NOTES AND CORRESPONDENCE

On the Depth of the Surface Layer in the Turbulent Ekman Layer

By Nobuyuki Kinoshita and Hiroshi Niino

*Meteorological Research Institute, 1-1 Nagamine, Tsukuba, Ibaraki 305, Japan**(Manuscript received 21 August 1990, in revised form 9 October 1990)*

1. Introduction

The surface layer is defined as that part of the boundary layer where the turbulent fluxes vary by less than 10 percent of their magnitude with height (Stull, 1988). By assuming that the surface layer is a "constant-flux layer" to a first-order approximation, one can derive various formulae which are useful for both field observations and numerical models. Therefore, it seems to be of practical importance to know the depth of the surface layer.

Haugen *et al.* (1972) showed that turbulent fluxes at $z=22.6$ m and those at $z=5.6$ m can be regarded as equal within the accuracy of their field measurements, where z is the height above the ground. However, little is known concerning how the depth of the surface layer responds to external parameters such as roughness of the ground, wind speed of the free atmosphere and so on. It is a vaguely statement that the depth of the surface layer is the bottom 10 percent of the boundary layer (Stull, 1988) or 30–70 meters (Takeuchi and Kondo, 1971).

In this note, we study the depth of the surface layer in a turbulent Ekman boundary layer in a neutral atmosphere by means of a second-order turbulence closure model. The model has enough resolution in the surface layer that the dependence of its depth on external parameters can be studied. The results of the model are discussed in the light of a simple theoretical consideration, and their extension to the case of a stably- or unstably-stratified atmosphere are suggested.

2. Model

Consider a turbulent Ekman boundary layer over a rough surface in a neutral barotropic atmosphere, where the geostrophic wind G is constant with height. If the surface is assumed to have a constant roughness length z_o everywhere, the structure of the turbulent Ekman layer does not vary in the horizon-

tal direction and depends only on the height from the surface. Thus, its structure may be studied by means of a one-dimensional model.

A detailed description of the present model is given by Kinoshita (1990). Here, only the key features of the model are described. The model is a one-dimensional version of the Level 4 second-order closure model incorporated with an equation for the master length scale (Mellor and Yamada, 1982). All the constants used in the model are adopted from Mellor and Yamada (1982).

In most of the previous models for the atmospheric boundary layer, the lower boundary conditions have been given at an appropriate height in the surface layer. Since the height of the surface layer is not known *a priori*, however, we prefer to give the boundary conditions at $z=z_o$. This is possible because the present model predicts a constant-flux layer right down to $z=z_o$ (see next section), though the flow field for $z_o < z < (\text{say}) 100 z_o$ is not physically meaningful. The lower boundary conditions at $z=z_o$ are such that mean wind vanishes and vertical gradients of turbulent fluxes except those of momentum fluxes vanish. The vertical gradients of the momentum fluxes balance with the horizontal gradients of the geostrophic pressure.

The upper boundary conditions at the top of the calculation domain $z=z_m$ are such that the vertical gradients of mean wind and turbulent fluxes vanish. The value of z_m is chosen so as to satisfy $z_m = 4f/u_*$, where f is the Coriolis parameter, u_* the friction velocity defined as the square root of the magnitude of the surface Reynolds stress per unit mass. The value of u_* has been estimated by the so-called Rossby-number similarity law (Csanady, 1967; Blackadar and Tennekes, 1968);

$$\sin \alpha = Bu_*/G\kappa, \quad (1)$$

$$G\kappa/u_* = \left[\{ \ln(R_o u_*/G) - A \}^2 + B^2 \right]^{1/2}, \quad (2)$$

Table 1. The parameters for the numerical experiments (f : Coriolis parameter, G : geostrophic wind speed, z_o : roughness length, a_2/a_1 : the ratio of the constants in Eq. (3), and heights of grid points in meters)

case	$f(\times 10^{-5}/\text{s})$	$G(\text{m/s})$	$Z_o(\text{m})$	$R_o(\times 10^6)$	a_2/a_1	$\xi = 1$	grid point height (m)							
							2	3	4	... 9	10	... 20	... 80	
C1	8.365	10.0	1.0	0.11	900	1.7	2.7	4.5	7.4	84	132	2143	27000	
C2	8.365	5.0	0.1	0.60	2500	0.20	0.41	0.83	1.7	47	83	1182	11200	
C3	8.365	5.0	0.01	5.98	14200	0.03	0.07	0.18	0.47	43	84	1090	9000	
C4	4.988	5.0	0.01	10.02	22000	0.03	0.07	0.20	0.55	62	126	1749	14500	
C5	4.988	10.0	0.01	20.05	37800	0.03	0.09	0.26	0.79	131	269	4258	27400	
C6	2.532	10.0	0.01	39.49	66000	0.03	0.10	0.33	1.1	246	520	6537	51300	
C7	2.532	30.0	0.01	118.5	172000	0.03	0.12	0.41	1.4	520	1205	17894	142300	

where G is the magnitude of the geostrophic wind G , α the angle between G and the surface stress vector, $R_o(\equiv G/fz_o)$ the surface Rossby number, κ von Karman's constant, and A and B universal constants. Preliminary calculations have shown that A and B are 1.8 and 2.2, respectively. Therefore these values are used to estimate u_* . The value of κ is fixed at 0.4 in the present study.

Eighty one grid points are distributed between z_o and z_m . For computational accuracy, a transformed coordinate ξ defined by

$$\xi = a_1(z - z_o)/z_o + a_2 \ln(z/z_o), \tag{3}$$

is introduced, where a_1 and a_2 are constants to satisfy $1 \ll a_2/a_1 \ll z_m/z_o$, and the height of the n -th grid point z_n is determined by $\xi = n$ ($n=0, 1, 2, \dots, 80$).

In the following experiments, initial values for all the turbulent momentum fluxes per unit mass were $1 \times 10^{-3} \text{ m}^2/\text{s}^2$ and those for the mean wind vector are equal to G everywhere except at $z=z_o$. The model was integrated numerically with respect to time by a Dufort-Frankel scheme until the model atmosphere became steady throughout the boundary layer.

Seven numerical experiments were carried out. The parameters for these experiments are listed in Table 1.

3. Results

Figure 1 shows the vertical profiles of momentum fluxes $\overline{uw}$ and $\overline{vw}$ for Experiment C3 after 96 hours, where the x -coordinate lies along the direction of the surface stress. Note that the ordinate is logarithm of the height. By this time the model atmosphere has become steady up to a non-dimensional height of 1.8 ($1.8 u_*/f$ in dimensional units). It is seen that a "constant-flux layer", where $\overline{uw}$ remains constant and $\overline{vw}$ is negligibly small, is formed near the ground.

Figure 2 shows the vertical profiles of the x - and y -components of the mean wind velocity and its magnitude. In this particular figure, the x -direction is aligned with the direction of the geostrophic wind

Fig. 1. Profiles of $\overline{uw}$ and $\overline{vw}$ after 96 hours for the case of Experiment C3 in Table 1. The x -axis is aligned to the direction of the surface stress.

vector. A logarithmic wind profile which is consistent with the presence of the constant-flux layer appears near the ground. As the height increases, the mean flow approaches the geostrophic wind G .

Defining the depth z_c of the surface layer by the height at which

$$\frac{\overline{uw}(z = z_o) - \overline{uw}(z = z_c)}{\overline{uw}(z = z_o)} = 0.1 \tag{4}$$

is satisfied (Stull, 1988), one can obtain the value of z_c for each experiment. In Fig. 3, the non-dimensional depth $z_c f / u_*$ of the surface layer is plotted by triangles against the surface Rossby number R_o . It is seen that $z_c f / u_*$ remains almost constant (~ 0.02), though it shows a very weak dependence on R_o .

Also shown in Fig. 3 are the values of the universal constants A and B obtained from the experiments

Fig. 2. Profiles of U and V , the x - and y -components of the mean wind velocity, and its magnitude $M = \sqrt{U^2 + V^2}$ after 96 hours for the same case as in Fig. 1. The x -axis is aligned to the direction of the geostrophic wind vector.

(see Eqs. (1) and (2)). It is seen that B remains almost constant (~ 2.2) when Ro is changed. However, A shows a somewhat systematic dependence on Ro .

In the present numerical experiments, 10 grid points are distributed in the surface layer (see Table 1). Table 2 shows the value of z_c for each experiment as well as the friction velocity, the angle between the surface wind and geostrophic vectors, and the non-dimensional height hf/u_* of the Ekman layer. It is seen from Table 1 and 2 that the ratio of z_c to z_o becomes larger as the case number Cn ($n=1, 2, \dots, 7$) is increased. This means that the resolution in the surface layer may be less sufficient for Cn with larger n . To check this possibility, Experiment C6 has been repeated by changing the number N of grid points in the surface layer. When N was increased, A and z_cf/u_* were decreased and B was increased. Plotted in Fig. 3 by solid marks are the values of A and z_cf/u_* when N was 15. These values are close to those obtained from C1 and C2. When N is increased to 17 for Experiment C1, on the other hand, the values of A , B and z_cf/u_* did not change at all. These facts suggest that, when one would have good resolution in the surface layer, the Rossby-number similarity law would hold and A , B and z_cf/u_* would not have dependence on the Rossby-number. The asymptotic values of A , B and z_cf/u_* seem to be 1.6, 2.2 and 0.02, respectively.

Table 2 also shows the depth h of the Ekman boundary layer non-dimensionalized by u_*/f for each experiment, where h has been defined as the

Fig. 3. The non-dimensional depth of the surface layer $z_c f / u_*$ (Δ), and the universal constants A ($\square$) and B ($\circ$) for the Rossby-number similarity law. The values of A and $z_c f / u_*$ when the number of grid points in the surface layer is 15 are indicated by $\blacksquare$ and $\blacktriangle$, respectively. The abscissa is the logarithm of the surface Rossby-number Ro .

lowest height at which the direction of the mean wind coincides with that of the geostrophic wind. At this height, the magnitude of the Reynolds stress decreases to about 12 % of its surface value. A weak dependence of hf/u_* on Ro seems to exist. Figure 4 shows the dependence of hf/u_* on $1/\log Ro$. An experimental formula

$$h = \frac{u_*}{f} \left[0.653 + \frac{0.598}{\log Ro} \right], \quad (5)$$

fits nicely to the data points. Since several calculations for C6 with different resolutions in the surface layer lead to the same value of hf/u_* , it is unlikely that this dependence of hf/u_* on $\log Ro$ is of numerical origin. If the second-order approximation of the asymptotic similarity analysis were to be developed properly, one might be able to show that the second-order correction is $O(\log^{-1} Ro)$. However, it is beyond the scope of the present study to develop such an analysis. When Ro becomes sufficiently large, Eq. (5) suggests that h is given by $0.653 u_*/f$.

The proportional constant of about 0.7 agrees well with the result of a direct numerical simulation by Coleman *et al.* (1990), but is somewhat larger than the results of the previous studies (*e.g.*, 0.45 from a second-order closure model (Wyngaard *et al.*, 1974), 0.6 from a $E - \varepsilon$ model (Duynderke, 1988) and 0.5 from a large-eddy simulation (Mason and Thomson, 1986)). The fact that hf/u_* is about 0.7 means that the surface layer occupies about 3 % of the whole depth of the Ekman boundary layer.

Table 2. The parameters obtained from the experiments (u_* : friction velocity, α : the angle between the geostrophic wind vector and surface stress vector, A , B : the universal constants for the Rossby-number similarity law, z_c : the depth of the surface layer, $z_c f/u_*$: the non-dimensional depth of the surface layer, hf/u_* : the non-dimensional depth of the Ekman boundary layer)

Case	U_* (m/s)	α (deg.)	A	B	z_c (m)	$z_c f/u_*$	hf/u_*
C1	0.533	16.8	1.580	2.166	127.5	0.0200	0.771
C2	0.225	14.2	1.602	2.179	54.3	0.0202	0.756
C3	0.184	11.6	1.674	2.178	45.3	0.0205	0.742
C4	0.177	11.1	1.695	2.177	73.2	0.0206	0.739
C5	0.337	10.6	1.738	2.175	140.2	0.0208	0.734
C6	0.321	10.1	1.785	2.174	265.0	0.0209	0.732
C7	0.896	9.3	1.857	2.169	743.2	0.0210	0.727

4. Discussion

In the previous section, it is found that the depth of the surface layer for a neutral atmosphere is given by $0.02 u_*/f$. Now we will discuss its theoretical implication and an extension to stratified atmospheres.

The equation of motion in the x -direction is

$$\frac{dU}{dt} = fG \sin \alpha - \frac{\partial}{\partial z} \overline{uw}, \quad (6)$$

where U is the x -component of the mean wind. The Coriolis force fV , where V is the y -component of the mean wind, has been neglected since V is small in the surface layer. Danard (1981) integrated Eq. (6) from $z=z_o$ ($\ll z_c$) to $z=z_c$ and obtained

$$z_o \approx \frac{\overline{uw}(z=z_o)}{-dU/dt + fG \sin \alpha} \left[\frac{\overline{uw}(z=z_o) - \overline{uw}(z=z_o)}{\overline{uw}(z=z_o)} \right] \quad (7)$$

If a relative variation of 10 percent in the momentum flux is allowed in the surface layer (Eq. (4)), Eq. (7) gives,

$$z_c \approx 0.1 \frac{-\overline{uw}(z=z_o)}{-dU/dt + fG \sin \alpha}. \quad (8)$$

Assuming a steady state and horizontal homogeneity, Eq. (8) reduces to

$$z_c = -0.1 \overline{uw}(z=z_o) / fG \sin \alpha. \quad (9)$$

After substituting Eq. (1) and $\overline{uw}(z=z_o) = -u_*^2$ into Eq. (9), we obtain

$$z_c = 0.1 \kappa u_* / Bf. \quad (10)$$

This shows that, if we define z_c by Eq. (4), the depth of the surface layer is scaled with u_*/f . Using the value of $\kappa=0.4$ and $B \sim 2.2$ from the results of the present model, we obtain $z_c f/u_* = 0.018$. This value is in close agreement with the one obtained directly from the model.

Fig. 4. The non-dimensional depth of the Ekman layer hf/u_* . The solid line is according to Eq. (5). The abscissa is a reciprocal of logarithm of the surface Rossby-number Ro .

So far, we have considered only a neutral atmosphere. If Eq. (10) holds for a neutral atmosphere, however, one may extend this expression to a stably or unstably-stratified atmosphere. For the stable or unstable atmosphere, B may be considered to be a function of non-dimensional number $\mu = \kappa u_* / fL$ (Zilitinkevich, 1975) where L is the Monin-Obukhov length. Although the functional form of $B(\mu)$ has been studied by field observations and numerical models, there still remains a large ambiguity in the value of $B(\mu)$. According to Clarke and Hess (1974), the value of $B(\mu)$ is about 20 for a very stable case, about 4 for a neutral case and about 0.5 for a very unstable case. If their value is adopted, the depth of the surface layer is expected to be 5 times thinner for the stable case and 8 times deeper for the unstable case, than that for the neutral case.

5. Summary and Concluding Remarks

The depth z_c of the surface layer for a turbulent Ekman layer is studied by means of a second-order turbulence closure model and a simple theoretical consideration, where z_c is defined as the height at

which the momentum flux $\overline{uw}$ decreases to 90 percent of its surface value. The value of z_c is found to be given by

$$z_c = Cu_*/f, \quad (11)$$

where C is a universal constant. C is found to be 0.02 according to the present model. The theoretical consideration suggests that $C=0.1\kappa/B$, where κ is von Karman's constant ($=0.4$) and B one of the universal constants for the Rossby-number similarity law (Csanady, 1967; Blackadar and Tennekes, 1968). According to the present numerical calculations, B is about 2.2. Thus, the value of C is 0.018, which agrees with the model result. By allowing B to be a universal function of non-dimensional number $\mu=\kappa u_*/fL$, Eq. (9) can be extended to a stably or unstably-stratified atmosphere.

The height h of the neutral turbulent Ekman layer non-dimensionalized by u_*/f shows a linear dependence on $\log^{-1} Ro$. To examine if this dependence is a second-order correction to the asymptotic similarity theory (Blackadar and Tennekes, 1968) remains as an interesting problem for the future.

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乱流エクマン境界層の接地境界層の深さについて

木下宣幸・新野 宏

(気象研究所)