Numerical Modeling of Dynamical and Microphysical Evolution
of an Isolated Convective Cloud
– The 19 July 1981 CCOPE Cloud –

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Abstract

A 3-dimensional, anelastic cloud model is applied to the simulation of the July 19, 1981 Cooperative Convective Precipitation Experiment (CCOPE) case study cloud. The model utilizes the bulk water microphysical parameterization technique where number concentrations of cloud ice and snow are taken into account in addition to the mixing ratios of six water species (water vapor, cloud water, cloud ice, rain, snow and graupel/hail). Cloud ice is initiated only by primary nucleation processes (deposition/sorption and heterogeneous and homogeneous freezing of cloud droplets) in the present model. The timing reference was established between the simulation and observations based on a remarkable change in the rise rates of both the observed and simulated cloud tops, and the model results are compared with the observations as a function of time and space.

The general features of the cloud (such as cloud top height, cloud size, arrival time of precipitation at the cloud base, radar first echo, etc.) seem to have been well reproduced. Furthermore, the model cloud simulated quite well the location of hydrometeors with respect to the updraft/downdraft structure, the number concentration of precipitating ice particles, updraft velocity and cloud water content along the King Air’s penetrating pass.

The main features which are not accurately reproduced are the cloud base height, the rise rate of the cloud top and the radar echo near the ground. The cloud base height is too low, which is attributed to the lack of representativeness in the input data taken from the closest radiosonde sounding, while the too rapid rise rate of the cloud top seems to be attributable to the way in which convection is initiated. The rapid decrease in radar reflectivity of the simulated cloud seems to be attributable to inadequate parameterization for rain and graupel.

1. Introduction

The size and number concentrations of ice crystals and snow particles (single snow crystals and aggregates) play an important role in precipitation formation in mixed phase clouds. They are a critical factor in the determination of the growth modes (depositional, aggregational and riming growth) of snow particles and control the timing and location of graupel formation.

Most of the models with bulk-water microphysical parameterizations have adopted the water mixing ratios as the prognostic variables for each category of hydrometeors. Two exceptions are the models developed by Cotton et al. (1986) and Ziegler (1985). In the models using the water mixing ratios, competitive processes such as depositional, aggregational and riming growth were not accurately simulated due to the lack of information about the sizes and number concentrations of ice crystals and snow particles. To overcome these deficiencies in the models used so far, an improved model should be able to deal with temporal and spatial evolutions in the sizes and number concentrations of ice crystals and snow particles without requiring a large additional computer capacity.

Numerical models are used for diagnostic studies to gain new insights into the cloud and precipitation formation processes and/or for prognostic studies to guide future field experiments. However, the results obtained from such numerical simulations depend upon the ability of the models. Thus, before using the models for such purposes, it should be confirmed from comparisons with field observations that the simulation model can faithfully reproduce the natural phenomena.

In this study, a model with a new microphysical parameterization was developed to more accurately simulate the ice initiation and consequent precipi-
tation growth. The microphysical parameterization adopted two prognostic variables, the water mixing ratios and number concentrations for cloud ice and snow, as well as a new and more realistic formulation for some components of the microphysical processes. This model was applied to the simulation of the July 19, 1981 Cooperative Convective Precipitation Experiment (CCOPE) case study cloud to check on its performance. The CCOPE case study cloud was intensively observed using four instrumented aircraft and a ground based radar unit (Dye et al., 1986) so that it is one of the most suitable cases to see if the model could faithfully simulate the dynamical and microphysical features of an isolated convective cloud.

In the following sections, we describe the formulation of the model, present the results and compare the model results with the observations in detail.

2. Model description

2.1 Basic equations

The model used in this study is a 3-dimensional, time-dependent cloud model with bulk-water microphysical parameterizations. The non-hydrostatic, anelastic equations by Yoshizaki (1985) are adopted as basic equations. The equations of motion are as follows:

\[
\begin{align*}
\frac{\partial u'}{\partial t} &= -w' + v' - \frac{\partial K}{\partial x} - C_p \frac{\partial \pi'}{\partial x} + D(u'), \\
\frac{\partial v'}{\partial t} &= -w' + v' - \frac{\partial K}{\partial y} - C_p \frac{\partial \pi'}{\partial y} + D(v'), \\
\frac{\partial w}{\partial t} &= -\zeta + v' - \frac{\partial K}{\partial z} - C_p \frac{\partial \pi'}{\partial z} + D(w) + g (\frac{\theta'}{\bar{\theta}} + 0.61 Q_v - Q_c - Q_R - Q_I - Q_S - Q_G),
\end{align*}
\]

where vorticity components \( \zeta, \xi, \eta \) and kinetic energy \( K \) are given by:

\[
\begin{align*}
\zeta &= \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \\
\xi &= \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \\
\eta &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \\
K &= \frac{1}{2} (u'^2 + v'^2 + w'^2).
\end{align*}
\]

Here, \( u, v \) and \( w \) are velocity components in the \( x-, y-, z- \) directions, and \( u \) are given by

\[
\begin{align*}
u &= \bar{u} + u' - u^*, \\
v &= \bar{v} + v' - v^*.
\end{align*}
\]

The overbars represent the basic state which is assumed to be a function of \( z \) alone. Primed variables are the deviations from the basic state. \( u^* \) and \( v^* \) are the velocities of the coordinate system relative to the ground. In Eqs. (1) to (3), \( g \) is the gravity acceleration, \( C_p \) the specific heat of air at constant pressure, \( \pi \) the non-dimensional pressure, \( \theta \) the potential temperature and \( \theta_v \) the virtual potential temperature. \( Q_v, Q_c, Q_R, Q_I, Q_S, Q_G \) in the buoyancy term denote the mixing ratios of water vapor, cloud water, rain water, cloud ice, snow and graupel. \( \pi, \theta \) and \( \theta_v \) are given as the sum of the basic state and the deviation from it:

\[
\begin{align*}
\pi &= \bar{\pi} + \pi', \\
\theta &= \bar{\theta} + \theta', \\
\theta_v &= \bar{\theta}_v + \theta_v'.
\end{align*}
\]

\( D \) is the diffusion term and is based on the parameterization of subgrid turbulence used by Klemp and Wilhelmson (1976), in which sub-grid scale turbulent kinetic energy can be predicted.

The continuity equation is given as

\[
\frac{\partial}{\partial x} (\bar{\rho} u') + \frac{\partial}{\partial y} (\bar{\rho} v') + \frac{\partial}{\partial z} (\bar{\rho} w) = 0,
\]

where \( \bar{\rho} \) is the air density in the basic state.

The non-dimensional pressure is obtained by solving the Poisson equation:

\[
\begin{align*}
\frac{\partial}{\partial x} \left( C_p \bar{\rho} \frac{\partial \pi'}{\partial x} \right) + \frac{\partial}{\partial y} \left( C_p \bar{\rho} \frac{\partial \pi'}{\partial y} \right) + \frac{\partial}{\partial z} \left( C_p \bar{\rho} \frac{\partial \pi'}{\partial z} \right) &= \sigma, \\
\end{align*}
\]

where

\[
\begin{align*}
\sigma &= \frac{\partial}{\partial x} \left\{ \bar{\rho} [RHS \ of \ (1)] \right\} \\
&\quad + \frac{\partial}{\partial y} \left\{ \bar{\rho} [RHS \ of \ (2)] \right\} \\
&\quad + \frac{\partial}{\partial z} \left\{ \bar{\rho} [RHS \ of \ (3)] \right\}.
\end{align*}
\]

Here RHS denotes the right hand side of the numbered equations, omitting the pressure term.

2.2 Equations for water species and potential temperature

The model contains five classes of hydrometeors; cloud water, rain, cloud ice, snow and graupel. The shapes of all hydrometeors are assumed to be spherical, but the shape is not a critical factor in this
model. In this model snow includes both single snow crystals having enough fall velocity to start riming growth and aggregates of snow crystals. The size distributions of rain, snow, and graupel are assumed to be inverse exponential and those of cloud water and cloud ice are assumed to be mono-disperse. Cloud ice and snow are expressed by using two prognostic variables, the mixing ratio and number concentration, respectively. The parameters for cloud ice, snow and graupel are determined referring to the observational studies by Heymsfield (1978), Locatelli and Hobbs (1974), Kajikawa (1975, 1978), Yagi et al. (1979) and Harimaya (1978). Details of parameterization for hydrometeors are shown in Table 1.

The cloud microphysical processes simulated in the model are illustrated in Fig. 1. The parameterizations of microphysical processes are mainly based upon Lin et al. (1983) and Cotton et al. (1986). Here new developments and some modifications will be described. The prognostic equations of water species and potential temperature are

\[
\begin{align*}
\frac{\partial Q_V}{\partial t} &= ADV(Q_V) + D(Q_V) \\
&\quad -VD_{VR} - VD_{VI} - VD_{VS} \\
&\quad -VD_{VG} - NUA_{VI}, \quad (16)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial Q_C}{\partial t} &= ADV(Q_C) + D(Q_C) + PR_{C} \\
&\quad -NUF_{CI} - NUH_{CI} - CN_{CR} \\
&\quad -CL_{CR} - CL_{CS} - CL_{CG} + ML_{IC}, \quad (17)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial Q_R}{\partial t} &= ADV(Q_R) + D(Q_R) \\
&\quad +PR_{R} + VD_{VR} - CL_{RI} - CL_{RS} \\
&\quad -CL_{RG} + CL_{CR} + CN_{CR} + ML_{SR} \\
&\quad +ML_{GR} + SH_{SR} + SH_{GR} - FR_{RG}, \quad (18)
\end{align*}
\]
\[ \frac{\partial \theta'}{\partial t} = ADV(\theta) + D(\theta') + \frac{L_v}{C_p}\eta VDV_R + \frac{L_s}{C_p}\eta(VDV_L + VDV_S + VDV_G) + NUA_{AV} + NUFCI + NUHCI + FRG + ML_{SR} - SH_{SR} - ML_{GR} - SH_{GR}. \]  

In addition to the prognostic equations for the mixing ratios of water species, prognostic equations for the number concentrations of cloud ice and snow are also formulated as follows;

\[ \frac{\partial}{\partial t} \left( \frac{N_i}{\rho} \right) = ADV \left( \frac{N_i}{\rho} \right) + D \left( \frac{N_i}{\rho} \right) + PR_NI + \frac{1}{m_{10}} (NUA_{AV} + NUFCI + NUHCI) - \frac{N_i}{\rho Q_{IS}} (CL_{IR} + CL_{IS} + CL_{IG} + ML_{IC}) - AG_{NI} - \frac{1}{m_{so}} CN_{IS}. \]  

\[ \frac{\partial}{\partial t} \left( \frac{N_s}{\rho} \right) = ADV \left( \frac{N_s}{\rho} \right) + D \left( \frac{N_s}{\rho} \right) + PR_{NS} + \frac{1}{m_{so}} CN_{IS} - \frac{N_s}{\rho Q_{IS}} (CL_{SC} + CL_{SR}(1 - \alpha_{RS}) + CN_{SC} + ML_{SR}) - AG_{NS}. \]  

Here \( L_f, L_v, L_s \) are latent heats of fusion, vaporization and sublimation, respectively. The \( ADV \) terms in Eqs. (16) to (24) represent the advection of the water species and the \( D \) terms represent turbulent diffusion developed using the eddy viscosity parameterization developed by Klemp and Wilhelmson (1978). Following the terminology of Cotton et al. (1986), source and sink terms used in the above equations are defined as \( VD \) for vapor deposition or evaporation, \( NUA \) for deposition/sorption nucleation, \( NUF \) for condensation-freezing nucleation, \( NUH \) for homogeneous nucleation below -40°C, \( CL \) for collection, \( CN \) for conversion, \( ML \) for melting, \( FR \) for precipitation and \( AG \) for aggregation. \( PG \) represents graupel growth by accretion of other water species in either the dry or wet growth mode (Lin et al., 1983). Each term, except for \( PR, PG \) and \( AG \), includes a double subscript, where the first subscript is the water species being depleted and the second subscript is the water species which is growing. The subscripts \( V, C, R, I, S \) and \( G \) refer to vapor, cloud water, rain, cloud ice, snow and graupel. For the exchange between water vapor and cloud water, the instantaneous procedure used by Klemp and Wilhelmson (1978) was adopted. Cloud ice is assumed to melt into cloud water instantly above the freezing point. \( 1 - \alpha_{RS} \) is the ratio at which the collisions between raindrops and snow result in graupel production. The equation chosen to express \( \alpha_{RS} \) was;

\[ \alpha_{RS} = \frac{\rho_s^2 \left( \frac{4}{\lambda_s} \right)^6 + \rho_w^2 \left( \frac{4}{\lambda_w} \right)^6}{\rho_s^2 \left( \frac{4}{\lambda_s} \right)^6 + \rho_w^2 \left( \frac{4}{\lambda_w} \right)^6} = \frac{m_s^2}{m_s^2 + m_r^2}. \]  

(a) Ice Nucleation

In the model cloud, cloud ice is produced through depositional/sorptive nucleation, heterogeneous freezing of cloud droplets, and homogeneous freezing of cloud droplets below -40°C. The secondary ice crystal production term (Hallett and Mossop, 1974) was turned off in the present simulation. In the following paragraphs, the parameterizations for the first two processes are described in detail.

The temperature dependency of deposition/sorption nuclei is given by Fletcher’s (1962) empirical equation;

\[ N_I = N_{10} \exp(\beta_2 T_s). \]  

Likewise, the supersaturation dependency of ice nuclei is given by Huffmann and Vali (1973);

\[ N'_I = A \left( \frac{S_I - 1}{S_0 - 1} \right)^B. \]  

Replacing \( A \) with \( N_I \), we obtain

\[ N'_I = N_{10} \exp(\beta_2 T_s) \left( \frac{S_I - 1}{S_0 - 1} \right)^B, \]  

where \( N_{10} = 1.0 \times 10^{-2} \) (m^-3), \( \beta_2 = 0.6 \) (K^-1), \( B = 4.5 \). \( (S_0 - 1) \) represents the ice supersaturation of a water-saturated cloud. It may be reasonable to assume that ice nucleation by deposition/sorption occurs in an ascending air mass in clouds. Assuming that the vertical change in humidity is negligibly small, we arrived at the following equation for ice nucleation rate in ascending cloud air;

\[ NUA_{AV} = m_{10} \frac{\partial N_I}{\partial t} \approx m_{10} \left( \frac{\partial N'_I}{\partial T_s} \frac{\partial T_s}{\partial z} \right) \frac{dz}{dt} = m_{10} \beta_2 N_{10} \exp(\beta_2 T_s) \left( \frac{S_I - 1}{S_0 - 1} \right)^B \frac{\partial T_s}{\partial z}. \]

For heterogeneous freezing of cloud droplets, we obtain the following equation by extrapolating the Bigg’s (1953) equation down to the cloud droplet size;

\[ NUF_{CI} = B'[\exp \left( A'(T_0 - T) \right) - 1] \frac{\rho Q_{CI}^2}{\rho_w N_C}, \]
where we choose $A' = 0.66 \ (K^{-1})$, $B' = 100.0 \ (m^{-3} s^{-1})$. $\rho_w$ is the density of liquid water, $N_C$ the number concentration of cloud droplets which is preset in this model.

(b) Conversion from cloud ice to snow

The conversion from cloud ice (pristine ice crystals) to snow takes place through two processes; depositional growth of cloud ice and aggregation between pristine ice crystals. The time needed for an ice crystal to grow from $\bar{r}_I$ to $r_{S0}$ in radius is

$$\Delta t_1 = \frac{r_{S0}^2 - \bar{r}_I^2}{2a_1} \rho_I,$$

(31)

where $r_{S0}$ is the smallest radius of particles in the snow class. $a_1$ is given by

$$a_1 = (S_I - 1) \left( \frac{L^2}{\kappa_a R_W T^2} + \frac{1}{\bar{q}_{VS} D_V} \right)^{-1}.$$  

(32)

Here $R_w$ is the gas constant for water vapor, $\kappa_a$ the thermal conductivity of air, $q_{VS}$ the water vapor density for ice saturation and $D_V$ the diffusivity of water vapor in the air. The conversion rate from cloud ice to snow due to depositional growth is given by

$$CN_{IS}^D = \frac{Q_I}{\Delta t_1}.$$  

(33)

The rate of collision-coalescence among a homogeneous population of ice crystals may be written by

$$\frac{dN_I}{dt}\bigg|_{\text{aggr.}} = \frac{1}{2} K_I N_I^2,$$

(34)

where

$$K_I = \frac{\pi}{6} \bar{D}_I^2 U_I E_{II} X.$$

(35)

Here $\bar{D}_I$ represents the mean diameter of ice crystals, $U_I$ the fall velocity of ice crystals, $E_{II}$ the collection efficiency between ice crystals, and $X$ the dispersion of the fall velocity spectrum of ice crystals. Using the following equation for the fall velocity

$$U_I = a_1 \bar{D}_I \left( \frac{\bar{\rho}_0}{\bar{\rho}} \right)^{\frac{3}{2}},$$

(36)

Eq. (35) is rewritten as

$$K_I = \frac{c_1}{N_I},$$

(37)

where

$$c_1 = \frac{Q_I \rho_4 E_{II} X}{\rho_I} \left( \frac{\bar{\rho}_0}{\bar{\rho}} \right)^{\frac{3}{2}}.$$  

(38)

Combining Eq. (34) and Eq. (37), we obtain

$$\frac{dN_I}{dt}\bigg|_{\text{aggr.}} = -\frac{c_1}{2} N_I.$$  

(39)

The time needed for cloud ice to grow by aggregation from $\bar{r}_I$ to $r_{S0}$ in radius is equal to the time needed for the cloud ice concentration to decrease from $N_I$ to $N_I (\bar{r}_I / r_{S0})^3$. Assuming that $\rho_I$ is constant yields

$$\Delta t_2 = -\frac{2}{c_1} \log \left( \frac{\bar{r}_I}{r_{S0}} \right)^3.$$  

(40)

The conversion rate from cloud ice to snow by aggregation is given by

$$CN_{IS}^A = \frac{Q_I}{\Delta t_2}.$$  

(41)

The total conversion rate from cloud ice to snow is given by the sum of $CN_{IS}^D$ and $CN_{IS}^A$.

(c) Conversion from snow to graupel

Snow is converted into graupel through the collection of supercooled cloud droplets (riming process). This conversion is assumed to occur when $CL_{CS}$ is greater than $VD_{VS}$ and the air temperature is below 0°C. The conversion rate is given by the sum of $CL_{CS}$ and the portion of snow mixing ratio which is converted into graupel in unit time. Assuming that the change of snow particle size due to riming is negligibly small, the time needed for snow to be converted into graupel through the riming process is given by

$$\Delta t = \frac{(\rho_G - \rho_S) \pi}{4} \frac{D_G^3}{(D_S^2 - 6D_S^2 E_{CS} Q_C)}.$$  

(42)

Therefore the snow mixing ratio which is converted into graupel in unit time is

$$CN_{SG} = CL_{CS}$$

$$+ \int_0^\infty \frac{\rho_S \pi}{6} D_S^3 n_{S0} \exp(-\lambda_S D_S) \frac{\rho_G - \rho_S}{\rho_G - \rho_S} dD_S$$

$$= CL_{CS} + \frac{\rho_G}{\rho_G - \rho_S} CL_{CS}$$

$$= CL_{CS} \frac{\rho_G}{\rho_G - \rho_S}.$$  

(43)

(d) Aggregation

The decrease in number concentration of cloud ice due to aggregation among then is given by Eq. (39), while the decrease in number concentration of snow due to aggregation among snow crystals (or aggregates) is obtained using an equation based on the analytical model of aggregational growth by Passarelli (1978).
\[ AG_{NS} = \frac{dN_S}{dt} \bigg|_{\text{agg.}} = -\frac{cE_{SS}}{4 \times 720} \frac{1}{\frac{x}{\lambda_S^3} + \frac{1}{\rho \frac{x}{\lambda_S^3} + \frac{1}{\rho S} + \frac{x}{\lambda_S^3}}} \frac{Q_{S}}{N_{S}^{2}} \frac{q_{S}^{2}}{N_{S}^{2}}, \]

where

\[ I(d) = \int_{0}^{\pi} \int_{0}^{\pi} \frac{\pi}{4} (D_{R} + D_{S})^{2} E_{RS} |U_{DR} - U_{DS}| \rho w \frac{D_{R}^{3}}{6} \times n_{R0} \exp(-\lambda_{R} D_{S}) n_{S0} \exp(-\lambda_{S} D_{S}) dD_{R} dD_{S}. \]

\[ (e) \text{Collision between the separate precipitating hydrometeors} \]

For collision between rain and snow, the rate of change of the rain mixing ratio is

\[ CL_{RS} = \frac{1}{\rho \omega} \int_{0}^{\pi} \int_{0}^{\pi} \frac{\pi}{4} (D_{R} + D_{S})^{2} E_{RS} |U_{DR} - U_{DS}| \rho w \frac{D_{R}^{3}}{6} \times n_{R0} \exp(-\lambda_{R} D_{S}) n_{S0} \exp(-\lambda_{S} D_{S}) dD_{R} dD_{S}. \]

In most models so far, the following approximation is used for the differential velocity;

\[ |U_{DR} - U_{DS}| \approx |\bar{U}_{R} - \bar{U}_{S}|. \]

This approximation causes a pronounced underestimation in \( CL_{RS} \) when the value of \( \bar{U}_{R} \) is close to \( \bar{U}_{S} \). To remedy this underestimation, we used the following approximation;

\[ |U_{DR} - U_{DS}| \approx \sqrt{(\bar{U}_{R} - \bar{U}_{S})^{2} + \alpha \bar{U}_{R} \bar{U}_{S}}. \]

We chose \( \alpha = 0.04 \). The approximation expressed by Eq. \((48)\) yields

\[ CL_{RS} = \pi x \omega E_{RS} \left( \bar{U}_{R} - \bar{U}_{S} \right)^{2} + \alpha \bar{U}_{R} \bar{U}_{S} \rho \omega \frac{D_{R}^{3}}{6} n_{R0} n_{S0} \times \left( \frac{5}{\lambda_{R}^{3}} + \frac{2}{\lambda_{S}^{3} \lambda_{R}^{2}} + \frac{0.5}{\lambda_{R}^{4} \lambda_{S}^{3}} \right). \]

where the collection efficiency of snow for rain (or that of rain for snow), \( E_{RS} \), is assumed to be unity. The rate of change of snow mixing ratio is

\[ CL_{SR} = \pi x \omega E_{RS} \left( \bar{U}_{R} - \bar{U}_{S} \right)^{2} + \alpha \bar{U}_{R} \bar{U}_{S} \rho \omega \frac{D_{R}^{3}}{6} n_{R0} n_{S0} \times \left( \frac{5}{\lambda_{S}^{3}} + \frac{2}{\lambda_{S}^{3} \lambda_{R}^{2}} + \frac{0.5}{\lambda_{R}^{4} \lambda_{S}^{3}} \right). \]

A similar approximation is used in used in deriving the rates involving collisions between graupel and snow and between graupel and rain.

2.3 Numerical techniques

The equations were solved over a 32 km × 32 km × 14 km domain with a 1000 m grid interval in the \( x \)- and \( y \)-directions and variable grids (100 m to 500 m grid intervals) were used in the \( z \)-direction to express boundary layer processes. The grid finite difference forms are the same as those of Yoshizaki (1985). For the time integration, we used the leapfrog scheme with \( \Delta t = 5 \) s and Asselin’s filter with \( \alpha = 0.1 \).

2.4 Boundary conditions

At the top boundary, \( w \) and the vertical gradients of other variables are assumed to be zero. At the lower boundary, the \( x \)- and \( y \)-wind components relative to the surface, \( u \) and \( v \), the \( z \)-component, \( w \), and the vertical gradients of other variables except for \( u \), \( v \), \( w \), \( \theta \), and \( Q \) are assumed to be zero. The sub-grid momentum, heat and moisture fluxes at the surface are given as

\[ u'' w'' = -u_{*} \frac{\bar{u}_{1/2}}{v_{1/2}} \frac{\bar{u}_{1/2}}{v_{1/2}}, \]

\[ \theta'' w'' = -u_{*} \theta_{*} \bar{Q}_{V}' \frac{w'' - u_{*} Q_{V}''}{w''}. \]

where \( u_{1/2}, v_{1/2} \) and \( |v_{1/2}| \) are \( \bar{u}, \bar{v} \) and total velocity at the lowest level. \( u_{*}, \theta_{*} \) and \( Q_{V}'' \) were calculated using the empirical formulations given by Barker and Baxter (1975). The formulation is based on the observational results of the flux-profile relationships and the Monin-Obukhov similarity theory. For the lateral boundary, we used a cyclic boundary condition.

2.5 Initial conditions

Initial conditions for this model simulation are taken from the 1440 MDT sounding of temperature, humidity, and wind at Mile City, Montana on July 19, 1981 (see Fig. 1 of Dye et al. (1986). This sounding represents the atmosphere approximately 1.5 hours prior to the formation of the observed cloud and was taken approximately 35 km to the east of the observation area. This sounding showed weak wind shear above 800 mb level, moderate thermal instability of 1.5 to 2°C from 625 to 425 mb. Cloud tops were expected to reach 11 km (−50°C level) and the 0°C level was at around 4 km msl.

In order to initiate convection, we used a warm and moist bubble with a maximum temperature excess of 1.6°C and a humidity excess of 20 %. The bubble was 4 km wide in the \( x \)- and \( y \)-directions and was located in the lower 2.5 km of the domain.

3. Result of numerical simulation

3.1 General cloud appearance and dynamics

The general cloud appearance and evolution are shown in Fig. 2. The figures depict cloud and precipitation fields and wind fields in the \( x-z \) plane at
Fig. 2. Cloud and wind field evolutions at 5-min intervals. In the left hand side panels, the solid line indicates the cloud outline, "-" cloud ice (>10^{-6} kg kg^{-1}), "o" snow (>10^{-4} kg kg^{-1}), "Δ" graupel (10^{-4} kg kg^{-1}) and "." rain (>10^{-4} kg kg^{-1}). The panels in the right hand side show the wind field (uw component).
Fig. 2. (Continued)
5-min intervals. Each cross section is selected to pass near the region of the maximum water content. The wind is relative to the cloud system. The heights in the model simulation are with respect to the ground level while all references to heights for the observed cloud are with respect to mean sea level. The elevation of the study area is about 800 m. Therefore 800 m must be added to the model height for a direct comparison of heights between the observations and the model domain.

Prior to 15-min simulation time, the cloud consists of cloud water alone, although the cloud top temperature is well below −15°C. At 20-min simulation time, the cloud is in a rapid (and explosive) growing stage and has a considerable amount (>10−5 kg kg−1) of cloud ice in its upper part. By 25-min simulation time, precipitation formation processes become active through an ice phase path, and snow and graupel occupy the middle level of the cloud. Snow and graupel begin to fall out on the downshear side of the updraft.

At 30-min simulation time, the cloud top has developed to about 10.3 km msl and an anvil is forming on the downshear side of the cloud. Also the upshear lobe of the anvil is beginning to form. A whole layer of the cloud has become occupied by solid hydrometeors (cloud ice, snow, and graupel) and some of the graupel begins to fall below the melting level (4 km msl). The amount of rain formed through the melting of graupel is well below the plotting threshold (10−4 kg kg−1). Secondary clouds have also formed on the both sides of the precipitation shaft. By 35-min simulation time, the precipitation processes erode away the lower portion of the cloud water while the anvil cloud continues to form the cloud ice and snow. Graupel falls well below the melting level and a considerable amount (just below the plotting threshold) of rain is produced.

At 40-min simulation time, rain does not yet reach the ground because of intense evaporation in the lowest 1.5 km layer. Light rain does, however, reach the ground between the simulation times of 43 and 54-min.

In order to help visualize the spatial distribution of cloud and precipitation, the x–y cross sections of hydrometeors and relative wind field at 30-min are shown in Fig. 3. By this time, precipitation has damped the main updraft as seen in Fig. 2. The locations of cloud and precipitation particles are identical to each other at the upper level (8.25 km AGL) of the cloud while the precipitation area shifted towards the southeast of the cloud area at the middle and lower levels (6.25 km and 4.25 km AGL). The erosion of cloud water by precipitation started from the southeast part at the middle level.

As shown in Fig. 4, the maximum updraft in the cloud is about 5 m s−1 during the first 8-min of simulation time. Then it rapidly increases to about 28 m s−1 at 20 min. By 16-min simulation time, the cloud top rises at a rate of 4 to 6 m s−1 but during the 17 to 23-min interval it rises at a rate of 11 m s−1.

3.2 Microphysical properties

The change in the domain maximum mixing ratio of each hydrometeor as a function of time is shown in Fig. 5. At 19-min simulation time (14 minutes after initial condensation), a significant amount of cloud ice appears in the upper level of the cloud and is rapidly transformed to snow and then to graupel. The domain maximum cloud water has a peak value of 5.6 g kg−1 at 19 min (1 minute prior to the time of updraft peak) and rapidly decreases after 27 min due to the precipitation process. At 27 min, the domain maximum mixing ratio of snow has almost reached the peak value of 3 g kg−1 and maintains this value for more than 30 minutes. In contrast to snow, graupel reaches its domain maximum value of 1.8 g kg−1 at 26-min and decreases rapidly due to fallout and melting after 40-min.

Figure 6 shows the evolution of cloud ice and snow concentrations in the x−z cross sections which correspond to those in Fig. 2. Ice initiation started from 19-min simulation time and by 30 min the cloud ice and snow concentrations increase explosively to 4x106 and 1x106 particles/m3, respectively. The simulated concentrations seem to be reasonable in comparison with the observational results obtained so far (Pruppacher and Klett, 1978, Heggli et al., 1983, etc.), although such a high concentration was not measured by the King Air because the flight levels were lower than 6 km msl. High concentrations of cloud ice were found in the region where there were a strong updraft and a high concentration of cloud water. As the updraft faded out, cloud ice concentrations rapidly decreased and were accompanied by a gradual decrease in snow concentration.

Rain shows up at 27 min and has a peak value of 0.33 g kg−1 at 41-min simulation time. Rain is not formed by coalescence of cloud droplets but by the melting of graupel and snow. A relatively cold cloud base temperature (≈7°C) and a high concentration of cloud droplets prevented cloud droplets from growing to the threshold size (r=10 μm) (see Cotton et al., 1986).

(a) Ice initiation mechanism

Cloud ice production rates due to each individual process were integrated over the entire domain, and are shown in Fig. 7. A significant amount of cloud ice produced by the freezing of cloud droplets and the activation of deposition/sorption nuclei appears at 18 min. During the 18 to 29 min interval, the freezing of cloud droplets is much more efficient than the activation of deposition/sorption nuclei in forming cloud ice. At the early stage of ice initiation, both processes are active near 8 km msl (shown
Fig. 3. Cloud and wind field in $x - y$ cross sections at 30-min simulation time.
in Fig. 8). After 29 min, ice initiation is mainly due to deposition/sorption nucleation and occurs near the cloud top.

Homogeneous nucleation does not occur in the cloud although the cloud top temperature is well below $-40^\circ C$. This means that many cloud droplets freeze by heterogeneous nucleation and consequent depositional growth of cloud ice consumes the rest of cloud droplets before being carried up beyond the $-40^\circ C$ level.

(b) **Graupel formation**

The production rates related to graupel initiation are shown in Fig. 9. The graupel initiation starts at 19 min through the riming growth of snow ($CN_{ag}$). Graupel formation by the riming growth of snow lasts only 8 minutes. As shown in Fig. 10, the early graupel formation occurs at around 7.5 km msl. The collision between snow and rain occurs in the melting layer after 28 min but does not contribute to the graupel formation in this simulation because all liquid water collected by snow and graupel is assumed
to be reclassified as rain when temperatures are above 0°C. Graupel grows through the dry growth mode.

4. Comparison with observations

4.1 General feature

Some sort of timing reference seems necessary for direct comparison between the observations and the model results, since the observations are a function of both space and time. One of the common and remarkable features for both the observed and simulated clouds was their explosive growth. The simulated cloud exhibited the explosive growth phase similar to the observed cloud, so that a timing correspondence between the observations and the model results was established on the basis of the onset of this explosive growth phase. The observed cloud began rapid growth at around 1623 MDT while the simulated cloud began to grow explosively near 18-min simulation time.

As for general cloud features, quantitative comparisons between the observations and the model results are summarized in Table 2. The cloud top height, updraft velocity and $Z_{\text{max}}$ are strongly dependent upon the temperature and humidity excess which are given to initiate convection, while other
characteristics of the cloud and microphysical processes (timing and mechanisms of ice initiation and graupel initiation, etc.) are not so dependent on them.

The cloud base height of the simulated cloud is 1 km lower than that of the observed cloud. A large discrepancy like this was also reported by Helsdon and Farley (1987) using a 2-D model. This discrepancy in cloud base height can most likely be attributed to the lack of representativeness of the radiosonde sounding used for input data. The aircraft data taken by the King Air in the vicinity of the observed cloud showed that the moisture structure in the sub-cloud layer was highly variable and was, in general, drier than that observed at the time of the sounding (Dye et al., 1986). The rise rate of the
The cloud top height, liquid cloud size, maximum cloud water content, and the timing and position of the first radar echo are well simulated in our model. Helsdon and Farley (1987) may give a more realistic rise rate. This approach will be incorporated in a future work.

The simulated cloud top varied in time from 5 to 11 m s\(^{-1}\) while the observed cloud exhibited a little slower rise rate (5~7 m s\(^{-1}\)). This discrepancy seems to be related to the way in which we initiate convection in our model. The use of a combination of random perturbations and a main warm bubble as used in Fig. 9. The domain total of graupel production rates which are related to graupel initiation as a function of time.

Simulated cloud top varied in time from 5 to 11 m s\(^{-1}\) while the observed cloud exhibited a little slower rise rate (5~7 m s\(^{-1}\)). This discrepancy seems to be related to the way in which we initiate convection in our model. The use of a combination of random perturbations and a main warm bubble as used in Helsdon and Farley (1987) may give a more realistic rise rate. This approach will be incorporated in a future work.

The cloud top height, liquid cloud size, maximum cloud water content, and the timing and position of the first radar echo are well simulated in our model. Helsdon and Farley (1987) may give a more realistic rise rate. This approach will be incorporated in a future work.

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Table 2. Comparison between the general features of the actual and simulated clouds.

<table>
<thead>
<tr>
<th>General feature</th>
<th>Model</th>
<th>Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloud top height (km msl)</td>
<td>10.3</td>
<td>10.5</td>
</tr>
<tr>
<td>Cloud base height (km msl)</td>
<td>2.8</td>
<td>3.8</td>
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<tr>
<td>Cloud top rise rate (m s⁻¹)</td>
<td>5~11</td>
<td>5~7</td>
</tr>
<tr>
<td>Liquid cloud size (km)</td>
<td>~6</td>
<td>~6</td>
</tr>
<tr>
<td>Max. Qc (gm⁻³)</td>
<td>2.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Updraft (ms⁻¹)</td>
<td>15</td>
<td>28</td>
</tr>
<tr>
<td>Radar first echo (km msl)</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>(time MDT)</td>
<td>1625</td>
<td>1623</td>
</tr>
<tr>
<td>Max. radar reflectivity (dBZ)</td>
<td>45</td>
<td>55</td>
</tr>
</tbody>
</table>

The simulated maximum updraft velocity is much greater than the observed one. This large discrepancy may lie mainly in the fact that the airplane observations were limited in space and time and did not measure the maximum value in the cloud, although it may lie partly in the way convection was initiated in the model. The arrival of appreciable amounts of graupel particles at the flight level of the Queen Air (3 km msl) was around $35-1640$ (simulation time - MDT) and shows a good agreement with the observations.

4.2 Radar echo evolutions

Figure 11 shows a comparison of radar reflectivity profiles between the simulated and observed clouds at three comparable times. The radar reflectivity (dBZ) is calculated by

$$dBZ = 10 \log Z,$$

where

$$Z = \frac{1}{\Gamma(7)N_{R_0}} \left[ \frac{\rho Q_R}{\pi N_{R_0}} \right]^{1.75} 10^{18}$$

$$+ \frac{\gamma(7)}{\epsilon_i + 2} \left[ \frac{\epsilon_i - 1}{\epsilon_i + 2} \right] \left[ \frac{\epsilon_w - 1}{\epsilon_w + 2} \right]^{1.75} \frac{\rho S}{\rho W} N_{S_0} \left[ \frac{\rho Q_S}{\pi N_{S_0}} \right]^{1.75} 10^{18}$$

$$+ \frac{\gamma(7)}{\epsilon_i + 2} \left[ \frac{\epsilon_i - 1}{\epsilon_i + 2} \right] \left[ \frac{\epsilon_w - 1}{\epsilon_w + 2} \right]^{1.75} \frac{\rho C}{\rho W} N_{C_0} \left[ \frac{\rho Q_C}{\pi N_{C_0}} \right]^{1.75} 10^{18}$$

Here $\epsilon_i$ and $\epsilon_w$ are the dielectric constants of ice and water. There are some similarities between the simulated and actual RHI's, although the reflectivity values of the simulated cloud is a little smaller than that of the observed cloud.

At 21-1626, the strongest echoes of both the simulated and actual RHI's are located near 7 km msl. At 26-1631, the strongest echoes are located at the same altitude, and the horizontal echo sizes are also about the same. The appendage on the left side of the echo and the weak-echo region between this appendage and the main precipitation shaft are well simulated in this model. The weak-echo region is associated with the strong updraft in the model cloud.

At 36-1641 there are two dissimilarities between the simulated and actual RHI's. One is the anvil-like echo on the upshear side near the cloud top in the actual RHI's. In the simulation, a weak vortex is developed in this region and the clouds consisting of cloud water and cloud ice are formed. However, these clouds hardly contribute to the radar reflectivities. Typical anvil clouds are developed in the northern and western parts of the cloud top in the simulated RHI's.

Another dissimilarity is the strong echo near the ground in the actual RHI while the simulated RHI shows no significant echo near the surface. The radar measurement hardly showed any rapid decrease in radar reflectivity in spite of dry air (less than 50% R.H.) below the 3 km msl. In an actual atmosphere, relatively large raindrops and graupel survive a dry subcloud layer and reach the surface in spite of their low mixing ratios. Due to the prescribed size distributions in this model, however, rain and graupel of low mixing ratios always consisted of small particles. Smaller raindrops and graupel have lower fall velocity and evaporate more rapidly. Therefore the prescribed size distribution may accelerate the evaporation of rain and graupel unrealistically. This problem should be solved by predicting the size distribution of rain and graupel using two prognostic variables in a future work.

4.3 Microphysics

Analyzing the timing, location and mechanisms of ice initiation is very important in understanding the precipitation formation processes. The simulation indicated that cloud ice is initiated through the freezing of supercooled droplets and the activation of deposition/sorption nuclei. However it is not clear from the observations which mechanisms are responsible for ice initiation in the actual cloud.

In the simulated cloud, the graupel is initiated through the riming of snow, which is in correspondence with the fact that no rain drops were observed above the 0°C level in the cloud. Furthermore, the fact that all graupel grew in the dry growth mode in the simulated cloud corresponds well with the fact that no hail was found in the observed cloud (Dye et al., 1986).

Dye et al. (1986) described the spatial change in dynamical and microphysical properties for the fifth pass of the King Air. This pass penetrated...
the maximum reflectivity region of the cloud between 1629 and 1630 at an altitude of 5.4 km msl. In Fig. 12(A) this pass is superimposed on the PPI taken at 1629: 13 at a 9.3° elevation angle (~5.5 km m.s.l.). To simulate the fifth pass of the King Air, the updraft and microphysical data along the simulation pass are plotted in Fig. 12(B). The upper panel in Fig. 12(A) shows the measured vertical velocity and liquid water content, while in the middle panel, the particle number concentrations from the 2D-C probe and the 2D-P probe are plotted. The lower panel is the graph of the potential temperature and equivalent potential temperature. The two vertical dash-dot lines indicate the approximate updraft boundaries. These data were extracted from Fig. 5 of Dye et al. (1986).

The maximum updraft along the simulated pass is 12 m s\(^{-1}\) and almost equal to that experienced by the King Air during the penetrating pass. In the actual cloud, the areas of updraft and cloud water were located in the central and left portions of the cloud, and the strong precipitation region was located around the right boundary of updraft and on the downshear side of updraft. In the strong updraft region, the concentration of precipitation particles was very low. These features are well simulated in our model except for a little higher concentration of snow near the left boundary of the updraft.

The 2D-C and 2D-P probes measured hydrometeors (pristine ice crystals, snow crystals, and graupels) from 25 \(\mu\)m to 800 \(\mu\)m and from 0.2 to 6.4 mm, respectively. In the model, the number concentrations of cloud ice and snow are predicted whereas the number concentration of graupel is diagnosed.
Fig. 12. Dynamical and microphysical properties along the actual King Air’s pass (A) and the simulated pass (B) at the equivalent time. Liquid water content, LWC, and vertical air velocity, VW (upper left); ice particle concentration measured by 2D-C, TWODC and 2D-P, TWODP (middle left); temperature, TRF and equivalent potential temperature, THEATAE (lower left); Cloud water content and vertical air velocity (upper right); number concentrations of cloud ice and snow (middle right); water content of cloud ice, snow and graupel (lower right). The PPI taken at 1629:13 at 9.3° elevation angle (~5.5 km m.s.l.) and the PPI of the simulated cloud at the equivalent time and height are shown at the bottom. The time periods (left) and the positions (right) for ice particle spectra in Fig. 13 are indicated.

using prognostic variable, \( Q_G \). The simulated concentration of snow is much greater than that of graupel. Therefore the number concentrations measured by the 2D-C and 2D-P probes roughly correspond to those of cloud ice and snow in the model. The simulated particle concentration on the downshear side of the strong updraft is about 10 particles/l and shows a good agreement with the observations.

The general patterns of spatial and temporal changes in the size distribution of precipitation particles were fairly well reproduced in the model. The model simulates the sorting of precipitation particles especially well. That is, larger precipitation particles (with higher fall velocity) fall out near the updraft boundary and smaller ones (with lower fall velocity) fall out in the downdraft region (see Fig. 13). The simulated size distributions, however, are steeper (narrower) than the observed ones because the simulated size distribution does not include graupel, which would broaden the size distribution, while the measured size distribution included both snow and graupel.

To make a strict comparison between the observations and the model results, the size distribution of graupel should also be expressed explicitly. This will be incorporated in a future work.

5. Summary and conclusion

A 3-dimensional, anelastic model with incorporating a double-variable for two hydrometeors (cloud
ice, $Q_C$ and $N_C$; snow, $Q_S$ and $N_S$) was developed to accurately simulate ice initiation and development in clouds and was applied to the July 19, 1981 CCOPE case study cloud. This cloud is suitable for checking the model performance because its dynamic and microphysical properties were intensively measured using four instrumented aircraft with the help of ground based radar (Dye et al., 1986). With a timing reference established, the model results were compared with the observations as a function of time and space.

The general features of the cloud (such as cloud top height, cloud size, timing and location of first echo, arrival time of precipitation at the cloud base level, etc.) seem to have been well reproduced. However, the simulated cloud base was 1 km lower than the observed one, the cloud top rise rate was a little higher than the observed one and, in contrast to observations, there was no significant simulated radar echo near the ground. The discrepancy in the cloud base height seems to be attributable to a lack of representativeness in the radiosonde data used for initial conditions. The higher rise rate of the simulated cloud top may be attributed to the way in which convection is initiated in the model. The rapid decrease in radar echo near the ground appears to be attributable to the too-rapid evaporation of rain and graupel in the simulated dry sub-cloud layer. This rapid evaporation seems to have resulted from the prescribed size distributions for rain and graupel.

In the model cloud, cloud ice is initiated mainly through the freezing of supercooled cloud droplets and secondarily through deposition/sorption nucleation. The number concentrations of cloud ice and snow are well simulated without any secondary ice crystal production. Graupel is initiated through the riming of snow and grows in the dry growth mode. These features of graupel formation are not inconsistent with actual observations.

The model cloud well simulated the location of hydrometeors with respect to the updraft/downdraft structure, the number concentration of hydrometeors, updraft velocity and cloud water content along the King Air’s penetrating pass. A sorting of large and small precipitation particles is well simulated, although their size distributions are narrower for the simulated cloud than for the observed one. This discrepancy may be attributed to graupel, whose size distribution was not predicted in the present model.

A good correspondence between the simulated...
and observed clouds in the timing and location of first echo, arrival time of precipitation (graupel) at the cloud base and the number concentrations of cloud ice and snow shows that this model properly simulated ice initiation and consequent precipitation growth (except for the size distribution of graupel).

In a future work, various ways to initiate convection (including a combination of random perturbation and main warm bubble used by Helsdon and Parley, 1987) will be considered in an effort to eliminate the too-rapid rise rate of the cloud top. Furthermore, to obtain a more accurate correlation between the simulated and observed clouds, rain and graupel will also be represented using two prognostic variables—namely water mixing ratios and number concentrations—as cloud ice and snow were represented with in the present model.

Acknowledgements

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Appendix

In this list, L-<i>n</i>n, CT-<i>n</i>n, CS-<i>n</i>n indicate that these term are given by the formula nn in Lin et al. (1983), Cotton et al. (1986), and Cotton et al. (1982).

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
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</thead>
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<td>a_C</td>
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<td>A’</td>
<td>constant in Bigg’s equation</td>
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<td>diameter of snow</td>
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<td>$m$</td>
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<td>$m$</td>
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<td>mean diameter of cloud ice</td>
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<td>diffusivity of water vapor in the air</td>
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<td>$m^2 s^{-1}$</td>
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\text{constant in empirical formula for } U_{DG} \quad m^{-1} f s^{-1}

E_{CG} \quad \text{collection efficiency of graupel for cloud water} \quad \frac{Stk^2}{(Stk + 0.5)^2}

E_{CR} \quad \text{collection efficiency of rain for cloud water} \quad \frac{Stk^2}{(Stk + 0.5)^2}

E_{CS} \quad \text{collection efficiency of snow for cloud water} \quad \frac{Stk^2}{(Stk + 0.4)^2}

E_{IG} \quad \text{collection efficiency of graupel for cloud ice} \quad 0.1

E_{II} \quad \text{collection efficiency among cloud ice} \quad 0.1

E_{IS} \quad \text{collection efficiency of rain for cloud ice} \quad 1.0

E_{RG} \quad \text{collection efficiency of graupel for rain} \quad 1.0

E_{RS} \quad \text{collection efficiency of snow for rain} \quad 1.0

E_{SG} \quad \text{collection efficiency of graupel for snow} \quad 0.001

E_{SS} \quad \text{collection efficiency among snow} \quad 0.1

f \quad \text{constant in empirical formula for } U_{DG} \quad 0.64

F_{RRG} \quad \text{probabilistic freezing of rain (L-45)} \quad s^{-1}

g \quad \text{gravitational acceleration} \quad 9.8 \quad m s^{-2}

L_{f} \quad \text{latent heat of fusion} \quad 3.34 \times 10^5 \quad J kg^{-1}

L_{s} \quad \text{latent heat of sublimation} \quad 2.83 \times 10^6 \quad J kg^{-1}

L_{v} \quad \text{latent heat of evaporation} \quad 2.5 \times 10^6 \quad J kg^{-1}

m_{IO} \quad \text{mass of the smallest cloud ice} \quad 1 \times 10^{-12} \quad kg

m_{R} \quad \text{mean mass of rain} \quad kg

m_{S} \quad \text{mean mass of snow} \quad kg

m_{SO} \quad \text{mass of the smallest snow} \quad 2.6 \times 10^{-10} \quad kg

M_{LSR} \quad \text{melting of snow to form rain (L-32)} \quad s^{-1}

M_{LGR} \quad \text{melting of graupel to form rain (similar to } M_{LSR}) \quad s^{-1}

N_{C} \quad \text{number concentration of cloud water} \quad 1.0 \times 10_{6} \quad m^{-3}

N_{GO} \quad \text{parameter of graupel size distribution} \quad 1.1 \times 10_{6} \quad m^{-4}

N_{I} \quad \text{number concentration of cloud ice} \quad 1.0 \times 10_{-2} \quad m^{-3}

N_{RO} \quad \text{parameter of size distribution} \quad 8.0 \times 10^{6} \quad m^{-4}

N_{S} \quad \text{number concentration of snow} \quad 8.0 \times 10^{6} \quad m^{-4}

N_{SO} \quad \text{parameter of snow size distribution} \quad 8.0 \times 10^{6} \quad m^{-4}

N_{UAVI} \quad \text{depositional/sorption nucleation to form cloud ice (29)} \quad s^{-1}

N_{UCI} \quad \text{freezing of cloud water to form cloud ice (30)} \quad s^{-1}

N_{UHCI} \quad \text{spontaneous freezing of cloud water to form cloud ice} \quad s^{-1}

P_{GDGR} \quad \text{dry growth of graupel (L-39)} \quad s^{-1}

P_{GWET} \quad \text{wet growth of graupel (L-43)} \quad s^{-1}

Q_{VS} \quad \text{saturation mixing ratio for water vapor with respect to water} \quad s^{-1}

Q_{VSI} \quad \text{saturation mixing ratio for water vapor with respect to ice} \quad s^{-1}

\bar{r}_{I} \quad \text{mean radius of cloud ice} \quad m

r_{SO} \quad \text{radius of the smallest snow} \quad 1.0 \times 10^{-4} \quad m

R_{w} \quad \text{gas constant for water vapor} \quad 461.5 \quad J kg^{-1} K^{-1}

S_{HSR} \quad \text{water shedding from snow to form rain (CT-40)} \quad s^{-1}

S_{HGR} \quad \text{water shedding from graupel to form rain (L-44)} \quad s^{-1}

S_{I} \quad \text{saturation ratio over ice} \quad s^{-1}

S_{O} \quad \text{saturation ratio} \quad s^{-1}

Stk \quad \text{Stokes number for mass-weighted mean sizes of cloud water and precipitation particles} \quad s^{-1}

T_{S} \quad \text{supercooled temperature} \quad K

U_{DC} \quad \text{terminal velocity of cloud water of radius } D_{C} \quad ms^{-1}

U_{DG} \quad \text{terminal velocity of graupel of radius } D_{G} \quad ms^{-1}

U_{DI} \quad \text{terminal velocity of cloud ice of radius } D_{I} \quad ms^{-1}

U_{DR} \quad \text{terminal velocity of rain of radius } D_{R} \quad ms^{-1}

U_{DS} \quad \text{terminal velocity of snow of radius } D_{S} \quad ms^{-1}

U_{C} \quad \text{mean terminal velocity of cloud water} \quad ms^{-1}

U_{G} \quad \text{mass-weighted mean terminal velocity of graupel} \quad ms^{-1}

U_{I} \quad \text{mean terminal velocity of cloud ice} \quad ms^{-1}

U_{R} \quad \text{mass-weighted mean terminal velocity of rain} \quad ms^{-1}

U_{S} \quad \text{mass-weighted mean terminal velocity of snow} \quad ms^{-1}

V_{DG} \quad \text{depositional growth of graupel (similar to } V_{DVS}) \quad s^{-1}
$VDV_I$ deposition growth of cloud ice
$VDV_R$ evaporation of rain
$VDV_S$ deposition growth of snow (L-31)
$X$ dispersion of the fall velocity spectrum of cloud ice 0.25
$\alpha$ parameter in the approximate equation for differential fall velocity 0.04
$\alpha_{RS}$ switching parameter for CLRS and CLSR
$\beta_2$ parameter in Fletcher’s equation 0.6 $K^{-1}$
$\kappa_a$ thermal conductivity of air $2.4 \times 10^{-2}$ $Jm^{-1}s^{-1}K$ 
$\pi$ non-dimensional pressure
$\lambda_G$ slope parameter in graupel size distribution $m^{-1}$
$\lambda_R$ slope parameter in rain size distribution $m^{-1}$
$\lambda_S$ slope parameter in snow size distribution $m^{-1}$
$\bar{\rho}$ air density of the basic state $kgm^{-3}$
$\bar{\rho}_0$ air density of the basic state at $z=0$ m $kgm^{-3}$
$\rho_G$ density of graupel $2.0 \times 10^{2}$ $kgm^{-3}$
$\rho_I$ density of cloud ice $5.0 \times 10^{2}$ $kgm^{-3}$
$\rho_S$ density of snow $8.4 \times 10^{1}$ $kgm^{-3}$
$\rho_W$ density of water $1.0 \times 10^{3}$ $kgm^{-3}$

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孤立した対流雲の力学的・微物理学的変化の数値モデリング——1981年7月19日CCOPE期間中に観測された雲について——
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三次元、非弾性モデルをCooperative Convective Precipitation Experiment (CCOPE)で1981年7月19日に観測された雲に適用した。モデルはパルクの微物理パラメタリゼーション手法を用いているが、このパラメタリゼーションでは子孫変数として6種類の水物質（水蒸気・雲水・雲水・雨・雪・霰）の混合比の他に、雲氷と雪の数濃度を用いている。今回のモデルでは、雲氷は一次水晶生成過程（昇華吸着核の活性化・雲粒の不均質凍結・雲粒の均質凍結）によって生成された。モデルと観測した雲の雲頂上昇速度の顕著な変化にもとづいて時間を照合し、モデルの結果を時間的空間的に観測結果と比較した。

雲の全般的な特徴（例えば、雲頂高度・雲の大きさ・降水の雲底到達時刻・初期のレーダーエコーなど）は良く再現されているようである。更に、モデルではKing Airが測定した上昇流・下降流に相当した雲粒子・降水粒子の位置・降水粒子の数濃度・上昇流の大きさ・雲水量を良く再現した。

正確に再現されなかった主なものは、雲底高度、雲頂の上昇速度それと地上付近のレーダーエコーであった。モデルで求まった雲底高度は低すぎたが、これは人力データとして用いた時間的にも最も新しいレーダーエンゾンデータが代表性を欠いていたことに起因する。一方、速すぎる雲頂上昇速度は対流の開始方法に起因しているようである。また、シミュレーションした雲の反射強度の急な緩和は、雨と霰の不十分なパラメタリゼーションに起因しているようである。