpha_0) \), then \( f \cdot I \) is nearly equal to \( \gamma/q \) from eq. (22), where \( q \), which is near unity but larger than 1, denotes a rectifying factor for albedo, e.g., if the entire land were to be covered with snow, then \( q \) becomes unity. Since \( f \) is equal to \( c \cdot \gamma \) for the case of \( \alpha_{l,\text{max}} \), then \( I = 1/qc \), where \( c \) denotes the fraction of newly ice-covered area to the total land surface. Here, one should note that \( f \) would be also interpreted as a glacier productive capacity of the earth at the epoch under consideration (see also, Fig. 5 and the discussion described below). Although in the above discussion we neglect a change in ocean albedo, this would not lead a fatal error for our results.

When \( \xi = \xi_c \), using eqs. (11) and (22), we obtain that \( f \cdot I = 0.708 \gamma \). If we tentatively assume that \( I = I_0 \), then \( f = 0.71c_0 \gamma \). In the actual condition, as \( f \) for \( \xi = \xi_c \) condition is expected to be slightly larger than \( I_0 \) which is at the present condition of \( \xi = 1 \), we could guess that \( f \approx 0.71c_0 \gamma \). However, this estimation would be cancelled by the effect that \( q \) becomes larger than unity by degrees for smaller \( \xi \). Thus, crudely, we obtain that \( f \approx 0.7c_0 \gamma \) or maybe larger than \( 0.7c_0 \gamma \). This would mean that the earth could not acquire an oscillatory climate system, that is a clear climate system composed of atmosphere-ocean-ice sheet, when supposed maximum ice-covered land could not get the ability which exceeds about 70% or so of the present glacial capacity, most likely owing to some geographical conditions: Continental drift could have made it
possible to produce such an oscillatory climate on the earth, by drift of lands such as the Antarctica or Greenland from warm low-latitude regions to ice possible high-latitudes.

The idea of glacier productive capacity can be clearly seen from Fig. 5. One will easily appreciate from Fig. 5 that, for the lands with the same area, the left-hand side case in Fig. 5 which shows a continent lying with an ample spread toward high-latitude regions has more glacier productive capacity, by the area with the hatch, than the right-hand side case which shows a continent with less spread toward the high-latitudes.

Furthermore, we obtain that, for example, the values of $f$ are equal to 0.44$cq\gamma$ and 0.64$cq\gamma$ for $\gamma_1 = 0.20$ and 0.24 ($< \gamma_1 = 0.2520461$ or $< \xi_0$), respectively. Since it is not plausible that the climate had been existing with the geographical condition of much less than one half of the present glacier capacity in the past, $\xi$ on the earth is speculated to have had a value near $\xi_0$, i.e., a slightly smaller value than $\xi_0$, perhaps for a long time even during the warm climate era, although $\xi$ is able to vary, mathematically, from 0 to 1. It is supposed from our discussion and the observed data in Fig. 1 that the value of $\xi$ on the earth would have barely exceeded the limit of $\xi_0$, probably in the Pliocene, and that, about 3 million years ago, one clear oscillatory climate system had been established with increase in amplitude of climatic oscillations and decrease in temperature, due to change in geographical condition from $\xi=\xi_0$ toward $\xi=1$.

A dependence of the temperature and ice sheet extent upon $\gamma_1$ for the $l_{\text{max}}$-curve is indicated in Fig. 4. Our climate might have entered into the present oscillatory system from a vicinity of the point B' or from a much smaller glacier state. It is probable that some externally forced initial conditions might have made the earth's climate system possible to jump from a "stable" climate state into a clearly oscillatory climate, e.g., from the point B' to B in Fig. 4. For, once a focus is established in the phase-space composed of climate parameters, the climate at the point B' moves abruptly toward the climate condition at the focus B and begins to draw a limit cycle around the focus. (Or it might be completely absorbed into a stable focus if a stable focus were to emerge under appropriate conditions for $C_{L}/C_T$, although the present earth's climate system seems not to have such a stable focus because the value of $C_{L}/C_T$ on the earth seems not to allow a stable focus to exist (e.g., Källén et al., 1979; Ghil and Le Treut, 1981; Moriyama, 1986)).

5. Conclusions

Our results suggest that the earth's climate system may have a self-sustained oscillatory system. Under this assumption and by introduction of the geographical parameter $\xi$, we can show the observed fact that during last million years it grew colder with a gradual increase of amplitude of climatic oscillations. The oscillatory climate seems to emerge on the earth when the earth obtains a clear climate system composed of atmosphere-ocean-cryosphere. Mathematically speaking, it is able to be explained by the fact that our climate system would fall into an un-
stable focus region in the phase space composed of climate variables when a clear cryosphere had been established on the earth, e.g., on the Antarctica or Greenland, and draw a gradually larger limit cycle around a focus with increase in $\xi$, accompanied with colder climate. The earth's geographical condition would have made the earth's oscillatory climate system emerge during last million years. “Plate tectonics” is supposed to give the most influential explanation for it. Our tentative calculations reveal that the oscillatory climate might have emerged, when the glacial productive capacity of the earth could occupy above 70% or so of the present capacity, owing to drift of such lands as the Antarctica or Greenland from warm low-latitudes to ice-possible high-latitude regions.

Of course, the present discussion largely depends upon the availability of the KCG climate model. However, what does it mean that the KCG type climate model could explain definitely some of important observed facts, as shown in the present and the previous our papers (Moriyama, 1986)? We think that it may be caused by the fact that this model would hold some essential parts of the mechanism for the actual earth's climate, and that the ice-albedo feedback affecting the radiation balance and the hydrological feedback for ice mass evolution would hold the important key to the solution of the mechanism for the glaciation cycles on the earth.

If our model would reflect the essence of the actual earth's climate, our results also suggest that the present earth has certainly an auto-oscillatory climate system composed of atmosphere-ocean-cryosphere. This oscillation field appears to have composed the basic climatic field during the last million years on the earth. As suggested by our previous paper (Moriyama, 1986), the present earth's glacial cycles may be governed by the most dominant oscillations composed of large free oscillations with 100 kyr period, and also by the CO₂ effect which would be strongly induced through a link between the biological action and the Milankovitch insolation variation with 40 kyr and 20 kyr periods, superposed non-linearly on the basic climate field with the 100 kyr oscillation period.

### Appendix

Although we use eq. (10) in order to simplify our discussion, eq. (10), in general, should be replaced by eq. (A), with the use of the power constant $a (>0)$ as follow:

$$L_{\text{max}} = \xi^a L_{\text{max}}.$$  \hspace{1cm} (A)

In that case, eqs. (11) and (15) come to eqs. (B) and (C), respectively;

$$\alpha'_{\text{max}} - \alpha_0 = \xi^{a+1} (\alpha_{\text{max}} - \alpha_0),$$  \hspace{1cm} (B)

$$\xi^{a+1} \leq \sqrt{\frac{1 - \kappa(T - T_0)/Q}{(1 - \gamma)\alpha_0(T) - \gamma \alpha_0}} = D.$$  \hspace{1cm} (C)

In this case, the position of the point B in Fig. 2 shifts more or less, when compared with the case of $a = 1$, i.e., eq. (10). However, one can easily understand that this does not damage our mathematical explanation about observation at all that the earth has experienced colder climate with a gradual increase of amplitude of climatic oscillations, because Fig. 2 is still valid without any change.

On the other hand, the value of $D$ which is the right-hand side of eq. (C) varies from 0.638 to 0.725 with increase of $T$, i.e., from $T_{e,1} (= 273K)$ to $T_{e,1} (= 283K)$. Since, using eqs. (B), (C) and (22), we obtain that $f' I$ is equal to $D'\gamma$ for the case of $\xi = \xi_{e}$, according to the discussion in the text, we can comprehend that such a discussion using $a = 1$ as in the text does not lead so wrong estimation for the glacier capacity problem discussed in the text. Thus, as $T$ at $\xi = \xi_{e}$ is supposed to have the value of 280K or thereabouts, we can still conclude that oscillatory climate had appeared on the earth when the past glacier productive capacity could have exceeded about 70% or so of the present one.

### References


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地球の氷期サイクルに関する一仮説：
(II) 更新世における振動気候の出現の原因

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簡単な気候モデルを使って、地球上に振動する気候状態の出現した原因を解析的に議論している。気候モデルに地理学的なパラメータを導入して、振動気候の出現と大陸移動の間の関係を調べた。我々の結果は、気候の振動する振幅が次第に増幅しつつ寒冷化して行ったという過去数百万年間の記録を与える観測事実を数学的に説明できるし、また、振動気候は地理学的な条件がその存在を可能にするようなことがあったときに地球上に出現したのだろうことを示すことができる。南極やグリーンランドのような陸地が暖かい低緯度から積雪可能な高緯度領域へと移動することによって地球上に明確な雪水圏が確立され、過去の地球の氷河生産能力が、現在の能力のおよそ70％かそれ以上を越え得るようになったときに、振動する気候が地球上に出現したかも知れないということを我々の試行的な計算が示唆している。