NOTES AND CORRESPONDENCE

**Spectral Analysis of Vertical Temperature Profiles Observed
by a Lidar in the Upper Stratosphere and the Lower Mesosphere****Takashi Shibata, Seiya Ichimori,****Tohru Narikiyo and Mitsuo Maeda***Department of Electrical Engineering, Kyushu University**Hakozaki, Higashiku, Fukuoka 812, Japan**(Manuscript received 20 July 1988, in revised form 6 October 1988)***Abstract**

The Rayleigh lidar with a XeF excimer laser provides accurate temperature profiles in an altitude range from 30 to 70 km. The power spectral density of the temperature fluctuation as a function of the vertical wavenumber is determined with these data. Both the stratospheric and mesospheric spectra can be obtained simultaneously. The slopes of the observed spectra are between -2.5 and -3.0 , and almost coincide with the published values obtained by the wind observations. The observed spectral density in the upper stratosphere is smaller than that in the lower mesosphere, while the saturation spectrum theory predicts that the saturation spectrum in the upper stratosphere is larger than the lower mesospheric one. Difference between the observed and theoretical spectra are discussed.

1. Introduction

Rayleigh lidar is an unique ground-based remote sensing method that is capable of observing dynamical parameters from the stratosphere through the mesosphere. The vertical atmospheric density profile can be determined by measuring the Rayleigh backscattering profile with the lidar at altitudes higher than about 30 km where the Mie scattering by aerosols can be neglected. The density profile is converted to the temperature profile, assuming that the atmosphere satisfies the perfect gas law and hydrostatic relations (Shibata *et al.*, 1986a). The spatial and temporal resolutions of the density or the temperature distribution determined by lidar are high enough to detect small scale atmospheric fluctuations such as internal gravity

waves.

It is still under debate as to what is the fundamental mechanism of small-scale atmospheric fluctuations in the middle atmosphere. However, the theory of internal gravity waves succeeded in explaining the relations in the spectra of the atmospheric horizontal wind fluctuation versus frequency, vertical wavenumber, and horizontal wavenumber (*e.g.*, VanZandt, 1981 and 1985). The theory also successfully predicted the power spectrum of the horizontal wind fluctuation versus vertical wavenumber (Yamanaka, 1985; Dewan and Good, 1986; Smith *et al.*, 1987).

Shibata *et al.* (1986b) reported the results of about one year of observation starting at the spring of 1985 with a new type of UV Rayleigh lidar using the XeF excimer laser. Since 1987, higher quality data are obtained through an improvement of the laser system adopting the

unstable resonator configuration. In this paper, the power spectral density functions of the temperature fluctuations versus vertical wavenumber are calculated for the upper stratosphere and the lower mesosphere simultaneously, utilizing the same lidar observed temperature profiles. The results are compared with the theoretical spectrum of saturated gravity waves.

2. Scaling of saturated spectrum

The theoretical spectrum of the temperature fluctuations is obtained as follows. As the convective instability threshold by linear theory without inertial effect, the power spectral density F_u^s of the horizontal velocity fluctuation (u) versus vertical wavenumber (m) was derived for an individually saturated wave packet as

$$F_u^s = \frac{N^2}{2m^3}, \quad (1)$$

(Yamanaka, 1985; Dewan and Good, 1986; Smith *et al.*, 1987). Here N is the background buoyancy frequency defined by

$$N^2(z) = \frac{g}{\bar{T}} \left[\frac{\partial \bar{T}}{\partial z} + \frac{g}{c_p} \right], \quad (2)$$

where $\bar{T}$ is mean temperature; g is the acceleration of gravity; c_p is the specific heat at constant pressure. Smith *et al.* (1987) pointed out that the observed relations of the horizontal wind spectra are consistent with the scaling of N^2 (eq. (1)) between in the troposphere and in the stratosphere, and between in the mesosphere and in the thermosphere, though they mentioned nothing about the relation between in the stratosphere and in the mesosphere.

The relation between the temperature and the velocity fluctuations is given by

$$\frac{T'(z)}{\bar{T}(z)} = \frac{N}{g} u, \quad (3)$$

where T' is the temperature fluctuations. From eqs. (1) and (3), the saturation power spectral density F_T^s versus vertical wavenumber of the relative temperature fluctuation $T'/\bar{T}$ is derived as

$$F_T^s = \frac{N^3}{2g^2 m^3}. \quad (4)$$

As shown in eq.(4), F_T^s scales as N^3 . Based on the US Standard Atmosphere, 1976, N is about 0.0033 /sec in the upper stratosphere, and is about 0.0027/sec in the lower mesosphere; the atmosphere is much more stable in the stratosphere than in the mesosphere because of the atmospheric background temperature structure. The stratospheric saturation spectral density should therefore be about two times larger than the mesospheric value, if the concept of saturation applies. Gravity waves need about 1.4 times larger amplitude for saturation in the stratosphere than in the mesosphere.

3. Lidar observation

Table 1 shows the performance of the lidar system. The relative errors of the observed temperature are about 0.2%, 1% and 4% at 30, 45 and 60 km in altitude, respectively, for an observation time of 2 hours and the altitude resolution of 150 m. The use of unstable resonator configuration for the XeF laser improved the measurement accuracy.

Table 1. Performances of lidar system.

Transmitter	
Laser	XeF, with unstable resonator
Wavelength	351, 353 nm
Energy per pulse	100 mJ
Pulse duration	20 ns
Pulse repetition	80 Hz
Beam divergence	≤ 0.5 mrad
Receiver	
Telescope	50 cm diameter
Field of view	≤ 1 mrad
Photomultiplier	EMI9558QB
Filter FWHM	3 nm
(transmission)	(30%)
Photon counter	
Vertical resolution	150 m
Channel number	1000

Twelve vertical temperature profiles observed from June, 1987 through April, 1988 were used in the calculation of the power spectral density. Table 2 shows the dates of the observations and the signal intergration time intervals. The lidar signal was accumulated for about 2 to 5

Table 2. Observation date and time intervals.

Date	Observation time (hour)	
4 June, '87	3.5	
27 Sept., '87	2	
28 Sept., '87	3	
22 Oct., '87	3.5	
9 Nov., '87	5.5	
20 Nov., '87	3	
24 Nov., '87	2	
7 Dec., '87	2	
14 Dec., '87	3	
16 Dec., '87	2	
10 Mar., '88	2	
14 Apr., '88	2.5	

hours to improve the signal to noise ratio. The altitude resolution is 150 m in the following procedure. The temperature fluctuations $T'(z)$ are defined as the difference between the observed temperature and the temperature $\bar{T}(z)$ that is the approximation of the temperature versus height z obtained by the first order least-square method; $\bar{T}(z)$ is calculated in the altitude range where the spectrum is calculated. The power spectral density of the relative temperature fluctuations ($T'(z)/\bar{T}(z)$) was calculated between 30 and 49.2 km in altitude as the upper stratospheric spectrum, and between 45 and 64.2 km as the lower mesospheric spectrum.

The power spectral density in each date is calculated by the maximum entropy method (MEM) between the wavenumber range 0.052 cy/km (cycles/kilometer) (vertical wavelength $\lambda_z=19.2$ km) and 6.7 cy/km ($\lambda_z=150$ m). The point number n of the prediction error filter in MEM was determined by Akaike's criterion; the final prediction error takes the minimum value at n (Hino, 1977). Finally the spectrum was obtained as an average of these twelve spectra. The spectrum was also independently calculated by a conventional FFT method in order to confirm the reliability of the computed spectra by MEM. The results by FFT almost coincided with those by MEM.

4. Results and Discussion

The solid lines in Fig. 1 show the mean spectral densities in the upper stratosphere (S) and in the lower mesosphere (M). The spectra

Fig. 1. Power spectral density of relative temperature fluctuations versus vertical wavenumber. The solid lines show the observed spectra, and the broken lines show the theoretical spectra by eq.(4) with US standard atmosphere, 1976. Marks M and S indicate the mesospheric and the stratospheric results, respectively.

were shown only in the wavelength regions where the values of the spectral density are more than about one order of magnitude larger than the noise levels. The spectra are obtained in the wavenumber range smaller than about 0.5 cy/km ($\lambda_z=2$ km) in the mesosphere, and than about 1.0 cy/km ($\lambda_z=1$ km) in the stratosphere. The standard deviations of the spectra are also shown in Fig. 1. The density in the mesosphere is about 2 or 3 times larger than the values in the stratosphere at the wavelength range from about 2 to 7 km. The slopes of the spectra with the wavenumber larger than 0.1 cy/km ($\lambda_z=10$ km) are between about -2.5 and -3 in both altitude ranges. These slopes coincide with the

slopes of the spectra obtained with the wind fluctuations (*e.g.*, Smith *et al.*, 1987).

The broken lines in Fig. 1 show the theoretical saturation spectrum in the stratosphere and the mesosphere for the model atmosphere obtained by eq.(4). According to the saturation wave theory, the spectrum in the upper stratosphere must be larger than the mesospheric spectrum. However, this relation in the observed results is in the reverse where the wavenumber is smaller than 0.5 cy/km ($\lambda_z = 2$ km), though the slopes of the spectra are near the theoretical value of -3 .

The observed temperature lapse rates ($-\partial\bar{T}/\partial z$) show negative and positive values in all the upper stratospheric and the lower mesospheric temperature profiles, respectively. The observed $\bar{T}$ values are in a similar temperature range in both altitude intervals. Therefore, from eq.(2), N takes larger values in the upper stratosphere than in the lower mesosphere; the relation of the theoretically estimated spectra between in the upper stratosphere and in the lower mesosphere does not change even though the observed N values are used; the estimated spectral density in the upper stratosphere is larger than that in the lower mesosphere.

Smith *et al.* (1987) showed that the saturated spectral density for superposing waves is a factor of 3 smaller than that for individually saturating waves, and that this agrees well with the observed wind spectra. Their results would explain the difference in the observed and the theoretical stratospheric spectra in Fig. 1. However, they are not able to explain the present relation between the observed stratospheric and mesospheric spectra.

The following explanation of the observed results is possible on the basis of the saturated spectrum theory; the observed spectra indicates that the gravity waves are not saturated in the upper stratosphere, though the slope is near the theoretically expected -3 . This slope of the spectrum near -3 where the waves are unsaturated can be still consistent with the saturation theory. If the waves are saturated in the lower and more unstable troposphere, the slope of the spectrum is about -3 there. If the shape of the

spectrum is not substantially changed when the waves propagate into the upper stratosphere from the troposphere, the slope of the spectrum can be also near -3 . These waves can be saturated again in the mesosphere (Lindzen, 1985).

In the above theory the inertial effect was neglected. The flowing explanation with the inertial effect and dynamical instability is also possible. As the Doppler-shifted gravity wave frequency ω (the frequency seen by an observer moving with the local mean flow) approaches the inertial frequency f , the shear instability saturation threshold amplitude of the gravity waves falls well below the convective instability threshold (Fritts and Rastogi, 1985). If the waves satisfy the condition that ω is so much nearer f in the stratosphere than in the mesosphere that the stratospheric saturation spectrum density is smaller than the mesospheric density, the observed relation of the spectra can be explained by the gravity waves saturated by the dynamical instability.

The possibility that the non-linear effects play some roles in determining the spectra also cannot be excluded in explaining the observed results. Therefore, as a conclusion, though the interpretation of the observed results with the simple linear theory without inertial effect suggest that the waves do not saturate in the upper stratosphere, further theoretical and especially observational efforts are urgently needed to understand the mechanism of the small-scale atmospheric fluctuations in the middle atmosphere.

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ライダーによって観測された
上部成層圏，下部中間圏鉛直温度分布のスペクトル解析
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XeFエキシマレーザーを用いたレイリーライダーにより高度域30から70km間の温度分布が正確に得られる。このデータを用いて温度変動の鉛直波数パワースペクトルを求めた。このとき、成層圏と中間圏の両スペクトルを同時に求めることができる。観測されたスペクトルの傾斜は -2.5 から -3.0 の間にあり、これまでに発表された風の測定による値とはほぼ一致する。観測された上部成層圏のスペクトルは下部中間圏のそれに比べて小さい、一方、飽和スペクトル理論では成層圏のスペクトルは中間圏のスペクトルより大きいと予想される。この観測と理論によるスペクトルの食い違いについて論ずる。