Katabatic Wind on Melting Snow and Ice Surfaces (II)
Application of a Theoretical Model

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Abstract

Theoretical consideration of the katabatic wind occurring on melting snow and ice surfaces, which is usually called a glacier wind, is made and is compared with observational results. The source of cooling is the constant $0^\circ$C surface; this is different from the usual katabatic wind which occurs as a result of radiation deficit at the surface. The katabatic wind model of Manins and Sawford (1979), which is a two-layer model incorporating entrainment at the top of the katabatic layer, will be applied to glacier wind. The needed parameters are simplified from the result of measurements of vertical profile of air temperature and wind speed in the present case of katabatic wind on melting snow and ice surfaces. The calculation result shows that on relatively large snow and ice masses, wind speed and thickness are $\propto s$, and sensible heat flux at the surface $\propto s^2$ where $s$ is the distance from the upper end of the snow or ice mass. This is different from the result obtained for katabatic wind occurring from a radiational deficit, for which the wind speed is usually $\propto s^2$ and thickness is $\propto s$. The effect of stability of the ambient atmosphere is such that when the stability is low, wind develops faster along the slope. However, there is no destruction of wind due to adiabatic warming of the katabatic wind layer. This modified model can explain the observed wind speed, depth and surface sensible heat flux at a certain site on snow and ice masses quantitatively and can explain the areal development qualitatively.

1. Introduction

On snow and ice masses (denoted as SIM hereafter) with inclined surfaces such as glaciers, ice caps, snow patches and snow fields, katabatic winds occur not only due to the radiative deficit at the surface but over a melting surface. It occurs due to the constant $0^\circ$C surface owing to the physical properties of the melting point of snow and ice. When the ambient air temperature is higher than $0^\circ$C, the air layer near the surface will be cooled by the surface, resulting in a temperature inversion near the surface. This will start a katabatic wind, which is commonly called a glacier wind or snow patch wind (in the present paper this type of wind will be called a glacier wind). These winds should be distinguished from the nocturnal katabatic winds, as these two types differ in the character of the cooling source.

Structures and characteristics of this type of wind have been studied from observation by Hoinkes (1954a, 1954b), Martin (1975), Munro and Davies (1977), Ohata and Higuchi (1979) and Ohata (1989a) and others. Their study has been limited to the vertical structures and frequency of glacier winds on different SIM. The vertical structure of the wind itself should not be different from that of nocturnal katabatic winds. However, the areal development of the wind would probably be different due to the different boundary conditions. There has been no simple model which has explained the katabatic wind on a constant-temperature slope, or in other words, a slope surface with varying surface heat fluxes.

In order to explain the glacier wind, a theoretical model which was developed for nocturnal katabatic wind will be used. The first model was the one-dimensional study of Prandtl (1942). Work was continued by Fleagle (1950) and others. In this paper the katabatic wind model of Manins and Sawford (1979) will be applied. The result will be compared with some observational results.

On a melting snow and ice surface, cooling of the air layer means heat supply to the surface resulting in melting. This means loss of SIM mass; this effect on the SIM is discussed in Ohata (1989b). The characteristics of the glacier wind treated here probably will not be applicable to a katabatic wind which occurs as a result of a radiation deficit.
2. Description of the model of glacier wind

The model which will be applied here is a two-dimensional steady state bulk model taking into account the dynamic equation, thermal equation and continuity equation (Manins and Sawford, 1979). The situation considered here is shown schematically in Fig. 1. An SIM of length \( L \) is located on an inclined surface of constant inclination \( \alpha \). The air layer is separated into only two layers, that is, the katabatic wind layer and the ambient atmosphere which is outside the thermal effect of the glacier. This ambient atmosphere is assumed to be calm in the present case. The potential temperature and wind speed of the katabatic wind layer are represented by single mean values.

The integrated equation shown in Manins and Sawford (1979) (Equations (2.23)-(2.26) can be expressed as follows.

\[
\frac{\partial}{\partial s} (V^2 h) = -\frac{\partial}{\partial s} \left( \frac{1}{2} S_1 \Delta h^2 \cos \alpha \right) + S_2 \Delta h \sin \alpha - C_M V^2 
\]

(1)

\[
\frac{\partial}{\partial s} (V \Delta h) = B - V h N^2 (\sin \alpha - E \cos \alpha) 
\]

(2)

\[
\frac{\partial}{\partial s} (V h) = EV 
\]

(3)

\[
E = A/(S_1 Ri + K) 
\]

(4)

\[
Ri = \Delta h \cos \alpha / V^2 
\]

(5)

\[
\Delta = \frac{\bar{\theta}'}{\bar{\theta}_a} 
\]

(6)

\[
B = \frac{g H_0}{\bar{\theta}_a \rho C_p} 
\]

(7)

\[
N^2 = \frac{g}{\bar{\theta}_a} \gamma 
\]

(8)

where

\[
U : \text{Mean wind speed of the katabatic layer} \\
h : \text{Height of katabatic layer} \\
\Delta \theta : \text{Buoyancy deficit} \\
\bar{\theta}' : \text{Mean potential temperature deficit of katabatic layer} \\
\theta_a : \text{Ambient potential air temperature} \\
C_M : \text{Drag coefficient} \\
\alpha : \text{Inclination of the surface} \\
H_0 : \text{Sensible heat flux to the surface} \\
E : \text{Entrainment factor} \\
Ri : \text{Richardson number} \\
A : \text{Constant (}= 2 \times 10^{-3}) \\
S_1 : \text{Constant (}=0.5) \\
S_2 : \text{Constant (}=0.9) \\
K : \text{Constant (}= 2 \times 10^{-2}) \\
s : \text{Coordinate along the SIM} \\
g : \text{Gravity acceleration force (=9.8m/s^2)} \\
\rho : \text{Density of air} \\
C_p : \text{Specific heat of air (= 1.0 J/g)} \\
N : \text{Atmospheric stability parameter} \\
\gamma : \text{Ambient potential temperature gradient}
\]

The characteristics of the temperature regime over a melting snow and ice surface are schematically shown in Fig. 1. The ambient potential air temperature taken as \( \theta_a = (\theta_a)_0 + \gamma z \). is the parameter showing the stability of the ambient air. The suffix 0 indicates the base level, which in the present calculation is taken at the level of \( z_a \) the upper end of the SIM where the katabatic wind starts. Potential temperature is a term normally used for the atmosphere, but it can be extended to the temperature of a surface in contact with the atmosphere. Therefore, the potential temperature of the melting
snow and ice surface, which is $0^\circ$C, will be expressed as $\theta_s$. This $\theta_s$ is shown in Fig. 1, and has a steeper gradient against height than $\theta_a$. $\theta_s$ can be written in the form $\theta_s = (\theta_s)_0 + \Gamma_d z$ where $\Gamma_d$ is the dry adiabatic lapse rate and takes the value $9.7^\circ$ C/km. If the SIM is large enough there will be a height where $\theta_a = \theta_s$. This height is denoted as $z_e$.

The mean potential air temperature of the katabatic layer will be expressed as $\bar{\theta}$. Under these conditions $\bar{\theta}$ can be expressed as:

$$\bar{\theta} = \theta_a - \theta_i$$

Some simplification will be made in the expression for $\bar{\theta}$. If there is some similarity in the air temperature profile, $\theta_a - \theta_i$ can be expressed as:

$$\theta_a - \theta_i = R(\theta_a - \theta_s)$$

$R$, called the temperature deficit factor, will be evaluated from observational results. As the katabatic wind thickness is small, air pressure is considered uniform, that is, $p_a = p_0 = p_s$. Then, eq. (10) can be written as follows, by changing $\theta$ into $T$:

$$R = \frac{T_a - T_i}{T_a - T_s}$$

$R$ can be calculated from this equation. $T_a$ is the temperature just above the glacier wind layer and $T_s$ is the temperature of the melting surface, that is $0^\circ$C. $T_i$ takes the following value.

$$T_i = \frac{\int_0^h T_s \, dz}{h}$$

where $T_s$ is the air temperature at height $z$ and $h$ is the glacier wind thickness. The top of the glacier wind is considered the height where the wind speed becomes $1/4$ of the maximum wind speed ($U_m$).

The value of $R$ will be obtained from observational results listed in the references shown in Table 1. In order to determine $T_i$ and glacier wind thickness $h$, the following empirical functions which proved applicable to glacier wind profiles in Martin (1975) and Ohata and Higuchi (1979) was applied to the observed profile.

$$U_z = A \ln(z/B) \exp(-z/C')$$

where $z$ is the height above the surface and $A, B, C, A', B'$ and $C'$ are coefficients. The values of the coefficients were determined by least square regression. The values $h$ and $R$ were derived from the above equations. The ambient air temperature $T_a$ and $R$ for 5 cases in which data are available are shown in Table 1. The values of $R$ vary from 0.15 to 0.20 with a mean value of 0.17. $R$ varies little, although these data are from glacier winds in different stages of development and in different external conditions such as inclination, roughness, ambient air temperature, etc. The constancy of these values is probably due primarily to the similarity of the katabatic boundary layer. Therefore, in the present calculation $R$ will be considered constant along the flow line, and $\bar{\theta}$ will be taken in the following form using $s$ instead of $z$.

$$\bar{\theta} = R(\theta_a - \theta_s)$$

Potential temperature in the present case will not be a dependent variable but will be a given parameter. In converting potential temperature to actual temperature, the air pressure profile of the NACA Standard Atmosphere will be used.

### Table 1. The parameter $R$ for 5 cases of glacier wind for which wind speed and air temperature profile are available.

<table>
<thead>
<tr>
<th>Name of SIM</th>
<th>$T_a$</th>
<th>$R$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peyto Gl.</td>
<td>16.5</td>
<td>0.15</td>
<td>Munro and Davies (1977)</td>
</tr>
<tr>
<td>Tsurugisawa (July)</td>
<td>11.7</td>
<td>0.15</td>
<td>Ohata and Higuchi (1979)</td>
</tr>
<tr>
<td>(Sept.)</td>
<td>13.7</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>San Rafael Gl. (deep case)</td>
<td>12.5</td>
<td>0.18</td>
<td>Ohata et al. (1985)</td>
</tr>
<tr>
<td>(shallow case)</td>
<td>11.5</td>
<td>0.17</td>
<td>Ohata (1989a)</td>
</tr>
</tbody>
</table>

$T_z = A' \ln(z/B') \exp(-z/C')$

where $z$ is the height above the surface and $A, B, C, A', B'$ and $C'$ are coefficients. The values of the coefficients were determined by least square regression. The values $h$ and $R$ were derived from the above equations. The ambient air temperature $T_a$ and $R$ for 5 cases in which data are available are shown in Table 1. The values of $R$ vary from 0.15 to 0.20 with a mean value of 0.17. $R$ varies little, although these data are from glacier winds in different stages of development and in different external conditions such as inclination, roughness, ambient air temperature, etc. The constancy of these values is probably due primarily to the similarity of the katabatic boundary layer. Therefore, in the present calculation $R$ will be considered constant along the flow line, and $\bar{\theta}$ will be taken in the following form using $s$ instead of $z$.

For further simplification, the first term on the right hand side of eq. (1) will be neglected. This term is approximately $10^{-2}$ m$^2$/s$^2$ compared to $10^{-1} - 10^{-2}$ m$^2$/s$^2$ for the second and third terms, according to observation. As the height drop of SIM from the top to the terminus is not large, $\theta_a$ and $p$ will be taken constant along the flow line, that is, $\bar{\theta}$ and $\bar{p}$ for $\theta_a$ and $p$. These values will be used in eqs. (6) to (8).

The following cases will be calculated. The first is for a small SIM in which all temperature parameters can be considered constant (Case 1). For a large SIM, two situations can be considered. One is when the top of the SIM ($z_u$) is higher than $z_e$; the glacier
wind starts at the height $z_e$ (Case 2). Another is the case when the top of the SIM ($z_u$) is lower than $z_e$. The glacier wind starts at the height $z_u$ (Case 3). The most general condition for SIM such as glaciers in the ablation (melting) season is case 3. In case 2, the wind starts where $\theta_s = \theta_a$, so the wind characteristic can be discussed by considering this as a special case of case 3.

Generally, the atmospheric stability of the ambient air is stable under natural conditions. First, an analytical solution for case 1 will be discussed, and then cases 2 and 3 will be considered together with varying stability.

3.1. Case 1

In this case the SIM is small so that $\theta_a$ and $\theta_s$ can be considered the same at every height. In this case, the $\theta' = \text{const}$ approximation is applicable. The boundary condition for this case is $U = h = 0$ at $s = 0$, and $\theta' = \text{const}$. The analytical solution for this case is:

$$V = \sqrt{\frac{S_2g\theta'}{(2 + 3C_Mb)\theta_a}} s \sin \alpha$$

$$h = \frac{2}{3} Es$$

$$H_0 = \rho Cp E \theta' \sqrt{\frac{S_2g\theta'}{(2 + 3C_Mb)\theta_a}} s \sin \alpha$$

$$E = \frac{A}{S_1 Ri + K}$$

$$Ri^2 + Ri \left( \frac{K}{S_1} - \frac{3C_MbS_1}{A} \right) - \left( 2\beta + \frac{3C_MbK}{2A} \right) = 0$$

$$\beta = \frac{2A}{2S_1S_2 \tan \alpha}$$

This result shows that the katabatic wind develops with $U \propto \sqrt{s}$, $h \propto s$ and $H_0 \propto s$. When taking the height drop from $s = 0$, that is $\delta z = s \sin \alpha$, it will be $U \propto \sqrt{\delta z}$, $h \propto \delta z$ and $H_0 \propto \sqrt{\delta z}$. This is similar to the result obtained for nocturnal katabatic winds (Reiher, 1936; Kondo, 1984) although the coefficients are slightly different. The analytical solution of Manins and Sawford (1979) was obtained by taking $B = \text{const}$, which comes from the assumption that the surface radiation deficit is constant along the slope and the sensible heat $H_0$ compensates for it. The result in the region where adiabatic warming was negligible was similar to this first result.

3.2 Cases 2 and 3

The boundary condition for case 3 can be expressed as:

$$U = h = 0, \theta' = (\theta')_0 \quad \text{at} \quad s = 0,$$

$$\theta' = Rrs \sin \alpha + (\theta')_0 \quad \text{at} \quad s \neq 0.$$

Case 2 can be considered a special case of case 3, where $(\theta')_0 = 0$. First, a simple analytical solution for case 2 under the condition $N = 0$ will be shown. The result is:

$$V = \sqrt{\frac{S_2gr R}{(3 + 2C_Mb)\theta_a}} s \sin \alpha$$

$$h = \frac{E}{2}$$

$$H_0 = \frac{3Ecp r R}{2} \sqrt{\frac{S_2gr R}{(3 + 2C_Mb)\theta_a}} (s \sin \alpha)^2$$

$$E = \frac{A}{S_1 Ri + K}$$

$$Ri^2 + Ri \left( \frac{K}{S_1} - \frac{3C_MbS_1}{A} \right) - \left( 3\beta' + \frac{2C_MbK}{A} \right) = 0$$

$$\beta' = \frac{2A}{2S_1S_2 \tan \alpha}$$

This result shows that $U, h \propto s$ and $H_0 \propto s^2$. In the case of using a height drop, we have $U, h \propto \delta z$ and $H_0 \propto (\delta z)^2$. This is a quite interesting result different from case 1. This is mainly due to the temperature field characteristics and constant $R$.

Case 3 is the most common case for SIM in a warm environment. There is no analytical solution. A calculation for conditions of $N = 0$ and $N \neq 0$ for case 2 and case 3 will be made. For calculation, the parameters in eqs. (1) to (5) will be taken equal to the values of Manins and Sawford (1979), which are shown in the equations in sec. 2. The topographical condition of the SIM will be taken as $z_u = 1300$ m and $\alpha = 3^\circ$. The solution with $\bar{p} = 950$ mb, $C_M = 0.01$, and $R = 0.17$ is shown in Figs. 2 and 3. $\theta_a$ depends on the ambient air temperature as explained below. Figure 2 shows the development of the wind along slope $s$, and Fig. 3 shows wind parameters as a function of $T_a$ where $s = 22$ km.

In Fig. 2, three cases (A, B and C) for different temperature ranges are shown. These correspond to the boundary condition $(\theta_a)_0 = 285.4$ K, 290.6 K and 296.9 K respectively. $(\theta_a)_0 = 285.4$ K. The corresponding values of air temperature ($T_a$) at $s = 0$ is 0°C, 5°C and 10°C, respectively. The stability parameter is taken to be $N = 0$, 0.008 and 0.012, which corresponds to vertical temperature gradients of 9.7, 6.9 and 5.5°C/km, respectively. $\theta_a$ is calculated from the above values. $T_a$ was selected as one
Fig. 2. Result of calculation for cases 2 and 3. Description is written in the text.
A: SIM in cold environment \((\theta_a)_0 = 285.4\) K,
B: SIM in warm environment \((\theta)_0 = 290.6\) K
C: SIM in warmer environment \((\theta_a)_0 = 296.9\) K
The numbers 1 to 3 indicate different atmospheric stabilities
1: \(N = 0\)  \(2: N = 0.008\)  \(3: N = 0.012\)

Fig. 3. Effect of atmospheric stability on glacier wind at \(s=22\) km on the SIM. The relationship to ambient air temperature \((T_a)\) at that site is shown. Calculation is made under same conditions as in Fig. 2. Numbers 1 to 3 correspond to same stabilities as in Fig. 2.
reference parameter in the figure since the occurrence of glacier wind is regulated by the temperature difference between ambient air and $0^\circ\mathrm{C}$, the melting surface, so $T_a$ is related better to the development of the glacier wind than potential temperature itself. The difference $\theta_a - \theta_s$ will be roughly proportional to $T_a$. Wind speed ($U$), height ($h$), sensible heat flux ($H_0$) and ambient air temperature ($T_a$) are shown as functions of distance ($s$) for three stabilities. $T_a$ in the graph seems to be linear but is slightly concave. Condition A in the figure corresponds to case 2, and B and C to case 3. Line A-1 is equivalent to the analytical solution eq. (22) - (27) shown at the beginning of this sub-section. For the case of $z_a > z_e$ in case 2, the result for A can be found by taking $s = 0$ at height $z_e$. Case C is in a warmer environment than B. The case of neutral stability $N=0$ will be compared first. Comparing B and C, $U$, $h$ and $H_0$ tend to increase faster along $s$ in C. In a warmer environment, the glacier wind develops faster to the stronger temperature contrast between ambient air and SIM surface. In the warmest case $C$, $U$ and $H_0$ exceed 5 m/s, 100 m and 300 W/m$^2$ at $s = 25$ km. The glacier wind is well-developed. This estimated sensible heat flux is equivalent to melting of the SIM surface by more than 7 cm of water per day. In the development of $U$, the change of $U$ from $*$ to $*$ and change of $H_0$ from $*$ to $*$ can be seen.

The effect of difference in stability $N$ will be discussed from calculated results shown in Figs. 2 and 3. Figure 2 shows that under stronger stability, development of the glacier wind along the slope is weaker, except for $h$ which does not vary. The difference between the three stabilities becomes smaller in a warmer environment. The reason for the difference in the three stabilities is that when $N$ is smaller, which means that the vertical air temperature gradient is larger, the potential temperature difference between the ambient air ($\theta_a$) and SIM surface ($\theta_s$) increases faster over the same distance $s$. In other words, the ambient air temperature rises fast. So under lower stability (small $N$), the glacier wind develops quickly.

The effect of stability ($N$) on the glacier wind at a fixed point on the SIM will be checked in relation to ambient air temperature at that point. The relation is considered, since the characteristics and variation of wind at a certain position on the SIM is important from the standpoint of heat transport to the SIM surface, resulting in melting of snow and ice, and mass balance of the SIM. Values of stability considered here are the same as in Fig. 2; the result at $s = 22$ km is shown in Fig. 3. At the top of the graph, the fetch ($F$) of the glacier wind in each case is also shown. This is shown because under lower ambient air temperature, $z_e$ becomes lower than $z_a$, so the starting point of glacier wind lowers and fetch decreases according to stability. From the figure, it can be said that the effect of stability on glacier wind is large below $T_a = 10^\circ\mathrm{C}$ when the fetch differs among the three. When SIM is in a warm environment, say in this case above $10^\circ\mathrm{C}$, the effect of stability is small. The differences of $U$, $h$ and $H_0$ at $11.3^\circ\mathrm{C}$ and $s = 22$ km between $N=0$ and $N=0.012$ are 10%, 18% and 4% respectively. So, when taking the ambient air temperature ($T_a$) as a function, the effect of atmospheric stability is weak, especially in $H_0$, as long as the fetch of glacier wind does not decrease.

The development of glacier wind also depends on other parameters. The effect of the change in the parameters $C_M$ and $R$ is shown in Table 2 for the case B-1 in Fig. 2. The relative difference in % is shown in the parentheses. As $C_M$ increases $U$, $h$ and $H_0$ decrease due to the larger friction between the atmosphere and the surface. Increase in $R$ increases $U$ and $H_0$, but $h$ is constant. Increase in $H_0$ is very large, so it can be said that the $R$ parameter has a strong effect on $H_0$. The effects of changing $C_M$ and $R$ in other cases are quite similar to this.

From the proceeding discussion it follows that important factors in understanding the characteristics of glacier wind and its variation are different from the nocturnal drainage wind. Besides the SIM parameters (length, inclination, roughness, etc.), sta-
Table 3. The comparison between observed values and the values calculated by the present model. Two cases for the snow patch at Tsurugisawa, Japan and one case for the San Rafael Glacier are shown. O: Observed value, C: Calculated value.

<table>
<thead>
<tr>
<th>Name of SIM</th>
<th>s (km)</th>
<th>δz (m)</th>
<th>α (deg)</th>
<th>$T_a$ (°C)</th>
<th>$N$</th>
<th>$C_M$ (K)</th>
<th>$\bar{p}$ (mb)</th>
<th>R (m/s)</th>
<th>U (m/s)</th>
<th>h (m)</th>
<th>$H_0$ (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSURUGISAWA 1</td>
<td>0.76</td>
<td>260</td>
<td>20</td>
<td>12</td>
<td>0.011</td>
<td>0.007</td>
<td>303</td>
<td>750</td>
<td>0.15</td>
<td>2.2</td>
<td>10.0</td>
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<td>0.17</td>
<td>2.6</td>
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<td>0.15</td>
<td>2.1</td>
<td>10.8</td>
<td></td>
<td>81</td>
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<tr>
<td>TSURUGISAWA 2</td>
<td>0.29</td>
<td>40</td>
<td>8</td>
<td>14</td>
<td>0.011</td>
<td>0.011</td>
<td>303</td>
<td>750</td>
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<td>0.20</td>
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<td>SAN RAFAEL</td>
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<td>0.17</td>
<td>4.0</td>
<td>108</td>
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<td>104</td>
</tr>
</tbody>
</table>

4. Comparison with observational results

4.1 Absolute values of $U$, $h$ and $H_0$

Observations of glacier winds have been made on many glaciers and snow fields, but the cases in which the values of parameters needed for calculation are available are limited. Observations made on snow patches of two different sizes at Tsurugisawa, Northern Japan Alps (Ohata and Higuchi, 1979) and observations on San Rafael Glacier on the Northern Patagonia Icefield, Chile (Ohata et al., 1985) which is 40 km long are used for comparison. The profiles selected here are ones in which the ambient wind was relatively weak. So, they are not necessarily the profiles shown in the papers cited above. The calculated results using the equations derived in sec. 3 and the observational results are shown in Table 3. Notations O and C in the table stand for observed and calculated results, respectively.

Parameters $s$, $\delta z$ and $\alpha$ were obtained from the glacier topography and observation position. $s$ was taken as the distance from the top of the snow patch for TSURUGISAWA1 and TSURUGISAWA2. However, for San Rafael Glacier, the upper accumulation area was very flat, and below 1300 m.a.s.l. inclination increased. It is reported that in the accumulation area (1500 m.a.s.l.) the predominant wind is not a down-glacier wind. So in the present analysis, the starting point of the glacier wind is taken at 1300 m.a.s.l.. Inclination was obtained from drop height $\delta z$ and distance $s$, that is,

$$\alpha = \sin^{-1}(\delta z/s)$$  \hspace{1cm} (28)

As for TSURUGISAWA1, inclination differed much at the higher and lower portions of the snow patch. So, only for this case, another inclination was taken as the mean value of inclination along distance $s$. This was $15^\circ$. $T_a$ and $\bar{p}$ were derived from observed values. The value of $C_M$ will be cited later.

$N$ was taken from other observational results. For TSURUGISAWA1 and TSURUGISAWA2, the vertical air temperature value of $6^\circ C/km$ was obtained for this area (Moribayashi and Higuchi, 1980), so $N = 0.011$ was taken. As for SAN RAFAEL, $5.3^\circ C/km$ (Inoue et al., 1987) was reported; a similar value was found by radio-sonde observation (Fujiyoshi et al., 1987). So, $N = 0.012$ is taken for SAN RAFAEL. The case of SAN RAFAEL corresponds to the calculated case in Fig. 2.

The values of mean wind speed $U$ and thickness $h$ of glacier wind were calculated from the analytical profile derived by adapting equations (13) and (14) to the observed data. $H_0$ and $C_M$ were calculated from the observed data of wind speed and air temperature data below the maximum wind speed level of the glacier wind. The equation of heat flux and vertical flux of horizontal momentum shown in Oke (1978), which considered stability and assumed equivalent value for diffusion coefficient for momentum and heat, was used. Further, a logarithmic profile of wind speed was assumed. $H_0$ is expressed as,

$$H_0 = \rho C_p \frac{k^2 U_s T_z (1 - 5 R_i)^2}{(\ln \frac{z_0}{z_s})^2}$$  \hspace{1cm} (29)

where $k$ is the Karman constant, $U_s$ and $T_z$ is the wind speed and air temperature at height $z$, $R_i$ is the Richardson number and $z_0$ is the roughness length. Turbulent diffusion decreases as $R_i$ increases. The bulk coefficient parameter $C_M$ was calculated from the following equation which was derived by assuming that vertical flux of horizontal momentum $(\tau_s)$ shown in the above reference is equivalent to $\rho C_M U^2$, where $U$ is the mean wind speed of the katabatic wind layer.

$$C_M = \frac{k^2 U^2 (1 - 5 R_i)^2}{(\ln \frac{z_0}{z_s})^2 V^2}$$  \hspace{1cm} (30)
Fig. 4. Relationship between the wind speed at 1 m height ($U$) and $\delta z$ for observational data for glacier wind on a snow patch (solid line) and calculated result (broken line). The observational data are from the Tsurugisawa snow patch (Ohata and Higuchi, 1979), and the values of parameters used for calculation are the same as for TSURUGISAWAI in Table 3.

$z_0$ was taken as 0.2, 0.7 and 10 cm for TSURUGISAWA1, TSURUGISAWA2 and SAN RAFAEL from observed wind profiles. The parameter $C_M$ was calculated as 0.007, 0.011 and 0.01, respectively, and used in calculation.

The observed and calculated values of $U$, $h$ and $H_0$ based on observation and the calculated values using the above model (same procedure as case 3) are shown in Table 3. As for $R$ values the mean value 0.17 and $R$ value for individual cases shown in Table 1 were both used. The values of main parameters are shown in the Table, and other constants take the same value as in sec. 3.

The result in Table 3 shows that the wind speed $U$ and thickness $h$ in three cases fit the observational results quite well. Two $R$ values are taken for TSURUGISAWAI and TSURUGISAWA2. The calculated results of $U$, $h$ and $H_0$ show better results when individual observed $R$ values are used, that is, 0.15 for TSURUGISAWAI, and 0.20 for TSURUGISAWA2, except for $H_0$ in TSURUGISAWAI. This is related to overestimation of $U$ and $h$. The possible cause of this discrepancy is, 1) the possibility that the top of the snow patch was near the ridge and the ambient wind was blowing there, and the starting point of the glacier wind was lower than the top of the snow patch, 2) the effect of change in the surface inclination on the snow patch which is more than 25° in the upper part and 5° in the lower part. Estimation becomes better when taking 15°, which is the mean inclination along the flow line, instead of 20° as shown in Table 3.

The above result shows that the model reproduced the katabatic wind fairly well as regards absolute value of $U$, $h$ and $H_0$. The calculated results are closer to the observational results when values of parameters such as $R$ and $\alpha$ are taken to fit each observation instead of taking constant values or bulk values. When these factors are considered, all calculated values of the elements in all three cases are within ±20% of the observed values.

4.2 Development along the flow line

Not many observations have been made on the variation of these winds along the flow line. Ohata and Higuchi (1979) have observed glacier wind on snow patches in Tsurugisawa, Japan. They measured wind speed at three sites (sites A, B and C) along the center of a snow patch more than 1 km in length many times. Measurements were made at 1 m height, since the wind speed at this height is considered to be proportional to the mean wind speed of the glacier wind layer. Only the case when glacier wind was steadily blowing is taken, and an average condition was derived. In Fig. 4, the relationship between wind speed $U$ and $\delta z$ is shown for an observation result and model calculation. The calculation was done under the same conditions as for TSURUGISAWAI in Table 3 with $R = 0.17$ and $\alpha = 20^\circ$. The difference in absolute value is due to the fact that observational data are measured at one height, 1 m, while calculated values are means over the whole layer. From the figure, both results show a similar trend.

5. Discussions and concluding remarks

It was shown in the preceding sections that the glacier wind can be explained by a modification of the Manins and Sawford katabatic wind model (1979), with special parametrization of the katabatic wind layer and boundary conditions at the surface. Due to the physical conditions, the glacier wind showed different characteristics compared with
a nocturnal katabatic wind of radiational type. For example, Manins and Sawford (1979) assumed that B (shown in eq. (6)) is constant. Their result, with a general tendency similar to previous studies, showed that U increased along the slope proportional to \(\sqrt{s}\) in the case \(N = 0\). But in the case \(N \neq 0\), when \(s\) increased, it weakened due to the adiabatic warming.

In the present model calculation, \(U, h\) and \(H_0\) of glacier wind were derived by assuming a constant temperature deficit factor \(R\) along the flow line. In case of small SIM or short distance (small \(s\)) on large SIM, wind developed as \(U \propto \sqrt{s} \), \(h \propto s\) and \(H_0 \propto \sqrt{s}\), similar to the radiation deficit case. However on a large SIM, wind speed \(U\) and depth \(h\) of the katabatic wind develop proportional to \(s\), and the sensible heat flux \(H_0\) at the surface is \(H_0 \propto s^2\). This differs from winds which result from radiational deficit. The effect of \(N = 0\) for the glacier wind was not the same as in the radiational case, as heating due to the adiabatic warming of the katabatic wind layer was used for sensible heat transfer to the surface. So there was no weakening of the wind at large \(s\) as in the radiational case.

In the case of a katabatic wind due to radiational deficit, the main regulating factor for development is the amount of radiational deficit at the surface and the ambient atmospheric stability. However, in the present case of glacier wind, the regulating external condition differs and the ambient air temperature structure (absolute value and vertical gradient which is equivalent to atmospheric stability) is the main controlling factor. So the effect of atmospheric stability does not work independently, but interacts with the absolute value of air temperature. This is different from the radiational deficit case.

There were not enough observational results to verify the areal development clearly, but the absolute values obtained from model calculations for \(U, h\) and \(H_0\) on three SIM show good agreement with observational results.

This model still has shortcomings. The physical process determining constant \(R\) and its limit of use is not clear yet. The turbulent structure of this wind must be investigated further. However, adoption of \(R\) made the formulation quite simple and it was able to explain the observed results fairly well. Further study of \(R\) and also more areal observations are needed to discuss the katabatic wind on SIM in general. In the present paper, the latent heat transport due to this wind system, but some systematic effect on vapor transfer and latent heat flux to the surface possibly occurs due to the glacier wind.

References

融解している雪氷面上での斜面下降風 (II)
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融解している雪氷面上の斜面下降風、いわゆる氷河風の理論的考察を行った。この斜面下降風は、放射冷却によって起こるものとは、冷源の種類が異なり、原因は 0℃の表面である。本論文ではエントレイメントを考慮した斜面下降風二層モデル (Manins and Sawford, 1979) を採用し、上記の条件をモデル化するとともに、パラメーターを簡略化し、計算を行った。氷河風は小規模な雪氷塊では放射冷却による斜面下降風と類似した特性を示すが、大規模な雪氷塊では sを雪氷塊の上端（風が始まる地点）からの距離とした場合、氷河風の風速と厚さは sに比例し、表面での顕熱輸送量は s～s²に比例する。自由大気の安定度の影響は、自由大気の温度が低い場合強く現れるが、温度が高い場合にはほとんど影響を与えない。本モデルの計算結果と雪氷塊上の特定点での観測結果を比較した所、定量的に良い一致を見た。また、この斜面に沿っての発達の過程を観測例と比較した所、定性的に一致した。