An Economical Scheme for the Vertical Integral of Atmospheric Emission in Longwave Radiative Transfer

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Abstract

A computationally fast scheme has been developed which accurately evaluates the vertical integral of the transmission function for longwave radiative transfer, incorporating with the multi-parameter random model. In this scheme transmittances are calculated only between the half-levels of the vertical coordinate, while the Planck fluxes are evaluated at both the half- and full-levels. The vertical integral in one layer can be represented with two terms; one is for a temperature profile with no inversion and the other for a profile with inversion.

The validity of this scheme has been investigated by calculating temperature profiles in radiative and radiative-convective equilibria. It has been found that this scheme can accurately evaluate the cooling rates at levels with extreme temperatures near the tropopause and stratopause, and that the temperature profiles contain no fluctuation due to two-grid noise.

1. Introduction

In atmospheric numerical models, the level for temperature calculations (full-level) is usually different from that of radiative flux calculations (half-level) as shown in Fig. 1, and divergences of radiative fluxes at half-levels are set to be one of the diabatic heating terms which change temperatures at full-levels. In such a vertical arrangement of variables, if we continue to integrate the atmosphere evaluating the Planck functions only from temperatures at half-levels, two-grid noise frequently occurs in the full-level temperature profile in a region where long-wave radiative cooling is dominant. A cause of the two-grid noise is that the temperatures at half-levels are calculated through an interpolation of those at adjacent full-levels under the assumption that there is no temperature inversion across the full-level. In other words, as long as the half-level temperatures are fixed, any profile of the full-level temperature results in nearly the same long-wave radiative fluxes. Therefore, once a two-grid noise is included in the full-level temperature profile by some cause, the noise will not be diminished by the longwave radiation scheme alone. Harshvardhan *et al.* (1987) showed that even in layer-mean temperature profiles there also appears two-grid noise both in the radiative and radiative-convective equilibria, and they ascribed the cause of the two-grid noise to the error of temperature interpolations used in

the longwave parameterization. On the other hand, Shine (1987) inferred that the two-grid noise was due to a small numerical instability that could not be diagnosed.

There is always a temperature inversion across the full-level where the temperature profile shows an extreme value. Hence, if we use only the half-level temperature in the calculation of radiative fluxes, similar errors will arise and the cooling rate will be under- or overestimated at maximum or minimum temperature, respectively. As will be demonstrated in the following section, the two-grid noise is certainly very large around extreme temperatures such as those at the tropopause and stratopause. The full-level temperature, therefore, should be considered in the calculation of longwave radiative fluxes.

There are some methods for decreasing the two-grid noise. One method is to treat the nearest layers with a finer resolution. Originally, the reason for this additional treatment for the nearest layers arose from the non-linear behavior of atmospheric absorptance. Manabe and Möller (1961) evaluated the entire integral of absorptance from the top or surface to the half-level concerned with the sum of the integrals between full-levels and the integral between the nearest full-level and the half-level concerned. In the integral, the Planck function at a half-level was used only in the nearest layer integral. This method was utilized in the following works of the thermal equilibrium (Manabe and Strickler, 1964; Manabe and Wetherald, 1967). Williamson *et al.*

Fig. 1. Vertical arrangement of variables. Dotted and solid lines represent half- and full-levels, respectively. Downward and upward radiative fluxes, $F\uparrow$ and $F\downarrow$, are calculated at half-levels, while temperature T , temperature tendency $(\partial/\partial t)T$, and absorber amount W are calculated at full-levels.

(1988) used a similar method, in which the entire integral was represented with the sum of the integral between half-levels. The nearest layer was subdivided by the nearest full-level into two layers, and the Planck function at full-level was used only in these two integrals.

Another method is to increase the number of layers and use temperatures at full-levels and those at the increased levels in the calculation of longwave radiation. In addition, by increasing the number of layers, this method has an advantage improving the accuracy in the evaluation of the non-linear transmission function (or absorptance). However, this method requires a much more computational time because the computational time is approximately proportional to the square of the number of layers. Ramanathan *et al.* (1983) and Chiba *et al.* (1986) doubled the number of layers, and Peng *et al.* (1986) tripled it. The key procedure in both methods decreasing the two-grid noise is to use the Planck fluxes at full-levels.

One of the most important factors to determine

the accuracy in the calculation of longwave radiation is how to calculate transmittances in an inhomogeneous atmosphere. One way is to use a scaling approximation, which can independently calculate transmittance between arbitrary levels and therefore computational time is rather short. However, this approximation, based upon the assumption that broadening of absorption line is due to molecular collision, cannot be used with reasonable accuracy in a region where the Doppler broadening dominates, such as in the middle stratosphere and above. On the other hand, Godson's method (1953) for an inhomogeneous atmosphere can be used with reasonable accuracy irrespective of the shape function of absorption (Shibata and Aoki, 1989). This method, however, cannot independently calculate transmittance between arbitrary levels. Thus, calculation of both transmittances between full (half)-levels and those between half- and full-levels is not suited for Godson's method.

In this work, without increasing the number of layers, we have developed a new scheme accurately evaluating the vertical integral of the transmission function, in which the Planck fluxes at half- and full-levels and transmittances at half-levels are used. Since this scheme requires transmittances between only half-levels, it is suited for the method of the Multi-Parameter Random model (MPR) of Shibata and Aoki (1989), in which transmittances are calculated sequentially in the vertical direction with a modification of Godson's method.

2. Vertical integral of the transmission function

We consider, similarly to Manabe and Möller (1961), a vertical coordinate system composed of half- and full-levels as shown in Fig. 1. Procedure for calculating radiative fluxes is the same as that of Shibata and Aoki (1989) except for the evaluation of vertical integral. Transmittances between half-levels are calculated from the variables such as pressure, temperature, and absorber amount at full-levels. Upward and downward radiative fluxes over spectral interval $\kappa_2 - \kappa_1$ are represented by

$$E\uparrow(P) = \pi B(T(P)) + [\pi B(Tg) - \pi B(Ts)]\tilde{\tau}(P, Ps) - \int_{Ts}^T dT' \tau(P, P')(d/dT') \times \pi B(T'(P')), \quad (1)$$

$$E\downarrow(P) = \pi B(T(P)) - \pi B(T(O))\tilde{\tau}(O, P) - \int_{T(O)}^T dT' \tau(P, P')(d/dT') \times \pi B(T'(P')), \quad (2)$$

where $\pi B(T)$ is the Planck flux spectrally integrated over $\kappa_a - \kappa_1$ at temperature T , $T(O)$ is the temperature at the top of the atmosphere, T_g and T_s are the surface temperatures of the ground and the atmosphere, respectively, and $\tau(P, P')$ and $\tilde{\tau}(P, P')$ are the mean "Planck-weighted" transmission functions between the pressure levels P and P' , which are defined as

$$\tau(P, P) = \frac{\int_{\kappa_1}^{\kappa_2} d\kappa \tau_{\kappa}(P, P') \frac{dB_{\kappa}(T)}{dT}}{\int_{\kappa_1}^{\kappa_2} d\kappa \frac{dB_{\kappa}(T)}{dT}}, \quad (3)$$

$$\tilde{\tau}(P, P') = \frac{\int_{\kappa_1}^{\kappa_2} d\kappa \tau_{\kappa}(P, P') B_{\kappa}(T)}{\int_{\kappa_1}^{\kappa_2} d\kappa B_{\kappa}(T)}, \quad (4)$$

where $\tau_{\kappa}(P, P')$ is the monochromatic transmission function, and B_{κ} the monochromatic Planck function. Transmission functions $\tau(P, P')$ and $\tilde{\tau}(P, P')$ for an inhomogeneous path are converted to those for a corresponding homogeneous path with the modified Godson's method.

Consider the upward flux at the half-level $j + 1/2$. From Eq. (1) the upward flux is given by

$$\begin{aligned} F \uparrow (P_{j+1/2}) &= \pi B(T_{j+1/2}) + [\pi B(T_g) \\ &\quad - \pi B(T_s)] \tilde{\tau}(P_{j+1/2}, P_{LT+1/2}) \\ &\quad - \sum_{i=j+1}^{LT} \int_{i+1/2}^{i-1/2} d\pi B(P') \\ &\quad \times \tau(P_{j+1/2}, P'), \end{aligned} \quad (5)$$

where LT is the number of layers. First we use the weak approximation of the 2-parameter random model in evaluating the transmission function. That is, the transmission function within one layer is assumed to be represented with an exponential function of absorber amount. Then, the transmittance at an arbitrary level P between the half-levels $i - 1/2$ and $i + 1/2$ is given by

$$\tau(P_{j+1/2}, P) = \tau(P_{j+1/2}, P_{i-1/2}) \times \exp(-KW), \quad (6)$$

where K is a proportional constant represented by the three quantities, the absorber amount contained in the layer, $\tau(P_{j+1/2}, P_{j+1/2})$, and $\tau(P_{j+1/2}, P_{i-1/2})$, and W is the absorber amount contained between the half-level $i - 1/2$ and the level P . In the remainder of the paper we will, if unnecessary, omit the letters "P" and "T" from indices to simplify the notation. To carry out the integral analytically yet simply, we assume that the Planck function and absorber amount can be represented with linear functions of height within one layer. This assumption was also used in line-by-line calculations of Aoki (1988). Inserting Eq. (6) into the third term

in Eq. (5) and integrating the resultant equation, we obtain

$$\begin{aligned} &\int_{i+1/2}^{i-1/2} d\pi B(P') \tau(P_{j+1/2}, P') \\ &= \int_{i+1/2}^i d\pi B(P') \tau(P_{j+1/2}, P') \\ &\quad + \int_i^{i-1/2} d\pi B(P') \tau(P_{j+1/2}, P') \\ &= [\pi B(i) - \pi B(i + 1/2)] \\ &\quad \times \frac{\tau(j + 1/2, i) - \tau(j + 1/2, i + 1/2)}{\ln \tau(j + 1/2, i) - \ln \tau(j + 1/2, i + 1/2)} \\ &\quad + [\pi B(i - 1/2) - \pi B(i)] \\ &\quad \times \frac{\tau(j + 1/2, i - 1/2) - \tau(j + 1/2, i)}{\ln \tau(j + 1/2, i - 1/2) - \ln \tau(j + 1/2, i)}. \end{aligned} \quad (7)$$

We assume here that the i th full-level is just the level which divides the absorber amount Wi in the layer by halves. Then, the transmittance between the levels $j + 1/2$ and i can be written as

$$\begin{aligned} \tau(j + 1/2, i) &= \tau(j + 1/2, i - 1/2) \exp(-KW i/2) \\ &= \sqrt{\tau(j + 1/2, i + 1/2) \tau(j + 1/2, i - 1/2)}. \end{aligned} \quad (8)$$

It follows from Eqs. (5), (7) and (8) that

$$\begin{aligned} F \uparrow (j + 1/2) &= \pi B(j + 1/2) + [\pi B(T_g) \\ &\quad - \pi B(T_s)] \tilde{\tau}(j + 1/2, LT + 1/2) \\ &\quad - \sum_{i=j+1}^{LT} \left\{ [\pi B(i - 1/2) - \pi B(i + 1/2)] \right. \\ &\quad \left. + \pi [B(i - 1/2) + B(i + 1/2) - 2B(i)] \right. \\ &\quad \times \frac{\sqrt{1 - \tau(j + 1/2, i + 1/2) / \tau(j + 1/2, i - 1/2)}}{\sqrt{1 + \tau(j + 1/2, i + 1/2) / \tau(j + 1/2, i - 1/2)}} \left. \right\} \\ &\quad \times \frac{\tau(j + 1/2, i - 1/2) - \tau(j + 1/2, i + 1/2)}{\ln \tau(j + 1/2, i - 1/2) - \ln \tau(j + 1/2, i + 1/2)}. \end{aligned} \quad (9)$$

The brace in the right hand side of Eq. (9) represents a substantial change $d(\pi B)$ in the Planck flux within one layer. The first term in that brace is the (first) difference in the Planck flux between half-levels. This term virtually contributes to the substantial change in the Planck flux for the case of no temperature inversion, as will be proved later. On the other hand, the second term in that brace, represents the "correction term" for the two-grid noise or temperature inversion across a full-level. The "correction term" is composed of a product of two terms. The first term is a second difference in the Planck flux, i.e., $B(i - 1/2) - B(i) - (B(i) - B(i + 1/2))$. The second term is a relative change in transmittance. When transmittances at full-levels are set

Fig. 2a. Vertical profiles of the relative change in transmittance for the first neighbor layer in the "correction term" for the US Standard Atmosphere. Abscissa is logarithmically scaled.

to be the arithmetic mean of those at the adjacent half-levels and the integral is evaluated with the trapezoidal formula, the second term becomes $[\tau(i - 1/2) - \tau(i + 1/2)] / [\tau(i - 1/2) + \tau(i + 1/2)]$, where index " $j + 1/2$ " is omitted.

Next we consider the magnitudes of these two terms. As seen from the functional form of the second difference in the Planck flux in Eq. (9), we can roughly estimate its magnitude. When there is no temperature inversion, the second difference in the Planck flux is very small because the half-level temperatures are calculated through an interpolation of the full-level temperatures. On the other hand, where there is an inversion in the full-level temperature, the second difference for that height becomes quite large compared with the corresponding first difference.

The relative change in transmittance in the "correction term" varies from zero to unity with atmospheric opacity. We cannot, however, easily estimate the relative change. To carry out a quantitative evaluation we actually calculated the relative changes in transmittance for the clear model atmospheres compiled by McClatchey *et al.* (1972). Figs. 2a and 2b show the vertical profiles of the relative changes in

Fig. 2b. As in Fig. 2a except for the second neighbor layer.

transmittance for the first neighbor and the second neighbor layers, *i.e.*, $i = j + 1, j + 2$ in Eq. (9), for four spectral intervals for the US Standard Atmosphere. The CO₂ 15 μ m band and the O₃ 9.6 μ m band are included in the spectral intervals 550 - 800 cm⁻¹ and 800 - 1200 cm⁻¹, respectively. The magnitude of the relative change is the largest for the first neighbor layer and the second largest for second neighbor layer, and so on, since the transmission function rapidly decreases with absorber amount. On the other hand, the relative changes decrease with height for H₂O and CO₂, while for O₃ they do not, but show maximum values in the middle stratosphere as seen in Figs. 2a and 2b. It is worth while to point out that the relative change is not more than unity. However, the magnitude of the relative change of itself does not directly relate with that of the "correction term". Indeed, the relative change in the upper atmosphere is very small compared with that in the lower atmosphere, but this does not mean that the "correction term" virtually works in the lower atmosphere. For example, in the stratosphere the first difference in the Planck flux is also very small. Hence, the "correction term" can be greater than or comparable to the first difference in the Planck flux, if there is a temperature inversion across full-level.

Next, we consider the magnitude of the "correc-

tion term" based upon the estimation of the magnitudes of the two terms, *i.e.*, the second difference in the Planck flux and the relative change in transmittance. Consider a contribution of the i th layer to the upward flux at the $j + 1/2$ th level (see Eq. (9)). When there is no inversion in the full-level temperature from the $i - 1$ th to $i + 1$ th levels, the second difference in the Planck flux in the i th layer becomes smaller by far than the corresponding first difference. In this case the relative change in transmittance, less than unity, does not matter, and hence the "correction term" also becomes very small. From this temperature profile we increase only the i th level temperature to be the highest of the three temperatures, holding the first difference in the i th layer nearly unchanged. Then, the second difference in the i th layer becomes larger in magnitude than the corresponding first difference, and thereby the "correction term" becomes much larger than when there is no inversion. Consequently, the i th layer emits more longwave radiation to cool itself. When the i th level temperature is decreased to be the lowest of the three, the longwave emission from the i th layer decreases to warm itself. In these cases, the relative change in transmittance plays a crucial role on the magnitude of the "correction term". In summary, without the "correction term", we will make a quite large under- or overestimation of the longwave emission from an inversion layer.

In Eq. (9), the "correction term" becomes very small when the full-level temperature coincides with the arithmetic mean of the adjacent half-level temperatures. If vertical resolution is so fine that the Planck flux can be linearly interpolated with temperature within one layer, the "correction terms" vanish. This comes from the assumption that the full-level coincides with the level of half absorber amount.

Here, we change the form of the second difference in the Planck flux to be consistent with the interpolation scheme used in determining the half-level temperature, by interpreting that the second difference correctly stands for a finite difference of vertical second derivative. Since we determine half-level temperatures through a linear interpolation in $\log P$, we can replace the second difference in the Planck flux as

$$\begin{aligned} & B(i - 1/2) + B(i + 1/2) - 2B(i) \\ & \simeq \frac{\ln P(i - 1/2) - \ln P(i + 1/2)}{2} \\ & \times \left[\frac{B(i - 1/2) - B(i)}{\ln P(i - 1/2) - \ln P(i)} \right. \\ & \left. - \frac{B(i) - B(i + 1/2)}{\ln P(i) - \ln P(i + 1/2)} \right]. \end{aligned} \quad (10)$$

The right hand side of Eq. (10) is then reduced

to zero when the i th level Planck flux is a $\log P$ -weighted mean of those at the adjacent half-levels $i - 1/2$ and $i + 1/2$.

Next, we evaluate the vertical integral of Eq. (5) by using the strong approximation of the 2-parameter random model. This is because the strong approximation can yield more accurate cooling rate in opaque layers near the earth's surface than the weak approximation when the MPR model is used (Shibata and Aoki, 1989). The vertical integral, however, cannot be represented by a simple form such as Eq. (8). Hence, we make two assumptions on the analogy of the weak approximation: firstly, the substantial change in the Planck flux within one layer is composed of the first difference in the Planck flux between half-levels and the "correction term"; secondly, the "correction term" can be represented with those of the weak approximation in Eqs. (9) and (10). Then, adding the "correction term" in Eqs. (9) and (10) to the equation of upward flux (see Eq. (24) in Shibata and Aoki (1989)), we obtain the upward flux at the half-level $j + 1/2$:

$$\begin{aligned} F \uparrow (j + 1/2) &= \pi B(j + 1/2) + [\pi B(Tg) \\ & - \pi B(Ts)] \tilde{\tau}(LT + 1/2) \\ & - \sum_{i=j+1}^{LT} \left\{ [\pi B(i - 1/2) - \pi B(i + 1/2)] \right. \\ & + \frac{\ln P(i - 1/2) - \ln P(i + 1/2)}{2} \\ & \times \left[\frac{\pi B(i - 1/2) - \pi B(i)}{\ln P(i - 1/2) - \ln P(i)} \right. \\ & \left. - \frac{\pi B(i) - \pi B(i + 1/2)}{\ln P(i) - \ln P(i + 1/2)} \right] \\ & \times \frac{\sqrt{1 - \tau(i + 1/2)/\tau(i - 1/2)}}{\sqrt{1 + \tau(i + 1/2)/\tau(i - 1/2)}} \left. \right\} \\ & \times 2 \left(\frac{[\ln \tau(i - 1/2) - 1]\tau(i - 1/2)}{[\ln^2 \tau(i - 1/2) - \ln^2 \tau(i + 1/2)]} \right. \\ & \left. - \frac{[\ln \tau(i + 1/2) - 1]\tau(i + 1/2)}{[\ln^2 \tau(i - 1/2) - \ln^2 \tau(i + 1/2)]} \right) \end{aligned} \quad (11)$$

where index " $j + 1$ " is omitted in $\tilde{\tau}$ and τ , *i.e.*, $\tilde{\tau}(LT + 1/2) = \tilde{\tau}(j + 1/2, LT + 1/2)$, $\tau(i + 1/2) = \tau(j + 1/2, i + 1/2)$, and so on. Similarly, we obtain the downward flux:

$$\begin{aligned} F \downarrow (j + 1/2) &= \pi B(j + 1/2) - \pi B(1/2) \tilde{\tau}(LT + 1/2) \\ & + \sum_{i=j}^j \left\{ [\pi B(i - 1/2) - \pi B(i + 1/2)] \right. \\ & + \frac{\ln P(i - 1/2) - \ln P(i + 1/2)}{2} \\ & \times \left[\frac{\pi B(i - 1/2) - \pi B(i)}{\ln P(i - 1/2) - \ln P(i)} \right. \end{aligned} \quad (12)$$

Table 1. Vertical coordinate system of full- and half-levels. The numbers of layers is 23 (LT = 23). Unit is 10^3 mb.

Level index	Full sigma level	Half sigma level
1	0.500E-4	0.100E-3
2	0.193E-3	0.299E-3
3	0.511E-3	0.750E-3
4	0.112E-2	0.153E-2
5	0.190E-2	0.228E-2
6	0.282E-2	0.339E-2
7	0.420E-2	0.505E-2
8	0.625E-2	0.752E-2
9	0.931E-2	0.112E-1
10	0.139E-1	0.167E-1
11	0.206E-1	0.248E-1
12	0.307E-1	0.370E-1
13	0.458E-1	0.551E-1
14	0.682E-1	0.820E-1
15	0.101	0.122
16	0.151	0.182
17	0.225	0.271
18	0.335	0.403
19	0.499	0.600
20	0.688	0.780
21	0.839	0.890
22	0.940	0.980
23	0.990	1.000

$$\begin{aligned}
& \left[\frac{\pi B(i) - \pi B(i + 1/2)}{\ln P(i) - \ln P(i + 1/2)} \right] \\
& \times \left\{ \frac{\sqrt{1 - \tau(i - 1/2)/\tau(i + 1/2)}}{\sqrt{1 + \tau(i + 1/2)/\tau(i + 1/2)}} \right\} \\
& \times 2 \left(\frac{[\ln \tau(i - 1/2) - 1]\tau(i - 1/2)}{[\ln^2 \tau(i - 1/2) - \ln^2 \tau(i + 1/2)]} \right. \\
& \left. - \frac{[\ln \tau(i + 1/2) - 1]\tau(i + 1/2)}{[\ln^2 \tau(i - 1/2) - \ln^2 \tau(i + 1/2)]} \right)
\end{aligned}$$

We will investigate the validity of this representation of radiative fluxes by calculating thermal equilibrium in the next section.

3. Thermal equilibrium calculations

We calculate temperature profiles in radiative and radiative-convective equilibria under clear sky conditions by the two schemes, with and without the "correction terms" in Eqs. (11) and (12), and investigate the effect of the "correction terms". We set 10 points on the meridian, the latitudinal interval of which is 18 degrees. That is, the northern most point is at 81°N and the next one at 63°N , and so on. The procedure for calculating the radiative equilibrium is the same as that of Manabe and Möller (1961), and that for calculating the radiative-convective equilibrium is the same as that of Man-

abe and Strickler (1964), and Manabe and Wetherald (1967). However, we omitted the convective adjustment for the layer which was not in contact with the earth's surface, since this adjustment may eliminate or decrease the two-grid noise.

We consider the following absorbers as H_2O , CO_2 ($15\ \mu\text{m}$) and O_3 ($9.6\ \mu\text{m}$) for longwave radiation, and H_2O and O_3 for shortwave radiation. The calculations of longwave and shortwave heating are based upon the scheme of Shibata and Aoki (1989) and that of Lacis and Hansen (1974), respectively. We use the vertical coordinate system depicted in Table 1, which is composed of 23 layers and has a top level of 0.05 mb; this system is used in a global spectral model for the study of the middle atmosphere in the Meteorological Research Institute. Water vapor amount at the surface is calculated from the smaller value of either 70 % relative humidity or 21 mb partial pressure, and at 100 mb level its amount is 2×10^{-6} g/g mixing ratio. Between the surface and 100 mb, water vapor amount is linearly interpolated in $\log P$, and above 100 mb the same value 2×10^{-6} g/g is used. The concentration of CO_2 is assumed to be 330 ppmv. Ozone distribution is taken from Klenk *et al.* (1983). For the solar radiation, the surface albedo is set to be 0.10 and the fractional length of daytime is assumed to be 0.5 in each latitude. The solar constant is taken to be $1367\ \text{W/m}^2$ (WMO, 1982). The time and date, which is used for the calculations of ozone distribution and solar zenith angle, is set to be at 9 hour on the vernal equinox. Then, effective values of $\cos(\text{zenith angle})$ for the 5 points in the Northern Hemisphere are 0.06, 0.18, 0.28, 0.35, and 0.39, respectively. Ordinarily, these values are set not to exceed the annual mean value for the hemisphere, 0.25, in thermal equilibrium calculations (*e.g.*, Manabe and Strickler, 1964; Ramanathan, 1976). Our concern, however, is not to simulate thermal equilibria under real conditions but to investigate the two-grid noise made by the longwave radiative scheme. Hence we may set the values larger than 0.25.

Figures 3 and 4 show the temperature profiles at the five latitudes in the Northern Hemisphere in the radiative equilibrium calculated by the scheme with the "correction term", and the two-grid noises, *i.e.*, the temperature differences between the two schemes, respectively. The scheme with "correction term" certainly erases the two-grid noise, while the other scheme does not. The two-grid noise shown in Fig. 3 is small in the middle stratosphere, where temperature changes gradually with height, while it is rather large at other heights. In particular, very large errors occur at extreme temperatures: an underestimate at the tropopause and an overestimate at the stratopause. On the other hand, as seen in Figs. 5 and 6, the temperature in the radiative-convective equilibrium contains no two-grid noise in

Fig. 3. Temperature profiles in the radiative equilibrium. Circles, triangles, pluses, crosses, and rhombi represent the temperatures in 81°N, 63°N, 45°N, 27°N, and 9°N, respectively.

the surface convective layer where temperature is adjusted to yield a critical lapse rate (6.5 K/km). Therefore, there remain two regions of large two-grid noise around the tropopause and stratopause. However, where the top of convective layer coincides with the tropopause, *i.e.*, $\cos(\text{zenith angle}) = 0.28$ and 0.35, the large errors of underestimation at the tropopause disappear because of the convective adjustment as seen in Fig. 6.

By a detailed examination of the temperature profiles in the thermal equilibria, we noticed that small yet common erroneous rises remain only around 6 mb level; the rises can be more clearly discerned in the higher three temperature profiles as seen in Figs. 3 and 5. The present scheme was expected to suppress the rises by the same way as it erased the two-grid noise through the increase/decrease in longwave radiation. So, we will consider the cause of these temperature rises.

In the thermal equilibrium treated in this paper the shortwave heating owing to ozone absorption is constant throughout the time integration, since the ozone amount is prescribed irrespective of temperature. Hence, the thermal equilibrium is the state that the longwave cooling coincides with the shortwave heating given initially, so far as the stratosphere is concerned. We calculate ozone profiles from the coarse vertical data set of Klenk *et al.*

Fig. 4. Temperature differences between the two schemes with and without the "correction term". The meanings of symbols are the same as in Fig. 3.

(1983) by using a cubic spline function as in Shine (1987) such that the shortwave heating rates yield very smooth curves as shown in Fig. 7. On the other hand, the longwave cooling rates calculated by the MPR method are not always very smooth. The MPR method yields an abrupt change with an error of about 0.2 K/day decrease from about 10 mb to 6 mb levels in the cooling rate calculation for the model atmospheres, while line-by-line calculations yield a rather smooth cooling rate. This is because different sets of parameters are used in the different pressure ranges; the critical pressure is 6.91 mb in this case (Shibata and Aoki, 1989). Therefore, in the thermal equilibrium the temperature near 6 mb level rises to make the longwave cooling rate so smooth as to exactly compensate the shortwave heating rate. If we vary the ozone amount with temperature considering the ozone chemical reactions as in the work of Fels *et al.* (1980), the circumstances may become different; the temperature smoothness in the thermal equilibrium depends upon both of the shortwave and longwave calculations.

4. Summary and concluding remarks

We have developed a new scheme which can accurately yet efficiently evaluate the vertical integral of the transmission function even if there are temperature inversions across full-levels. Transmittances are calculated with the MPR model between half-levels.

Fig. 5. As in Fig. 3 except for the radiative-convective equilibrium.

Fig. 6. As in Fig. 4 except for the radiative-convective equilibrium.

The vertical integral of the transmission function from the top or the surface to a half-level is represented by the sum of those integrals through one layer, which are composed of two terms; one which corresponds to the integral for no temperature inversion uses the Planck fluxes at half-levels alone, and the other which corresponds to the "correction term" for temperature inversion uses the Planck fluxes at both half- and full-levels.

We have investigated the effect of the "correction term" by calculating temperature profiles in the thermal equilibria. It has been found that the equilibrium temperatures calculated by the scheme with the "correction term" contain no two-grid noise, while those calculated by the scheme without the "correction term" do contain two-grid noises. In particular, the largest errors in cooling rate calculations of an overestimate at the tropopause and an underestimate at the stratopause are removed by the "correction term".

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Fig. 7. Shortwave heating profiles used in the calculations of the radiative-convective equilibrium. The meanings of symbols are the same as in Fig. 3.

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大気放射伝達式の鉛直積分のエコノミカルスキーム

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長波放射の放射伝達式中に現れる透過率の鉛直積分を速く、正確に評価するスキームを開発した。マルチパラメータランダムモデルで表現された透過関数はハーフレベル間のみで計算され、プランクラックスはフル、ハーフの両レベルで計算される。或る層の鉛直積分は層内に温度の逆転が無い場合の積分と逆転に対する補正項の2つで評価される。

このスキームの有効性を調べるため放射及び放射-対流平衡時の鉛直温度分布を計算した。その結果、このスキームは層内で必ず温度の逆転を伴う温度の最低、最高点(対流圏界面、成層圏界面)付近での放射冷却率を正確に表現でき、しかも2-グリッドノイズのような振動を含まない温度分布を与えることが分かった。