NOTES AND CORRESPONDENCE

Parameterization of the Accretion Process between Different Precipitation Elements

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1. Introduction

The accretion process is one of the most important precipitation formation processes. Particle-particle accretions may transfer large amounts of water mass between different classes of cloud and precipitation elements.

Several researchers reported observational results describing the accretion between different precipitation elements. Magono and Oguchi (1955) presented photographs of aggregates produced by graupel-stellar collisions. Kikuchi (1972) and Kikuchi and Uyeda (1979) observed snow crystals with small raindrops in the Showa Station in Antarctica and Inuvik, N.W.T., Canada, and reported the size distribution of small raindrops attached to snow crystals. In addition, Sakurai and Ohtake (1981) observed small raindrops frozen onto the surface of dendritic or plane snow crystals. They showed that the collision process between supercooled drops and snow crystals would take place in cold clouds.

The mathematical formulation for particle-particle accretion is still incomplete. Wisner *et al.* (1972) first introduced the accretion rate in a numerical model for hail. They derived the accretion rate using an arbitrary assumption that precipitation particles fall at mass-weighted mean terminal velocities. The effect of the velocity spectrum of particles on the accretion rate was ignored because of the mathematical difficulty in solving a double integral in the accretion equation. Subsequently, Passarelli and Srivastava (1979) and Sasyo and Matsuo (1985) showed the importance of the particle velocity spectra for aggregation. Although the shortcoming was recognized, a number of researchers (*e.g.*, Orville and Kopp, 1977; Cotton *et al.*, 1982; Lin *et al.*, 1983; Lord *et al.*, 1984; Yoshizaki, 1986; Cotton *et al.*, 1986; Ikawa *et al.*, 1987) have used the ac-

cretion rate by Wisner *et al.* (1972) in various cloud models. Recently Murakami (1990) has used another parameterization for accretion rate. However, the accuracy of his parameterization was not evaluated. According to Lin *et al.* (1983), the accretional growth of hail is the biggest source term. At the present time there is no strict parameterization for the accretion process.

The purpose of this study is to present the strict accretion rate by solving the accretion equation with a double integration and to propose a parameterization for the accretion process.

2. Accretion equations

Now we consider the accretion between different precipitation elements as is shown in Fig. 1. Two types of precipitation particles are indicated by open and solid circles. Here, two assumptions are adopted regarding the size distribution and the terminal velocity of precipitating particles.

The size distribution is as follows,

$$N_P(D_P) = N_{OP} \exp(-\lambda_P D_P), \quad (1)$$

where D_P is the diameter, $N_P(D_P)$ the particle concentration of diameter D_P per unit volume per unit size interval, N_{OP} the intercept parameter, λ_P the slope parameter, and the subscript P denotes precipitation type. The inverse exponential form in Eq. (1) is assumed according to many measurements of particle-size distributions (*e.g.*, Marshall and Palmer (1948) for raindrops, Imai *et al.* (1955), Gunn and Marshall (1958), Passarelli (1978), Harimaya (1978), Houze *et al.* (1979) and Yagi *et al.* (1979) for snow, and Federer and Waldvogel (1975), Harimaya (1978), Kajikawa and Kiba (1978), and Yagi *et al.* (1979) for graupel). The mixing ratio Q_P can be obtained from Eq. (1) by multiplying it by particle mass and integrating over all diameters. Using Q_P , λ_P can be expressed as

Fig. 1. Accretion between precipitation elements. The letters S and R distinguish between two types of precipitation particles.

$$\lambda_P = \left(\frac{\pi \rho_P N_{OP}}{\rho Q_P} \right)^{1/4}, \quad (2)$$

where ρ_P is density of precipitating particle and ρ is air density.

The terminal velocity of precipitating particles is represented as follows, according to the equation of motion where the precipitation particle's weight is balanced by aerodynamic drag,

$$V_P(D_P) = a_P \left(\frac{\rho_o}{\rho} \right)^{1/2} D_P^{1/2}, \quad (3)$$

where

$$a_P = \left(\frac{4g\rho_P}{3C_{DP}\rho_o} \right)^{1/2}, \quad (4)$$

C_{DP} is the drag coefficient of particles, ρ_o is surface air density, and g is gravitational acceleration. C_{DR} is for raindrops and depends on its diameter (Pruppacher and Klett, 1978). C_{DS} is for snow particles and is assumed to be 1.2 (Matsuo and Sasyo, 1981). C_{DG} is for graupel and is assumed to be 0.6 as used in hail cloud models (Lin *et al.*, 1983).

Next we consider the accretion rate between precipitation types S and R (see Fig. 1). The accretion rate for total mass P_{SACR} is calculated using a sweep-out concept as follows,

$$\begin{aligned} P_{SACR} &= \int_0^\infty \int_0^\infty E_{SR} \frac{\pi}{4} (D_R + D_S)^2 |V_R(D_R) - V_S(D_S)| \\ &\quad \times N_R(D_R) \frac{\pi}{6} D_R^3 \rho_R N_S(D_S) dD_R dD_S, \end{aligned} \quad (5)$$

where E_{SR} is the collection efficiency of particle-S for particle-R. P_{SACR} is the total mass increase rate of the S particles due to the accretion of the R particles.

Before going into the calculation of the accretion rate, the accretion rate P_{SACR}' obtained by Wisner *et al.* (1972) should be referred to. They made the following assumption for simplicity,

$$|V_R(D_R) - V_S(D_S)| = |\bar{V}_R - \bar{V}_S|. \quad (6)$$

This means that R and S particles fall at their mass-weighted mean terminal velocities, $\bar{V}_R$ and $\bar{V}_S$, respectively. Substituting Eq. (6) into Eq. (5) they obtained the following accretion rate,

$$\begin{aligned} P_{SACR}' &= \pi^2 \rho_R E_{SR} N_{OR} N_{OS} |\bar{V}_R - \bar{V}_S| \\ &\quad \times \left(\frac{5}{\lambda_R^6 \lambda_S} + \frac{2}{\lambda_R^5 \lambda_S^2} + \frac{0.5}{\lambda_R^4 \lambda_S^3} \right). \end{aligned} \quad (7)$$

This shows that P_{SACR}' approaches zero as $\bar{V}_R$ becomes close to $\bar{V}_S$. In fact, particles of S and R have a broad velocity spectra, as is shown by Eq. (3), and the actual rate must have some finite value even in the case of $\bar{V}_R \simeq \bar{V}_S$. It is easily understood that the rate P_{SACR}' has a shortcoming in the case of $\bar{V}_R \simeq \bar{V}_S$.

The exact mass accretion rate can be obtained as follows, with the double integration in Eq. (5) strictly integrated,

$$\begin{aligned} P_{SACR} &= \frac{\pi^2}{24} E_{SR} N_{OR} N_{OS} \rho_R \left(\frac{\rho_o}{\rho} \right)^{1/2} \\ &\quad \times \sum_{i=1}^3 C_i \left[\frac{a_R \Gamma \left(7 - i + \frac{1}{2} \right) \Gamma(i)}{\lambda_R^{7-i+\frac{1}{2}} \lambda_S^i} \right. \\ &\quad \left. - \frac{a_S \Gamma(7-i) \Gamma \left(i + \frac{1}{2} \right)}{\lambda_R^{7-i} \lambda_S^{i+\frac{1}{2}}} \right. \\ &\quad \left. + 2 \left(\frac{\alpha}{\alpha+1} \right)^{7.5} \Gamma(7.5) \left\{ \frac{a_S F \left(1, 7.5; 8-i; \frac{\alpha}{\alpha+1} \right)}{(7-i) \alpha^{i+\frac{1}{2}} \lambda_R^{7-i} \lambda_S^{i+\frac{1}{2}}} \right. \right. \\ &\quad \left. \left. - \frac{a_R F \left(1, 7.5; 8-i+\frac{1}{2}; \frac{\alpha}{\alpha+1} \right)}{\left(7-i+\frac{1}{2} \right) \alpha^i \lambda_R^{7-i+\frac{1}{2}} \lambda_S^i} \right\} \right], \end{aligned} \quad (8)$$

where F represents Gauss' hypergeometric function, $C_1 = C_3 = 1$, $C_2 = 2$ and $\alpha = (a_S/a_R)^2 \lambda_R/\lambda_S$. This presumes, of course, that E_{SR} is constant for all types of S particle-R particle interactions as well as Wisner *et al.* (1972).

Here, we make a comparison of the strict accretion rate with the rate by Wisner *et al.* (1972). Figure 2 shows the ratio of Wisner's accretion rate P_{SACR}' to the strict rate P_{SACR} as a function of the raindrop mixing ratio Q_R . Some discrepancy between the two rates appears around a mixing ratio of raindrops $Q_R \simeq 10^{-6} (\text{kg kg}^{-1})$ where Wisner's value is discontinuous. A dip is created because P_{SACR}' approaches zero around this value. This originates from the fact that around the dip the mass-weighted mean terminal velocity of raindrops is almost equal to that of the snow particles. This result clearly shows that Wisner's accretion rate has a shortcoming, especially in the case of $\bar{V}_R \simeq \bar{V}_S$.

Fig. 2. The ratio of the total mass accretion rate by Wisner *et al.* (1972) and the precise parameterization proposed here, $PSACR'$, to that by the exact one of Eq. (8), $PSACR$, as a function of mixing ratio of raindrops. The conditions used here are as follows: $Q_S = 10^{-3}(\text{kg kg}^{-1})$, $N_{OS} = 3 \times 10^6(\text{m}^{-4})$, $\rho_S = 50(\text{kg m}^{-3})$, $N_{OR} = 8 \times 10^6(\text{m}^{-4})$, $\rho_R = 1000(\text{kg m}^{-3})$, $E_{SR} = 1.0$, $p = 800 \text{ mb}$, and $T = 0^\circ\text{C}$.

3. Parameterization

In the previous section the strict accretion rates between different precipitation elements were derived, and it was shown that the accretion rate by Wisner *et al.* (1972) has a shortcoming when $\bar{V}_R \simeq \bar{V}_S$. However, the strict accretion rates are so complicated to utilize in cloud models that they should be simplified. Here we will parameterize the accretion process between different precipitation elements.

A better parameterization for accretion rate will be proposed here. As the terminal velocity difference between different particles influences the accretion rate, the terminal velocity difference should be evaluated accurately. It can be considered as follows:

$$|V_R(D_R) - V_S(D_S)| = \sqrt{(\alpha\bar{V}_R - \beta\bar{V}_S)^2 + \gamma\bar{V}_R\bar{V}_S}, \quad (9)$$

where $\alpha = 1.2$, $\beta = 0.95$ and $\gamma = 0.08$. In Murakami (1990) $\alpha = 1.0$, $\beta = 1.0$ and $\gamma = 0.04$ were adopted. It is to be noted that Eq. (9) does not produce any zero value for any mixing ratios of particle- R and particle- S , in contrast with Eq. (7) by Wisner *et al.* (1972). Substituting Eq. (9) to Eq. (5), we obtain the parameterization of the accretion rate for total mass P'_{SACR} as follows,

$$P_{SACR}' = \pi^2 E_{SR} \sqrt{(\alpha\bar{V}_R - \beta\bar{V}_S)^2 + \gamma\bar{V}_R\bar{V}_S} \times \rho_R N_{OR} N_{OS} \left(\frac{5}{\lambda_R^6 \lambda_S} + \frac{2}{\lambda_R^5 \lambda_S^2} + \frac{0.5}{\lambda_R^4 \lambda_S^3} \right). \quad (10)$$

We now compare the accretion rate for total mass obtained by Eq. (10) with that by the strict accretion rate of Eq. (8). Figure 2 shows the ratio of

the accretion rate by Eq. (10) to that by Eq. (8) as a function of raindrop mixing ratio Q_R . The ratio shows almost unity over the wide range of raindrop mixing ratio. This shows that the parameterization of Eq. (10) is a more precise expression than that by Wisner *et al.* (1972).

4. Summary and conclusions

The accretion process between different precipitation elements and the aggregation process play an important role in precipitation formation. These produce a large mass transfer between precipitation particles and a broadening of the size distribution. In cloud models, so far, the parameterization of the accretion rate obtained by Wisner *et al.* (1972) has been used without any detailed evaluation. Here, the exact accretion rate was obtained by solving analytically an accretion equation without any arbitrary assumption. It was shown that the accretion rate by Wisner *et al.* (1972) had a shortcoming especially in the case where two different particles become close in mass-weighted mean fall velocity. As an improvement, a more accurate and simpler parameterization for the accretion rate was proposed based on the exact accretion rate. It was demonstrated that this parameterization was valid for various conditions of the accretion process.

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異降水粒子間の衝突併合による成長速度式のパラメタリゼーション

水野 量

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