Radiative Transfer to Space Through a Precipitating Cloud at Multiple Microwave Frequencies Part III: Influence of Large Ice Particles

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(Manuscript received 12 April 1989, in revised form 18 June 1989)

Abstract

By the use of a vertically and angularly detailed, plane-parallel microwave radiative transfer model, we have conducted a series of numerical experiments in conjunction with a cloud model simulation to investigate the impact of time-dependent cloud microphysical structure on the transfer to space of passive microwave radiation at several frequencies across the EHF-SHF spectrum. Our overall objective is to explore the detailed physics of using multi-channel, passive-microwave retrieval techniques for the estimation of precipitation from space-based platforms. This paper is a continuation of a previous sensitivity study, which we have published in two-parts (Mugnai and Smith, 1988; Smith and Mugnai, 1988).

The impact of large ice particles on passive microwave brightness temperatures over an evolving model rain cloud are examined at 10 separate frequencies in the EHF/SHF spectrum. Three separate cloud model designs are considered for both hard ice and low density ice freezing modes. The results emphasize how the range of frequencies between 10.7 and 231 GHz differentially respond to the various ice models and freezing modes.

It is shown how frequency-dependent, vertically distributed generalized emission/scattering weighting functions, which we have introduced to vertically resolve the contributions by individual cloud and precipitation layers to the brightness temperatures, can be used to identify the specific layers responsible for regulating the magnitude of top-of-atmosphere brightness temperatures. The weighting function is also coupled to a fractional contribution by scattering function which will exhibit the relative magnitude of the scattering source within the generalized weighting function itself. This enables a thorough understanding of how brightness temperatures are modulated by hydrometeors in individual layers.

Additional salient results of the study are the identification of the importance of incorporating a mixed layer at intermediate and higher frequencies (37 GHz and higher), and quantifying the influence of freezing mode at 85 GHz. It is also shown that the vertical scale of the cloud column, for a fixed equivalent liquid water path, is a major determining factor in generating brightness temperatures consistent with actual observations. This result is based on a case study of a severe thunderstorm monitored with a 2-channel airborne microwave radiometer during the 1986 COHMEX experiment in northern Alabama.

1. Introduction

This paper is the third in a series in which we explore the influence of time-dependent cloud microphysics on top-of-atmosphere (TOA) brightness temperatures at multiple microwave frequencies across the EHF-SHF spectrum. The first two parts (Mugnai and Smith, 1988; Smith and Mugnai, 1988) will be hereafter referred to as Paper I and II, re-
respectively. The primary focus of this study is to investigate the influence of clouds on brightness temperature-rain rate relationships (hereafter, \( T_B-RR \)) within the context of precipitation retrieval from passive microwave measurements. The time-dependent cloud model of Hall (1980) is used to provide the cloud medium in which radiative transfer calculations are carried out.

In Papers I and II, a variety of ice canopies corresponding to different ice-crystal size distributions and concentrations were also included in the model to demonstrate how the ice features differentially decrease the \( T_B \)'s (primarily by scattering) across the intermediate-to-high frequencies (37 to 231 GHz). No large ice particles were involved in these initial experiments. We noted that when only ice crystals are considered, it is far most important to correctly specify the properties of the ice-and-water mixed layer than the total concentration of ice crystals above it.

In this investigation we explore that time-dependent influence of large ice particles on the \( T_B \)'s that would be observed from space above severe thunderstorms. Graupel-size ice particles are expected to occur within limited updraft domains of intense thunderstorms, and since their size parameters at microwave frequencies as low as 6.6 GHz lie within the Mie size regime, they play an important role in determining the microwave appearance of such clouds. Although it is unclear as to what degree large-footprint, satellite-borne radiometers can effectively detect and resolve the mesoscale features of precipitating thunderstorms, this issue remains as an important topic in microwave precipitation retrieval because of the intense interest in heavy rainfall from severe storms (see Spencer et al., 1987).

The framework in which we have carried out the numerical experiments is basically analogous to that described in detail in Paper I. We now examine three different large-particle ice models, including a case in which a mixed-phase layer is incorporated, and also explore the impact of freezing mode (high density versus low density or porous ice particles).

The procedures for incorporating the various ice models and freezing modes are described in Section 2. Section 3 discusses the \( T_B-RR \) relationships. In Section 4 we define a "Generalized Weighting Function" which is used to vertically resolve the contributions by individual cloud and precipitation layers to the \( T_B \)'s; an accompanying profile referred to as the "Fractional Contribution by Scattering" denotes the degree to which multiple scattering influences the weighting functions. Based on this analysis we describe how precipitation retrieval can be formulated as a classic multispectral inversion problem. In this regard we point out both the importance and the advantage of considering the complete hydrometeor column (liquid cloud and precipitation drops along with ice particles) when considering precipitation retrieval as a vertical integration problem. In Section 5 we emphasize the importance of the ice column depth on the \( T_B \)'s; this section includes an intercomparison between model results and measurements obtained from a high-resolution airborne two-channel microwave radiometer. Conclusions are presented in Section 6.

2. Incorporation of time-dependent cloud microphysics

The motivation of this investigation is to explore the role of large ice particles in conjunction with cloud and rain liquid water on the \( T_B \)'s. Large ice particles have considerable influence at the intermediate and high frequencies (\( v \geq 18 \) GHz) and thus it is important to understand how their role changes during the time-evolution of the cloud.

To facilitate this analysis, we have designed three separate microphysical ice models for the precipitating cloud over a Lambertian land surface which is assigned a fixed emissivity of 0.9, invariant with respect to frequency, viz:

1) \( T_B \) does not contain ice particles at any altitude;
2) \( T_B \) contains water drops below a pre-defined freezing level (273*K, or 4.4 km altitude; \( z_f \)); water drops and ice particles in height-dependent proportions in a mixed layer between the freezing level and a pre-defined glaciation level (263*K, or 6.2 km altitude: \( z_g \)); and only ice particles above the latter layer. In the mixed layer, the mass fractions of ice and liquid water (\( m_i \) and \( m_{w} \), respectively) have been taken to be linearly dependent on altitude, \( z \):

\[
m_i = \frac{(z - z_f)}{(z_g - z_f)} \tag{1.1}
\]

\[
m_{w} = 1 - m_i = \frac{(z_g - z)}{(z_g - z_f)} \tag{1.2}
\]

3) \( T_B \) contains only water drops below the freezing level and only ice particles above it.

We refer to the above models as: 1) warm rain; 2) cold rain with mixed layer; 3) cold rain without mixed layer. In addition, a no ice (warm rain) ocean-background case has been included for purposes of intercomparison.

The ice particles are obtained by freezing the appropriate portions of the water drop distributions (generated by the Hall cloud model) that are situated above the freezing level. Calculations have been carried out for both hard and porous ice freezing modes. In this process, the radius change in the drop size spectra via density change has been taken into account; hard ice and porous ice are assumed to have a density of 0.91 g/cm\(^3\) and 0.45
g/cm³, respectively. Hard ice is treated as a pure medium with index of refraction corresponding to pure ice. Porous ice is treated as if the ice were a host matrix to air inclusions; its dielectric properties have been obtained by weighting those of air and ice according to their volume fractions. Porous ice calculations have been made for only ice model 3.

The difficulty with incorporating porous ice is the provision for an adequate treatment of the dielectric properties of a non-solid medium. There is experimental data on the dielectric properties of snow dating back to the early 1950’s; e.g. Cumming (1952), Evans (1965), Ulaby et al. (1981). It is difficult to apply these data directly to our problem where we assume that the porous graupel is a matrix of frozen ice crystals which can retain water or air similar to a sponge — see Rasmussen and Heymsfield (1987a). Bohren and Battan (1982) have discussed how this problem can be treated as the inclusion of a foreign substance into a host matrix (in our present case air into ice). In their study, they examined the problem in which the formulation for the complex dielectric coefficient did not assume symmetry with respect to the inclusion and the matrix. In our experiments we adapt the first order approximation which implies symmetry and which involves an air-ice mixture.

In the following notation we treat the ice as the host matrix and the air as the inclusion. Thus:

\[ n_{mix} = f_{air} n_{air} + (1 - f_{air}) n_{ice} \]  

(2)

Where \( f_{air} \) is the volume fraction of air, \( n_{air} \) and \( n_{ice} \) are the dielectric functions of air and ice respectively, and \( n_{mix} \) is the dielectric function of the mixture. This is always adequate for the case where \( |(\varepsilon_1 - \varepsilon_2)/\varepsilon_1| << 1 \), however, if \( \varepsilon_1 \neq 0(\varepsilon_2) \), improvements in the formulation may be required — see Bohren and Battan (1982).

Taking the density of pure ice (\( \rho_i \)) as 0.91 gm·cm⁻³ and porous ice (\( \rho_p \)) at 1.45 gm·cm⁻³ — see Rasmussen and Heymsfield (1987b) — we can find \( f_{air} \) by assuming no mass difference between a solid sphere and the associate low density sphere; i.e.:

\[
(4/3)\pi r_i^3 \rho_i = (4/3)\pi r_p^3 \rho_p \\
(\rho_i/\rho_p)^3 = r_p/r_i
\]

(3.1)

(3.2)

when \( r_i \) and \( r_p \) are the radii of the solid and porous ice spheres respectively. Now since \( f_{air} \) is related to the volumes of the solid and porous ice (\( V_i \) and \( V_p \)):

\[
f_{air} = (V_p - V_i)/V_i \\
= (r_p^3 - r_i^3)/r_p^3 \\
= 1 - \rho_p/\rho_i
\]

(4)

Therefore:

\[
f_{ice} = 1 - f_{air} = \rho_p/\rho_i
\]

(5)

The refractive index of moist air is given by Battan (1973):

\[
n_{air} = \left[ \frac{77.6}{T} \left( P + 4810 \cdot \frac{e}{T} \right) \right] \cdot 10^{-6}
\]

(6)

where \( e \) is the vapor pressure and \( T \) and \( P \) are the atmospheric temperature and pressure. For sea level conditions \( n \approx 1.003 \).

Since the magnetic permeability is sufficiently close to 1.0, then:

\[ m^2 = \varepsilon \]

(7)

and the complex index of refraction (\( m \)) of the mixture is:

\[
m_{mix} = \left[ f_{air} \cdot n_{air}^2 + (1 - f_{air}) \cdot m_{ice}^2 \right]^{1/2}
\]

(8)

Ulaby et al. (1981) and Bohren and Huffman (1983) have suggested alternate mixing formulae based on air as the host and ice as the inclusion; the real part of \( \varepsilon \) is not significantly altered, however, there are small changes of the imaginary part. We have not tested the impact of these changes on our results.

Using the drop spectra radii abscissa for the cloud drops from our original study, we generate average cross sections (\( \sigma \)) for solid ice from the original drop distributions through a simple integral transformation of the efficiencies (\( Q \)):

\[
\sigma_i = \frac{1}{N} \int \sigma_i dN = \frac{1}{N} \int \sigma [r_i(r)] \frac{dN}{dr} dr
\]

(9)

where \( r \) and \( r_i \) are the radius of the water and ice spheres respectively. The radii of the ice spheres exceed that of the liquid spheres according to:

\[
r_i = (r_i/\rho_i)^{1/3}
\]

(10)

Replacing \( r_i \) with the above and taking \( \rho_w \) as 1 gm·cm⁻³:

\[
\sigma_i = \frac{\pi}{N} \rho_i^{-2/3} \int Q [r_i(r)] r^2 \frac{dN}{dr} dr
\]

(11)

where \( N \) is the particle number density. Since the drop spectra are expressed as dN/ dlnr, and:

\[
\frac{dN}{dlnr} = r \frac{dN}{dr}
\]

(12)

then:

\[
\sigma_i = \frac{\pi}{N} \rho_i^{-2/3} \int Q [r_i(r)] r \frac{dN}{dlnr} dr
\]

(13)

By the same token, for porous ice:

\[
\sigma_p = \frac{\pi}{N} \rho_p^{-2/3} \int Q [r_p(r)] r \frac{dN}{dlnr} dr
\]

(14)
Table 1. Cases for which computations have been carried out.

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Cold rain over land-cloud with mixed layer-hard ice</td>
</tr>
<tr>
<td>B</td>
<td>Cold rain over land-cloud without mixed layer-hard ice</td>
</tr>
<tr>
<td>C</td>
<td>Cold rain over land-cloud without mixed layer-porous ice</td>
</tr>
<tr>
<td>D</td>
<td>Warm rain over land</td>
</tr>
<tr>
<td>E</td>
<td>Warm rain over ocean</td>
</tr>
</tbody>
</table>

Table 2. Rain rates associated to each cloud development time.

<table>
<thead>
<tr>
<th>$t_c$ (sec)</th>
<th>1000</th>
<th>1100</th>
<th>1200</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
<th>1900</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR (mm/hour)</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
<td>1.5</td>
<td>5.0</td>
<td>10.0</td>
<td>20.0</td>
<td>25.0</td>
<td>30.0</td>
</tr>
<tr>
<td></td>
<td>very light</td>
<td>light</td>
<td>medium</td>
<td>heavy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The five cases for which computations have been carried out are summarized in Table 1. The model cloud is analyzed at the same ten time steps utilized in Papers I and II, from a cloud development time $t_c = 1000$ sec to $t_c = 1900$ sec in steps of 100 sec. The model cloud only generates hydrometeors above cloud base (1.0 and 1.5 km over land and over ocean, respectively). Accordingly, a most probable rain rate below cloud base has been associated to each growth stage of the cloud, as indicated in Table 2. According to Paper II, we have selected for the worst probable rain rate the “potential average rainfall rate” for the cloud (see also Paper I). The rain layers between cloud base and surface have been simulated by means of vertically-homogeneous Marshall-Palmer (1948) drop distributions corresponding to these same rain rates. Note that the rain rates in Table 2 can be classified as very light, light, medium, or heavy.

As the cloud development time increases, the cloud grows in the vertical. The cloud top does not reach the freezing level until $t_c = 1400$ sec; therefore no ice particles can exist prior to this time. Thus, results from model cases A, B, and C for $t_c > 1400$ sec can be used to analyze the role of graupel-size particles.

3. Influence of ice phase on BT-RR relationships

Having associated a most probable rain rate below cloud base to each cloud development stage (see Table 2), we can now explore the $T_B$-$RR$ relationships using cloud development time as the abscissa. We point out that this scale is preferable to a log ($RR$) scale (which has been adopted by a number of investigators) because it resolves the rain rate process more uniformly. This has been illustrated for three frequencies in Fig. 1 based on the results of a case A experiment. Note that the dashed lines, which refer to cloud development time, have more gradual roll-off than the solid lines which are associated with of the common logarithm of rain-rate.

Figure 2 shows $T_B$-$RR$ relationships at six frequencies for the three cold rain cases over land, and the two warm rain cases; over land and over ocean. The main feature of this diagram is that both background surface and cloud ice differentially influence the $T_B$'s depending on frequency and rain rate. For the lower frequencies (up to 22.235 GHz), it is the background surface, not the ice physics, that mainly determined the $T_B$'s; at virtually all rain rates for 10.7 GHz; at least up to medium rain rated for the higher frequencies. It is apparent that at the highest rain rates the specification of ice physics begins to play a significant role (about a 10°C effect at 22.235 GHz).

At 37 GHz, the surface background is most important for the light rainfall rates (up to a few mm/hour, or $t_c$ ~ 1400–1500 sec), after which the ice physics model dominates — i.e., there are significant differences between the no-ice, mixed-layer, and no-mixed-layer cases. On the other hand, the freezing mode (hard or porous ice) has negligible influence on the $T_B$'s at this frequency.

At 85 GHz and beyond, the background surface plays no significant role in influencing the $T_B$'s once the cloud has formed. On the other hand, the ice model and the freezing mode are important factors in determining the $T_B$'s — provided the cloud can actually develop and ice phase (i.e. for $t_c$ > 1400 sec, or, equivalently, beyond the very light rain rates). The specification of the ice model is by far of greatest importance at all rain rates; there are major differences among the no-ice and various ice cases. However, starting from the medium rain rates ($t_c > 1600$ sec), the freezing mode plays a considerable role; for the heavy rain rates, the porous ice $T_B$'s are about 20°C warmer than their counterparts for hard ice. Furthermore, the presence of a mixed layer becomes less and less important as either rain rate or frequency increase (i.e., curves A and C tend to converge), while the freezing mode still modulates the $T_B$'s.

It is apparent from these calculations that if 37
GHz and higher microwave frequencies are to be incorporated into precipitation retrieval schemes, then cloud ice and under certain conditions the freezing mode should be accounted for in the precipitation model.

4. Generalized emission/scattering weighting function

When using radiometric data to estimate rainfall from space, it is advantageous to obtain as much information on the vertical structure of the cloud as feasible. This is because the microphysical nature of the main rain layers is not independent of the remaining liquid, mixed-phase, and ice layers which extend up to cloud top. Therefore, knowledge of the microphysical and radiative behavior of these layers can be used to directly aid precipitation retrieval.

Our results shown in Paper II indicate that the frequencies under study are differentially responsive to the mid and upper cloud layers, and that the radiation signals upwelling from the rain layers are often modified significantly by these cloud layers. At the intermediate frequencies (18 to 37 GHz), this occurs once a rain cloud reaches its mature state; at the high frequencies (85 to 231 GHz), radiation from the rain layers is virtually obscured by the upper layers. We show here that only the 6.6 GHz frequency is essentially unaffected by the upper cloud layers.

As a consequence, since the vertical column containing the precipitation retains an internally consistent microphysical stratification, which can largely be resolved by a set of discrete frequencies, we conclude that a most probable rainfall rate can be objectively assigned from a set of multi-channel measurements.

The framework for this approach is to define a height-dependent radiative structure function representing the vertical source of the upwelling radiation to the measuring radiometer. The utility of this type of function was first recognized by Wu and Weinman (1984). Results of our own preliminary study were reported by Mugnai and Smith (1988b). Since microwave radiative transfer through a precipitating cloud involves both emission and scattering in the radiative source function, the radiative structure function must intrinsically contain contributions from both emission and scattering processes. Furthermore, since at microwave frequencies the brightness temperature can be considered to be directly proportional to the upwelling radiation since the Rayleigh-Jeans approximation is satisfactory, the radiative source function can be computed by means of the radiative contribution to the measured $T_B$ from each individual atmospheric layer.

A solution for this problem can be derived directly.
Fig. 2. Top-of-atmosphere brightness temperature as a function of cloud development time for three cold rain cases over land (curves A, B, C), and two warm rain cases: over land (curve D) and over ocean (curve E). Each panel refers to a different microwave frequency from 10.7 to 231 GHz.

From the equation of transfer:

$$\frac{dI}{d\tau} + I = (1 - \omega_0) B(T) + \frac{\omega_0}{2} \int_{-1}^{1} \bar{P} (\mu, \mu') I (\mu') d\mu'$$

(15)

where \( I, \mu, \tau, \omega_0, \bar{P} \) are radiance, cosine zenith angle, optical depth, single scatter albedo, and average scattering phase function respectively, and the R.H.S. is the source term \( J \) at level \( z \):

$$J(z, \mu) = [1 - \omega_0(z)] B(T') + \frac{\omega_0(z)}{2} \int_{-1}^{1} \bar{P} (z : \mu, \mu') \times I (z; \mu') d\mu'$$

(16)

Integrating across a thin layer \([z - \delta z/2, z + \delta z/2]\) of optical thickness \( \delta \tau \):

$$I \left(z + \frac{\delta z}{2}, \mu\right) - I \left(z - \frac{\delta z}{2}, \mu\right) = -\frac{\delta \tau}{\mu} I(z, u) + \frac{\delta \tau}{\mu} J(z, \mu)$$

(17)
The intensity loss and gain terms due to the thin layer are:

Loss = $\frac{\delta \tau}{\mu} I(z, \mu)$

Gain = $\frac{\delta \tau}{\mu} J(z, \mu)$  \hspace{1cm} (18)

The contribution of the thin layer to the radiance at an arbitrary level ($z = z^*$) is:

$C_I(z, \mu) = \frac{\delta \tau}{\mu} J(z, \mu) \exp \left[ -\frac{\tau(z)}{\mu} \right]$  \hspace{1cm} (19)

where $\tau(z)$ is expressed in terms of volume extinction $(\beta_{ext})$:

$\tau(z) = \int_z^{z^*} \beta_{ext}(z') \, dz'$  \hspace{1cm} (20)

The contribution to the upwelling flux at $z^*$ is then given by:

$C_F(z) = 2\pi \int_0^1 C_I(z, \mu) \mu \, d\mu$  \hspace{1cm} (20.1)

$= 2\pi \delta \tau \int_0^1 J(z, \mu) \exp \left[ -\frac{\tau(z)}{\mu} \right] \, d\mu$  \hspace{1cm} (20.2)

The contribution to the upwelling brightness temperature is:

$C_{BT}(z) = \int_{\Delta \lambda} B^{-1}_{RJ} \left[ \frac{C_F(z)}{\pi} \right] \, d\lambda$  \hspace{1cm} (21)

where $\Delta \lambda$ is the channel band pass’ of interest and $B^{-1}_{RJ}$ is the inverse of the Rayleigh-Jeans form of the Planck function. This is the flux equivalent expression of what Wu and Weinman (1984) referred to as the “contribution of atmospheric layers to upwelling radiance”. If we take $\delta z$ as 1 km we can express $C_F(z)$ as:

$C_F(z) = 2\pi \beta_{ext}(z) \int_0^1 J(z, \mu) \exp \left[ -\frac{\tau(z)}{\mu} \right] \, d\mu$  \hspace{1cm} (22)

and thus the relative contribution of a thin layer to the upwelling Rayleigh-Jeans brightness temperature at the top-of-atmosphere ($z_R$), i.e., $T_{RJ}(z_R)$ is:

$W_{BT}(z) = C_{BT}(z)/T_{RJ}(z_R)$  \hspace{1cm} (23)

where $C_{BT}(z)$ is given in deg-km$^{-1}$, $W_{BT}(z)$ in km$^{-1}$ and $T_{RJ}(z_R)$ is expressed according to Eq. 21 by using the flux at level $z_R$. Since $z_R$ is the reference level, $\tau(z)$ in Eq. 22 is the optical depth from $z$ to $z_R$. We call $W_{BT}(z)$ the “Generalized Weighting Function” for upwelling brightness temperatures. Note that $W_{BT}(z)$ incorporates the effects of both emission and scattering. It is a normalized quantity in the sense that:

$\int_0^R W_{BT}(z) \, dz + W_{BT,S} = 1.0$  \hspace{1cm} (24)

where $W_{BT,S}$ is the relative contribution of the surface (both emission and reflection) given by:

$W_{BT,S} = \frac{C_{BT,S}}{T_{RJ}(z_R)}$  \hspace{1cm} (25.1)

$C_{BT,S} = \int_{\Delta \lambda} \left[ B^{-1} \left\{ 2 \left[ \varepsilon B(T_s) + 2(1-\varepsilon) \right] \cdot \int_0^1 I(z = 0, -\mu') \cdot \mu' \, d\mu' \right\} \right] \, d\lambda$  \hspace{1cm} (25.2)

where $\varepsilon$ and $T_s$ are the beam emissivity and skin temperature of a Lambertian surface. It should be recognized that generally, there is a discontinuity between $W_{BT}(0)$ and $W_{BT,S}$.

If weighting functions are obtained at several microwave frequencies, the vertical profile of hydrometeor concentration begins to emerge. Fig. 3 presents frequency-dependent weighting function for both warm-rain/no-ice and cold-rain/mixed-layer land experiments (cases A and D from Table 1) at three cloud-model time steps ($t_c = 1500$, 1700, and 1900 sec) associated with light, medium, and heavy precipitation stages throughout the cloud simulation.

It is important to recognize that these weighting functions behave very similarly to those defined in the context of thermal sounding of the atmosphere, in which vertical penetration of the atmosphere is achieved by selecting a set of frequencies along the wing of a strong absorption band of a uniformly mixed gas and exploiting pressure broadening as a means to select the depth of penetration. In our case, the frequency selection is obtained by ranging across the five microwave windows arising from the superposition of the two strong H2O lines, the strong O2 line and the strong O2 line complex within the EHF/SHF spectrum.

The depth of penetration can be obtained by virtue of the differential emission/scattering properties of hydrometeors as a function of frequency and particle size. Regardless of the physics controlling the behavior of the weighting functions, the result is the same in both cases — the atmosphere is vertically partitioned according to its influence on the upwelling radiances. We thus conclude that the mathematics of precipitation retrieval can be cast into the form of classic multispectral inversion, in which physical iterative retrieval methods can be applied. A recent article by Kummerow et al. (1989) has also addressed this issue.

Due to the normalization condition (24), the weighting functions $W_{BT}(z)$ cannot be divorced from the associated surface relative contributions $W_{BT,S}$. Fig. 4 shows the valued of $W_{BT,S}$ corresponding to the same model cases, time steps and
Fig. 3. "Generalized Weighting Functions" for upwelling brightness temperatures at ten microwave frequencies from 6.6 to 231 GHz. Results are shown for cloud-model time steps $t_c = 1500, 1700, 1900$ sec (light, medium, heavy rain rates, respectively), for both a warm-rain (no ice) case over land (left column) and the mixed-layer cold-rain over land (right column). The vertical structure of the cold-rain cloud consists of a vertically inhomogeneous liquid column (cloud and precipitation drops), a vertically inhomogeneous mixed layer (liquid cloud, precipitation drops and frozen graupel), and a vertically inhomogeneous pure ice layer containing only frozen graupel. The cloud base (CB), freezing level (FL), glaciation level (GL), and cloud top (CT) are indicated with horizontal lines.

Note that the surface has a considerable influence on the $T_b$'s up to 19 GHz at all rainfall intensities; up to 22.235 for medium rain rates; and up to 37 GHz for light rain rates. This is an important issue because it means that iterative retrieval schemes involving a relaxation procedure cannot neglect the surface. It is instructive to monitor the frequency dependent influence of both surface and subcloud layer processes on TOA brightness temperatures as the precipitation model is changed. Fig. 5 illustrates the impact on the relative contribution by the surface to the $T_b$'s as the
microphysics change from warm to cold rain over a land background. Since the ice particles do not appear in the simulation until \( t = 1400 \) sec for the cold rain case, no differences can arise until that time. After that time the ice layer depths increase monotonically with time. The \( T_B \)'s are insensitive to precipitation microphysics in regards to surface influence at frequencies higher than 22.235 GHz. By 18 GHz the microphysics have a negligible impact on the surface influence to the \( T_B \)'s. Below 18 GHz, for heavy rain rates, the differences are significant; 2.5\% and 4\% for 10.7 and 6.6 GHz respectively. This means, for example, that neglect of the ice mechanism at 10.7 GHz in conjunction with a TOA brightness temperature in the vicinity of 260 K, imparts an error of approximately 6.5° C. Therefore, it is important to recognize that variations of the microphysics impact the contributions by the surface to the upwelling TB’s more at the higher rain rates. This is because the cloud is undergoing much greater
changes at the high rainrates that at the light rainrates.

It is just as important to consider the sensitivity in the weighting functions to variations in precipitation intensity below cloud base, particularly when surface rainfall is often the key parameter of interest.

Numerical results show that the weighting functions are only responsive to variations in rain intensity below cloud base at the 2 lower frequencies and then only under certain conditions. Fig. 6 shows differences in the amplitudes of the 10.7 GHz weighting functions as the below cloud precipitation intensities...
are varied across realistic ranges — see Paper II for discussion of these ranges. However, the differences are only significant for the light and intermediate rain-rates and then only for the ocean background case. For the over land background case, modulating the below cloud precipitation intensities has no effect on the weighting functions at 10.7 GHz or any higher frequency.

To facilitate the understanding of the role of scattering in determining the $T_B$'s for each frequency, we have calculated the vertical profile of an expression we refer to as the "Fractional Contribution by Scattering" to the weighting function. This quantity is arrived at by first considering only the emission portion of the atmospheric source term, i.e.:

$$J_e(z,\mu) = [1 - \omega(z)] B_T$$

(26)

Defining the associated contribution to upwelling brightness temperature by emission as $C_{BTE}(z)$, we can express the fractional portion of the weighting function due solely to scattering [$F_{SCA}(z)$]:

$$F_{SCA}(z) = \frac{C_{BT}(z) - C_{BTE}(z)}{C_{BT}(z)}$$

(27)

We refer to this as the "Fractional Contribution by Scattering Function". For consistency, we define $F_{SCA,S}$ to be:

$$F_{SCA,S} = \frac{C_{BT,S} - C_{BTE,S}}{C_{BT,S}}$$

(28)

where $C_{BTE,S}$ is given by:

$$C_{BTE,S} = \int_{\Delta \lambda} B^{-1} \left\{ 2[\xi \cdot B(T_s)] \right\} \times \int_0^1 \exp \left[ -\frac{\tau(z = 0)}{\mu} \right] \mu d\mu \, d\lambda$$

(29)

Figure 7 shows the vertical profiles of the $F_{SCA}(z)$'s associated with the $W_{BT}(z)$'s given in Figure 3. These two figures should be considered together in determining the scattering influence on the $T_B$'s. In illustrating the vertically distributed scattering functions, the dashed portions designate those layers where the total contributions to the brightness temperature are less than 1°C. This is helpful in interpretation because in the lower atmosphere where the fractional scattering contributions often exceed those in the upper atmosphere, the overall influence on top-of-atmosphere measurements is small. At the two lowest frequencies, scattering plays a minor role throughout the vertical column of the precipitating cloud. On the contrary, at the three highest frequencies, scattering by the ice particles above the freezing level has a dominating influence; by comparing the warm rain and cold rain results, one sees that scattering by water drops in the upper regions of the cloud plays a considerable role.

For the cold rain case at the four frequencies between 18 and 22.235 GHz, the weighting functions are maximum just above the freezing level and then tail off to zero near the glaciation level in accordance with the continuously decreasing concentration of emitting liquid particles. On the other hand, the associated scattering functions reach their maximum values well below the freezing level, with a secondary local maximum near the glaciation level, the significance of which is mitigated by the low values of the $W_{BT}(z)$'s at that level. As a result, at these frequencies, scattering from layers both above and below the freezing level contributes significantly to the upwelling $T_B$'s. Finally, at 37 GHz the scattering function is highly dependent on rain rate. At moderate rain rates, it shows a broad maximum from well below the freezing level up to the mid portions of the mixed layer; at the heavy rain rates, it has a dominant peak at the glaciation level. If one considers that the weighting functions peak within the middle of the mixed layer and tail off to zero just above the glaciation level, it is obvious that scattering from the ice particles within the mixed layer plays a considerable role in determining the upwelling $T_B$'s at 37 GHz.

It must be recognized that the selection of the cloud model used within this type of inversion framework is of fundamental importance. Specifically, our results show that the incorporation of a mixed layer within the cold rain model prevents a transmission discontinuity across the freezing level, and thus prevents highly unrealistic spikes in the weighting functions $W_{BT}(z)$. This is particularly important for the intermediate frequencies, which are perhaps the most critical ones for precipitation retrieval. When the mixed layer is neglected and only ice particles are allowed above the freezing level, the scattering functions must spike toward 1.0 discontinuously once the freezing level is crossed into the nearly non-emitting medium, as is clearly illustrated in Fig. 8. These spikes occur in the vicinity of the levels where the weighting functions take on near-maximum values. The end result is that the weighting functions become discontinuously large at the freezing level from a scattering contribution that totally misrepresents the layers which would, under normal conditions, contain a mixture of water and ice particles. It follows that since mixed layers represent a realistic feature in cold rain physics, it is essential to correctly describe them in order to reproduce their impact on the weighting functions.

A fundamental issue in interpreting the role of scattering on the weighting functions, is to recognize that scattering redistributes the microwave radiation emitted by the cloud and precipitation layers and from the surface. Thus, in a sense, multiple scattering disguises the emission process in such a fashion that the weighting functions do not identify
the original sources of the emitted radiation, but only the last layers in which the radiation interacts with the medium and is eventually directed towards the radiometer. In order to enable a more thorough understanding of the transfer process we are currently developing the mathematical framework for an “Emission Source Weighting Function”. We believe this quantity will be useful in combination with the generalized weighting function to understanding how a mixture of hydrometeor species gives rise to the original radiation streams that eventually reach the satellite radiometer through a combination of emission, transmission, absorption, reemission, reflection and scattering.

5. Sensitivity of brightness temperatures to ice columnar depth

Since frozen graupel-size particles have such a dominating influence on intermediate and high frequency $T_B$s at the advanced growth state of the
Fig. 8. Impact of neglecting the incorporation of a mixed layer on the weighting functions at low and intermediate microwave frequencies. Note the discontinuities and unrealistic magnitudes at the freezing level and the misrepresentation of the relative source of layer influence at 37 GHz.

Fig. 9. NASA ER-2 pass over two severe Alabama thunderstorms on July 11, 1986 during COHMEX. The two-channel Microwave Precipitation Radiometer (MPR) shows moderate $T_B$ depressions at 18 GHz and considerable $T_B$ depressions at 37 GHz. Cloud tops for the Eldridge storm are at about 15.25 km.

Cloud, it is of considerable interest to determine how the depth scale of the cloud influences the $T_B$'s. Measurements obtained by the NASA Microwave Precipitation Radiometer (MPR), flown over thunderstorms during the 1986 Cooperative Huntsville Meteorological Experiment (COHME) in northern Alabama (see Dodge et al., 1986), indicate that $T_B$'s at 18 and 37 GHz drop considerably above the central cores of the storms with respect to the cloud-free background — see Fig. 9. The magnitudes of these $T_B$ depressions represent the influence of large graupel-size ice particles, which largely backscatter the upwelling radiation — see also Hakkarinen and Adler (1988) and Heymsfield and Fulton (1988).

The essential question in interpreting these $T_B$'s is — What is the central factor that controls the magnitude of the depressions: the size, concentration or the vertical distribution of the ice hydrometeors? Since the former two parameters present difficult retrieval problems in conjunction with remote
sensing, it is essential to understand the significance of their role. Thus, we have conducted a numerical experiment designed to investigate how the vertical extent of the layer over which a given concentration of ice particles are distributed impacts the upwelling $T_B$'s. Specifically, $T_B$ calculations have been made for the last six time steps of the cloud simulation in which the equivalent water masses as each time step are distributed over twice the cloud column depth (doubled case). For this experiment, the reference case assumes no mixed layer and the freezing level is held fixed at 4.4 km. Since the doubled case uses the same freezing level, the resultant hydrometeor profiles contain an ever increasing proportion of ice particles above the freezing level as the time step is advanced [see Smith and Mugnai (1988b) for a preliminary analysis]. We invoked the doubled case to obtain a storm depth scale corresponding to the COHMEX storm. Since the Hall model does not produce its own ice phase, a realistic depth scale cannot be obtained directly.

The results of this experiment suggest that the

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**REFERENCE CASE**

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<th>Brightness Temperature (K)</th>
<th>$\lambda$ (cm)</th>
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**DOUBLING CASE**

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<td>202 133 111 109 110</td>
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Fig. 10. Intercomparison of frequency dependent $T_B$'s for cloud thickness doubling experiment at 6 cloud model time steps. Note the consistency between the brightness temperatures at 18 and 37 GHz for the doubled cloud column and the Eldridge storm (Fig. 9) when the vertical scale of the model cloud is matched to that of the observed cloud. The temperature directly above the schematic clouds are the time-dependent thermometric cloud top temperatures.
exact size and distribution of the ice particles may not be a first-order factor. The computed $T_B$'s are shown in Fig.10. At the $t_c = 1900$ sec, when the depth scale of the model cloud (15 km) is equivalent to that of the Eldridge storm shown in Fig.9 (about 15.2 km), the 18 and 37 GHz temperatures are in reasonable agreement — 208 K and 233 K at 18 GHz, 135 and 136 K at 37 GHz. On the contrary, the $T_B$'s for the reference case, in which the ice portions of the columns are considerably smaller than those of the doubled case, indicate considerably higher values. This means that at $t_c = 1900$ sec, when the precipitating cloud exhibits steady-state behavior, there is an appropriate amount of frozen particles within the correctly scaled ice column to provide the essential equivalence between model and real cloud. We can conclude from this intercomparison that it is the vertical scale of the ice column of the mature rain cell which is the main factor in determining the upwelling $T_B$'s.

Of further interest is that the 85 GHz $T_B$'s achieve the lowest values when considering all frequencies. This frequency reaches lower temperatures than the two higher frequencies (130 and 231 GHz) because its associated imaginary part of the complex index of refraction for ice is smaller. Since these three frequencies are essentially dominated by scattering, the cloud tops appear coldest at the least absorbing frequency.

6. Conclusions

Our principal interest in this study is to demonstrate that precipitation retrieval is a problem that is intrinsically related to the complete vertical hydrometeor profile including the ice phase. We have shown that the detailed vertical hydrometeor structure of a precipitating cloud can be effectively resolved by means of a Generalized Weighting Function (a function that includes the effects of both emission and scattering) at a set of discrete microwave frequencies ranging from 6.6 to 231 GHz. The behavior of the weighting functions is very similar to that of weighting functions employed in conventional retrieval problems, thus suggesting that the classic multispectral inversion technique is a feasible approach for precipitation retrieval.

It is evident from the numerical experiments that freezing mode (hard or porous ice), through its influence on complex index of refraction and size of the ice particles, is of primary significance to the $T_B$ results except at the highest frequencies. On the other hand, we have shown that a mixed-phase layer is an important feature of any cloud model used in inversion techniques because it prevents unrealistic discontinuities from occurring in the weighting functions across the freezing level. In an ongoing study, we are investigating this issue in more detail by means of a cloud model which explicitly resoves a hierarchy of liquid and ice phases, that develop during the life cycle of a super-cell thunderstorm that has evolved from a multi-cell environment.

It has also been shown through the use of a vertical profile of "Fractional Contribution by Scattering" that the scattering process is vertically distributed in the same manner as a weighting function, and that at the 18–37 GHz frequencies the behavior of the peaks of the weighting functions are highly influenced by the scattering contribution in proximity to the freezing level.

Finally, we have demonstrated that it is the relative distribution of hydrometeor mass into ice and water components along the vertical axis, rather than the total equivalent water mass concentration, which is the crucial factor in determining the magnitude of $T_B$ depressions over a precipitating cloud. By matching the depth scale of a simulated mature precipitating cell to that of a severe COHMEX thunderstorm, we have shown that the computed $T_B$'s at 18 and 37 GHz are in good agreement to measurements at these frequencies obtained from an airborne microwave radiometer.

Acknowledgements

The authors thank Dr. Roy Spencer of the NASA/Marshall Space Flight Center who provided the MPR data and to Nancy Schudalla and Enrico LoCascio for their contributions in preparing the manuscript. Support for this research has been provided by NASA Grant NAGW-991, and by Agenzia Spaziale Italiana and Gruppo Nazionale per la Difesa dalle Catastrofi Idrogeologiche of Italy. NATO provided essential travel support during the preparation of the manuscript under its Grants Program for Collaboration in Research (0217/87). Computational support has been provided by the Florida State University, Supercomputer Computations Research Institute, under DOE Contract DE-FC05-85ER250000.

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この研究のもう一つの顕著な成果は、中間または高周波数（37GHz以上）に対する、混合相の存在の重要性を明らかにしたこと、及び85GHz での氷結モードの影響を定量的に示したことである。更に、ある一定の全液体成分量に対しては、雲の鉛直スケールが、実際の観測と合わせ輝度温度を与える上での、主な支配的要因であることが示された。この結果は、北部アラバマでの1986年のCOHMEの実験における、航空機搭載の2チャンネルマイクロ波放射計による激しい雷雲の観測の実例解析に基づいている。