Observation of the Falling Motion of Early Snowflakes.

Part II: On the Variation of Falling Velocity

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Abstract

The variations of vertical and horizontal velocity of early snowflakes, which are composed of two to six crystals, were analysed by a stereo-photogrammetric method. The following main results were obtained:

- 1) The standard deviation of variation in vertical velocity is usually smaller than 2 cm s^{-1} (3 to 5 % of the mean vertical velocity). On the other hand, the standard deviation of variation in horizontal velocity is considerably larger (1 to 8 cm s^{-1}) and increases with the mean horizontal velocity. These results are independent of the main type of component snow crystals. Accordingly, it seems likely that the variation of horizontal velocity plays an important role in the aggregation process of snowflakes.
- 2) The mean horizontal velocity can be estimated from the non-dimensional amplitude of the spiral or rotational falling motion of snowflakes.
- 3) The non-dimensional amplitude is inversely related to the non-dimensional frequency of spiral or rotational motion.
- 4) The non-dimensional frequency can be estimated from the Reynolds number with respect to the mean vertical velocity of snowflakes.

1. Introduction

The aggregation of snow crystals to form snowflakes is an important process in the growth of precipitation particles. Information on the falling motion of snowflakes in the atmosphere is required to calculate their growth by the aggregation process, in addition to the falling velocity of individual snow crystals.

The falling motion of snowflakes is accompanied by the horizontal motion and both vertical and horizontal velocity variations (*e.g.*, Sasyo, 1971, 1977; Zikmunda and Vali, 1972; Kajikawa, 1976). Those velocity variations and horizontal motion are responsible for an increase in the collision efficiency of snowflakes, as pointed out by Rogers (1974), Sasyo (1977), Passarelli (1978, 1979), and Sasyo and Matsuo (1985).

Sasyo (1971) observed the motion of snow particles falling through still air by a stereophotographic method and investigated statistically the features of oscillatory unstable motions. In his observations, however, the correlation of snow particle type to its trajectory on photographs was not performed. Sasyo (1977) also investigated the relationship between oscillatory angular motion and the mass of

snowflakes.

In the former observations described above, the available data of early snowflakes, which are composed of a few or several snow crystals, are not sufficient to study the aggregation process. In Part I (Kajikawa, 1982), the free-fall pattern of early snowflakes in still air and the number and shape of component snow crystals were analysed. Early snowflakes are composed of two to six crystals, of which about 80 % fall with a spiral or rotational motion. In this paper, the correlations between velocity variations and features of early snowflakes will be analysed and discussed.

2. Method of observation

Since the method of measurement of falling motion is the same as in Part I, the procedure will be described briefly. One by one, suitable early snowflakes were picked up with a piece of wood from many snow particles collected on a black cloth and gently dropped into a four-cornered apparatus of about 120 cm in height. For the measurement of three-dimensional motion, a falling snowflake was illuminated by stroboscopic light and photographed by means of a stereoscopic camera system. The positions of the snowflake were calculated from successive corresponding points (the center of images) on

Fig. 1. Relationships between the mass (M) of early snowflakes and the mean vertical ($\bar{V}_Z$) and horizontal ($\bar{V}_H$) velocity. Symbols show the main type of component snow crystals.

the pair of photographs.

The fallen snowflake was caught on a sampling glass plate, which was covered with a thin film of white vaseline, and was microphotographed to determine the shape, size and number of its component snow crystals. Then the snowflake was melted and the hemispherical droplet obtained was microphotographed again to calculate its mass. In this paper, data obtained from the analysis of the fall pattern of the spiral type involving rotational were used to reveal the variations of vertical and horizontal velocities. About 80 % of early snowflakes were composed of two or three crystals of plane type.

3. Results

The relationships between the mass (M) and the falling velocity (mean vertical velocity, $\bar{V}_Z$) and the mean horizontal velocity ($\bar{V}_H$) for dendritic type snowflakes are shown in Fig. 1. For the classification of snowflakes, the main type of component crystals (the largest component crystal) was described, according to Magono and Lee (1966). It is evident from this figure that $\bar{V}_H$ has no relation to M . On the other hand, $\bar{V}_Z$ has reasonable correlation to M . Empirical equations were obtained by a least-squares technique and are summarized in Table 1, along with other relevant data. The values of $\bar{V}_Z$ of snowflakes composed of dendritic crystals with plates at ends (P2a) are slightly larger than those of ordinary dendritic crystals (P1e). This is primarily

due to the difference in aerodynamic drag. The diameter (d : the diameter of circumscribed circle) of P1e snowflakes is larger than that of P2a snowflakes for a given mass on average as shown in Fig. 2, so that the particle cross-sectional area of the former in the direction of its fall is larger than the latter.

Figure 3 shows that non-dimensional amplitude ($a' = a/d$) of the spiral motion, where a is the amplitude of the spiral motion. It is clear that $\bar{V}_H$ is proportional to a' and is independent of the main type of component snow crystals. On the other hand, there is some trend toward smaller $\bar{V}_Z$ with increasing a' .

The non-dimensional frequency ($n' = nd/\bar{V}_Z$) of spiral motion increases as the Reynolds number ($Re = \bar{V}_Z d/\nu$) increases and is independent of the main type of component crystals, as seen in Fig. 4. Here n and ν are the frequency of spiral motion and the kinematic viscosity of air, respectively.

Figure 5 is the relationship between a' and n' . It is evident that a' is inversely proportional to n' and is dependent to some degree of the main type of component snow crystals.

Figure 6 shows an example of the variation in vertical velocity (V_Z) and horizontal velocity (V_H), obtained by the analysis of stereophotographs of Fig. 2 in Part I. In this figure, the extent of velocity variation is expressed by the standard deviation (σ_Z and σ_H) in the distribution of velocity. The variation of V_H is considerably larger than that of V_Z . The

Table 1. Summary of empirical equations.

Main type of component snow crystals	Ordinary dendritic crystal (Ple)	Stellar crystal with plates at ends (P2a)	Plate with simple extensions (P2e)	Stellar crystal with spatial dendrites (P6d)	Rimed stellar crystal (R1d)
Diameter d (cm) range	$0.08 - 0.68$	$0.14 - 0.70$	$0.08 - 0.46$	$0.16 - 0.66$	$0.16 - 0.58$
Mass M (g)– d (cm) relationship	$M = 4.82 \times 10^{-4} d^{1.97}$ $N = 77, r = 0.74$	$M = 8.30 \times 10^{-4} d^{2.09}$ $N = 97, r = 0.76$	$M = 3.96 \times 10^{-4} d^{1.40}$ $N = 40, r = 0.75$	$M = 1.02 \times 10^{-3} d^{2.22}$ $N = 26, r = 0.78$	$M = 5.28 \times 10^{-4} d^{1.76}$ $N = 43, r = 0.68$
Falling velocity $\bar{V}_Z$ (cm s ⁻¹)– M relationship	$\bar{V}_Z = 102 M^{0.11}$ $N = 77, r = 0.52$	$\bar{V}_Z = 94 M^{0.089}$ $N = 97, r = 0.34$	$\bar{V}_Z = 68 M^{0.053}$ $N = 40, r = 0.21$	$\bar{V}_Z = 135 M^{0.13}$ $N = 26, r = 0.60$	$\bar{V}_Z = 286 M^{0.18}$ $N = 41, r = 0.48$
Non-dimensional frequency n' –Reynolds number Re relationship	$n' = 0.0096 e^{0.0069 Re}$ $N = 40, r = 0.49$	$n' = 0.010 e^{0.0057 Re}$ $N = 59, r = 0.43$	$n' = 0.0078 e^{0.0094 Re}$ $N = 19, r = 0.42$	$n' = 0.0092 e^{0.0054 Re}$ $N = 16, r = 0.59$	$n' = 0.0096 e^{0.0052 Re}$ $N = 23, r = 0.36$
Non-dimensional amplitude a' – n' relationship	$a' = 0.0092 n'^{-1.1}$ $N = 40, r = -0.80$	$a' = 0.027 n'^{-0.86}$ $N = 59, r = -0.65$	$a' = 0.013 n'^{-1.0}$ $N = 19, r = -0.71$	$a' = 0.024 n'^{-0.85}$ $N = 16, r = -0.73$	$a' = 0.0091 n'^{-1.1}$ $N = 23, r = -0.74$
Mean horizontal velocity $\bar{V}_H$ (cm s ⁻¹)– a' relationship	$\bar{V}_H = 5.2 a'^{0.22}$ $N = 40, r = 0.43$	$\bar{V}_H = 5.7 a'^{0.25}$ $N = 59, r = 0.54$	$\bar{V}_H = 4.9 a'^{0.26}$ $N = 19, r = 0.51$	$\bar{V}_H = 5.7 a'^{0.30}$ $N = 16, r = 0.53$	$\bar{V}_H = 5.2 a'^{0.41}$ $N = 23, r = 0.67$
Standard deviation in vertical velocity σ_Z – $\bar{V}_Z$ relationship	$\sigma_Z = 0.18 \bar{V}_H^{0.79}$ $N = 48, r = 0.63$	$\sigma_Z = 0.28 \bar{V}_H^{0.57}$ $N = 62, r = 0.50$	$\sigma_Z = 0.26 \bar{V}_H^{0.62}$ $N = 24, r = 0.63$	$\sigma_Z = 0.41 \bar{V}_H^{0.40}$ $N = 19, r = 0.37$	$\sigma_Z = 0.34 \bar{V}_H^{0.39}$ $N = 29, r = 0.47$
Standard deviation in horizontal velocity σ_H – $\bar{V}_H$ relationship	$\sigma_H = 0.70 \bar{V}_H^{0.80}$ $N = 48, r = 0.79$	$\sigma_H = 0.76 \bar{V}_H^{0.72}$ $N = 62, r = 0.78$	$\sigma_H = 0.70 \bar{V}_H^{0.84}$ $N = 24, r = 0.83$	$\sigma_H = 0.89 \bar{V}_H^{0.61}$ $N = 19, r = 0.70$	$\sigma_H = 0.68 \bar{V}_H^{0.81}$ $N = 29, r = 0.85$

N is the number of early snowflakes, and r is the correlation coefficient for the relationship.

Fig. 2. Relationship between the diameter (d) and the mass (M) of P1e and P2a snowflakes.

Fig. 3. Relationships between the non-dimensional amplitude (α') of the spiral or rotational motion and the mean vertical ($\bar{V}_Z$) and horizontal ($\bar{V}_H$) velocity.

Fig. 4. Relationship between the Reynolds number (Re) with respect to the mean vertical velocity and the non-dimensional frequency (n') of the spiral or rotational motion.

Fig. 5. Relationship between the non-dimensional frequency (n') and the non-dimensional amplitude (a').

Fig. 6. An example of the variation in vertical velocity (V_Z) and horizontal velocity (V_H). σ_Z and σ_H are the standard deviation of variation in V_Z and V_H , respectively.

Fig. 7. Relationship between the mean horizontal velocity ($\bar{V}_H$) and the standard deviation (σ_Z) in vertical velocity.

relationship between $\bar{V}_H$ and the standard deviation (σ_Z) of the variation in V_Z is shown in Fig. 7. Although σ_Z increases slightly with $\bar{V}_H$, these values are usually smaller than 2 cm s^{-1} (3 to 5 % of $\bar{V}_Z$) and are independent of the main type of component snow crystals. The values of σ_Z are smaller than the mean value (2.83 cm s^{-1}) observed by Sasyo (1971). This is primarily due to the difference in $\bar{V}_Z$ of the snowflakes between the two studies.

The relationship between $\bar{V}_H$ and the standard deviation σ_H is shown in Fig. 8. It can be seen that σ_H increases with increase in $\bar{V}_H$ and is independent of the main type of component crystals. The value of σ_H amounts to about 50 % of $\bar{V}_H$ and is larger than σ_Z . These values of σ_H are comparable to the mean value (3.0 cm s^{-1}) observed by Sasyo (1971). From the results described above, it seems reasonable to conclude that the variation of the horizontal velocity plays an important role in the aggregation process

of snowflakes.

4. Concluding remarks

The falling motion in still air of early snowflakes, which are composed of two to six crystals, was analysed by means of stereo-photogrammetry.

The variations of vertical and horizontal velocity in the spiral or rotational falling motion were empirically obtained, as shown in Figs. 7 and 8. The standard deviation of the variation in vertical velocity is small. On the other hand, the standard deviation of the variation in horizontal velocity is considerably larger and increases with mean horizontal velocity. Accordingly, the variation of horizontal velocity plays an important role in the aggregation process of snowflakes.

The mean horizontal velocity can be estimated from the non-dimensional amplitude of the spiral or rotational motion. The non-dimensional ampli-

Fig. 8. Relationship between the mean horizontal velocity ($\bar{V}_H$) and the standard deviation (σ_H) in horizontal velocity.

tude is inversely related to the non-dimensional frequency of spiral or rotational motion. And the non-dimensional frequency can be estimated from the Reynolds number with respect to the mean vertical velocity of snowflakes.

In order to generalize the present results, more precise comparison with the falling motion of larger snowflakes, which are composed of many crystals, is important.

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初期雪片の落下運動の観測
第2部：落下速度の変動について
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初期雪片（2ないし6個の雪結晶で構成された雪片）について、落下速度の鉛直成分と水平成分の変動を立体写真法で観測した。

鉛直成分の変動の標準偏差は2 cm/sより小さく、平均落下速度（鉛直成分の平均値）の3ないし5%にすぎなかった。一方、水平成分の変動の標準偏差はかなり大きく（1ないし8 cm/s）、平均水平速度（水平成分の平均値）の約50%にも達した。このような特性は、雪片を構成している主な雪結晶の形にほとんど依存しなかった。

従って、雪片の形成過程においては、落下速度の水平成分の変動が鉛直成分の変動よりも重要な役割をしていると考えられる。