Modelling the Evaporation from Bare Soil
with a Formula for Vaporization in the Soil Pores

By Junsei Kondo

Geophysical Institute, Tohoku University, Sendai, 980, Japan

and

Nobuko Saigusa

Institute of Biological Sciences, University of Tsukuba, Tsukuba, Ibaraki, 305, Japan

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Abstract

A multi-layer soil model is constructed taking into consideration the diffusion of water vapor and vaporization in the soil pores. The resistance to vaporization in the pores is introduced as a function of the volumetric soil water content. The soil layer is treated as a 'porous canopy' and the vapor density profile in the soil is calculated by an equation for vapor diffusion. The model simulates the evaporation rate, the soil temperature, and the soil water content profile rather well when compared to measurements from field observations and pan experiments.

Recent numerical models usually assume that the water vapor found in the soil pores is in equilibrium with the soil water at each depth, with the equilibrium humidity expressed by thermodynamical theory (Philip, 1957). However, the present model calculations reveal that the relative humidity in the pores close to the ground surface is different from that at equilibrium, since the water vapor is continuously transported to the atmosphere during the drying stage.

1. Introduction

Land surface evaporation plays an important role in the circulation of both heat and water over the earth's surface. The weather and climate over semiarid land surfaces are strongly influenced by the efficiency of surface evaporation. However, the calculation of evaporation from a non-saturated soil has proved difficult, due to such problems as the estimation of surface wetness, the inhomogeneity of water content, differences among various soil types, to name a few.

Several soil models have been developed for estimating the heat and water transport between the air and land surfaces. The bucket model employed by Manabe (1969) and Hansen et al. (1983) has been useful in climate models, and the force-restore method has been used in meso- and large-scale models (Noilhan and Planton, 1989). Sellers et al. (1986) made use of a matric potential-based formulation, but the flux of water vapor in the soil was neglected in their model. These soil models perform rather well for a relatively wet soil, but are unsuitable for dry soil since the evaporation from a dry soil is, for the most part, controlled by the rate of water vapor diffusion in the soil.

A number of soil models are based on the theory of moisture flow resulting from the matric potential gradient in the liquid and vapor phases (Philip, 1957, Sasamori, 1970, McCumber and Pielke, 1981, Camillo et al., 1983). These models assume that a local equilibrium always exists between the liquid and vapor phases within the soil pores, with the relative humidity \( h \) in the pore being expressed as

\[
h = \exp \left( \frac{\psi g}{R_w T} \right),
\]

where \( \psi \) is the soil water potential at the soil surface, \( g \) the acceleration of gravity, \( R_w \) the gas constant for water vapor, and \( T \) the soil temperature. Equation (1) is derived from thermodynamic theory and relates the water vapor pressure immediately above the soil water to the surface tension of the water.

These models also assume that a local equilibrium
exists between the liquid and vapor phases of the soil water, even at the ground surface. The evaporation rate can be expressed by a bulk formula as

$$E = \rho C_e u (h q^*(T_s) - q_a),$$  \hspace{1cm} (2)

where $E$ is the evaporation rate, $\rho$ the air density, $C_e$ the bulk transfer coefficient for water vapor, $u$ the wind velocity, $q^*(T_s)$ the saturation specific humidity at the soil surface temperature $T_s$, $q_a$ the specific humidity of the air, and $h$ is obtained by Eq. (1).

However, the use of Eq. (1) to calculate the relative humidity at the ground surface has been questioned by Wetzel and Chang (1987) and Kondo et al. (1990). (See Figs. 4a and 5 in Kondo et al., 1990). Although Eq. (1) is adequate for calculating the relative humidity of the air immediately above the water surface within the soil pores, the extension to calculating the relative humidity at the ground surface is questionable.

In this paper, the process of water vapor transport in the soil is considered as follows. As schematically shown in Fig. 1, water vaporizes in the soil pores and is diffused from the interior consisting of 'small pores' to 'larger pores', where the 'large pore' extends upward, being exposed to the ground surface. With soil drying, the water is vaporized at deeper depths in the soil, with the vapor passing through the dry surface layer to the atmosphere. In the soil model presented here, the vapor transport in the soil is regarded as diffusion in a porous material, and the diffusion equation is solved by employing the specific humidity at each depth as a dependent variable. Evaporation to the atmosphere is expressed by the bulk formula (Eq. 2), where the ground surface specific humidity is also a dependent variable determined by the diffusion equation.

The formulation of both the vaporization and diffusion, and the multilayer soil model are presented in Section 2. The comparison between field observations and model calculations is described in Section 3. In Section 4, several problems concerning the relative humidity profile in the soil are discussed.

2. The soil model

2.1 Formulation of soil water vaporization

When the soil contains a sufficient amount of water, the water vapor in the soil 'large' pore is almost saturated, but is less than the saturation value of a free water surface when a limited amount of water remains in the soil, because the saturation vapor pressure is reduced due to the large surface tension on the small amount of soil water when the soil is dry. Another reason is that the water vapor in the pores close to the ground surface is continuously removed to the atmosphere during the drying stage, so that the upward diffusion of vapor exceeds the
vaporization within the soil and the diffusion of vapor from the underlying layer.

Figure 2 schematically displays the process of moisture diffusion in the liquid and vapor phases during the drying stage as determined by the resistance expression. In the figure, \( q \) is the specific humidity in the 'large pores' exposed to the ground surface, \( \theta \) the volumetric soil water content, \( T \) the temperature, and \( i \) represents the soil layer. Water is vaporized in the interior of the soil 'small pore' where the specific humidity is expressed as \( hq^*(T) \), and diffused to the 'larger pores'. Here, \( h \) is expressed as Eq. (1).

The vapor diffusion in the soil pore should be analogous to that in the vegetated canopy. In the soil, water vapor is carried from the 'small pore' to the 'large pore' by molecular diffusion and, in the case of a vegetated canopy, vapor is carried from stomata in the leaf to the air by molecular and turbulent diffusion. The rate of vaporization from the water in the 'small pores' at each depth in the soil is expressed as

\[
E_{\text{soil}} = \rho_D \frac{h}{R} 
\]

where \( E_{\text{soil}} \) is the mass of water vaporized per unit volume and unit time, and \( D_{ \text{atm} } \) is the molecular diffusion coefficient for water vapor. \( R(\theta) \) is the resistance to vaporization and vapor transport from the 'small pores' to 'larger pores' per unit volume. The value of \( R(\theta)/D_{ \text{atm} } \) has the same role as the stomatal resistance in the vegetated canopy.

The resistance \( R(\theta) \) is expressed as a function of soil water content, being very small when sufficient water exists, with \( q \) equal to \( hq^*(T) \) at the given depth. \( R(\theta) \) has a very large value when small amounts of water exist. For this case, vaporization hardly occurs at the given depth (see the top dry layer in Fig. 1). The value of \( R(\theta)/D_{ \text{atm} } \) is experimentally estimated in the following sections.

Since evaporation is a continuous diffusion process as shown in Fig. 2, the specific humidity in the 'large pore' \( q \) is not always in equilibrium with the water at each depth. The value of \( q \) is controlled by the evaporation rate, the vaporization at each depth, and the diffusion of vapor from underlying soil layers. In this model, the specific humidity \( q \), the vaporization in the soil \( E_{\text{soil}} \), the vapor diffusion, and the liquid water diffusion are estimated by solving the liquid water and vapor diffusion equations in the soil.

2.2 Equations

The heat and water transport within the soil is expressed by the equations for heat conduction, liquid water diffusion, and vapor diffusion. The rates of liquid water, water vapor, and temperature change are expressed as

\[
\rho_D \frac{\partial \theta}{\partial t} = -\frac{\partial q_{\text{vap}}}{\partial z} - E_{\text{soil}},
\]

\[
\frac{\partial \left( (\theta_{\text{sat}} - \theta)q \right)}{\partial t} = -\frac{\partial q_{\text{vap}}}{\partial z} + E_{\text{soil}},
\]

and

\[
C \frac{\partial T}{\partial t} = -\frac{\partial q_{\text{vap}}}{\partial z} - \rho_p K.
\]

respectively. Here, \( \rho_D \) is the density of the liquid water (=10^3 kg m^-3), \( t \) the time, \( z \) the depth, \( \theta_{\text{sat}} \) the saturated volumetric soil water content, \( C \) the volumetric heat capacity, and \( L \) the latent heat of vaporization. \( q_{\text{vap}} \) is the flux of liquid water, \( q_{\text{vap}} \) the water vapor flux, \( q_{\text{vap}} \) the heat flux, and \( (\theta_{\text{sat}} - \theta)q \) expresses the mass of water vapor per unit volume of soil. If \( \theta \) approaches \( \theta_{\text{sat}} \), the volume of gas in the soil will be zero, then the water is carried only as the liquid phase expressed by Eq. (4).

The variables \( q_{\text{vap}} \), \( q_{\text{vap}} \), and \( q_{\text{vap}} \), are positive in upward flux, and given by

\[
q_{\text{vap}} = -\rho_D \frac{\partial \theta}{\partial z} - \rho_p \frac{\partial T}{\partial z} - \rho_p K,
\]

\[
q_{\text{vap}} = -\rho_D \frac{\partial \theta}{\partial z},
\]

and

\[
q_{\text{vap}} = -\lambda \frac{\partial T}{\partial z}.
\]

Here, \( D_{\text{vap}} \) and \( D_{\text{vap}} \) are the diffusion coefficients for liquid water, \( K \) is the hydraulic conductivity, \( f(\theta) \) the porosity and tortuosity factor, and \( \lambda \) the thermal conductivity of the soil. The heat flux due to the movement of liquid water and air parcels in the soil is neglected, for the heat conduction by the solid soil should be far larger than these. The diffusion coefficients \( D_{\text{vap}} \) and \( D_{\text{vap}} \) can be written as

\[
D_{\text{vap}} = K \frac{\partial \psi}{\partial \theta},
\]

and

\[
D_{\text{vap}} = K \frac{\partial \psi}{\partial T},
\]

where \( \psi \) is the matric potential. According to Clapp and Hornberger (1978), the matric potential \( \psi \) and hydraulic conductivity \( K \) can be expressed as

\[
\psi(\theta) = \psi_{\text{sat}} \left( \frac{\theta}{\theta_{\text{sat}}} \right)^{-b},
\]

and

\[
K(\theta) = K_{\text{sat}} \left( \frac{\theta}{\theta_{\text{sat}}} \right)^{+c},
\]

where \( \psi_{\text{sat}} \) and \( K_{\text{sat}} \) are the matric potential and hydraulic conductivity at saturation, while the constants \( b \) and \( c \) depend on the soil type. The
potential energy of soil water is customarily expressed in units of length, corresponding to the height of a water column (1 m corresponds to 9.8 J kg\(^{-1}\)).

In the present study, Eq. (12) is modified as

\[
\psi(\theta) = \begin{cases} 
\psi_{sat} \left( \frac{\theta}{\theta_{sat}} \right)^2, & (\theta \geq \theta_c) \\
\psi_{a1} \exp(-a1 \theta) + \psi_{a2} \exp(-a2 \theta), & (\theta < \theta_c)
\end{cases}
\]

(14)

in order to estimate the relation between \(\psi\) and \(\theta\) correctly even for small \(\theta\) (Kondo, 1993). \(\psi_{a1}, \psi_{a2}, a1, a2,\) are constants determined experimentally for each soil type. The values of the constants used in the present study are listed in Table 1.

The relation between \(\psi\) and \(T\) is assumed by Hopmans and Dane (1985) to be

\[
\frac{\partial \psi}{\partial T} = \psi \gamma(T)
\]

(15)

\[
\gamma(T) = -1.48 \times 10^{-5} T + 2.26 \times 10^{-3} \ [K^{-1}]
\]

(16)

The function \(f(\theta)\) given in Eq. (8) determines the volumetric reduction of pores available for vapor diffusion due to the presence of soil particles and liquid water. The factor \(f(\theta)\) has been expressed by Penman (1940) as

\[
f(\theta) = 0.66(\theta_{sat} - \theta),
\]

(17)

which is used in the present study.

Thermal conductivity \(\lambda\) (unit in J s\(^{-1}\) m\(^{-1}\) K\(^{-1}\)) normally depends on the soil water content, but the dependence has not been cleared yet. In this study, \(\lambda\) is assumed empirically as

\[
\lambda = 0.251 + 0.5\theta^{3/4}.
\]

(18)

The volumetric heat capacity \(C\) is expressed as

\[
C = (1-\theta_{sat})C_{soil} + \theta C_{water},
\]

(19)

where \(C_{soil}\) and \(C_{water}\) are the heat capacity of the soil particles and that of water, respectively. Here, \(C_{soil} = 1.26 \times 10^6\) J m\(^{-3}\) K\(^{-1}\) and \(C_{water} = 4.20 \times 10^6\) J m\(^{-3}\) K\(^{-1}\) are used as typical values. The heat capacity of air is omitted, since it is small when compared with those of other soil constituents.

Kondo et al. (1992) determined the resistance of water vapor transport from a 0.02 m thick soil layer, \(F\), for several soil types. The term \(F\) can be related to the evaporation from the 0.02 m thick soil layer, \(E\), by

\[
E = \rho D_{atm} \frac{q_s(T_s) - q_s}{F}.
\]

(20)

Another resistance, \(F'\), can be related to \(E\) by

\[
E = \rho D_{atm} \frac{F'}{F}.
\]

(21)

The value of \(F'\) can be estimated in a similar way for Narita sand and loam from another pan experiment and wind tunnel experiment, and is expressed as follows.

For Narita sand:

\[
F' = 0.04 \exp(-200 \theta) + 0.0003 \exp(-10 \theta^2).
\]

(22)

And for loam:
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\[ F' = 0.044 \exp(-100 \theta^2) + 0.002 \exp(-50 \theta^2). \]  
(23)

Figures 3 (a) and (b) show the experimental relationship between \( F' \) and the soil water content for Narita sand and loam, respectively. The values indicated by the open circles were obtained from the experiments, while the lines represent calculations using Eqs. (22) and (23).

The resistance \( R(\theta) \) introduced as Eq. (3), in units of m\(^2\), is assumed in the present model to be given by

\[ R(\theta) = 0.02 F'. \]  
(24)

Here, \( \theta \) and \( T \) are assumed to be uniform in the 0.02 m thick soil. Eqs. (22), (23) and (24) are provisional expressions and further investigation of the relationship between the resistance \( R(\theta) \) and the water content for various soil types needs to be conducted.

Equations (7)-(9) for liquid water, water vapor, and heat transport are transformed to finite difference approximations and solved with the use of a backward-differencing scheme. The time step of the integration is 60 s, and the soil is divided into 1 cm layers from the surface to a depth of 30 cm.

2.3 Boundary Conditions

The ground surface boundary conditions are

\[ Q_{b,\text{liq}} = 0, \]  
(25)

\[ Q_{b,\text{eap}} = E = \rho C_E u(q_s - q_a), \]  
(26)

and

\[ Q_s = -((1 - a) S' + L' - \sigma T_s' - c_e \rho C_v u(T_s - T_a)), \]  
(27)

where \( q_s (= q_1) \) is the specific humidity in the 'large pores' of the ground surface, \( a \) the albedo of the ground, \( S' \) the solar radiation, \( L' \) the atmospheric radiation, \( \sigma T_s' \) the upward long-wave radiation from the surface, \( c_e \) the specific heat of air, \( C_v \) the bulk transfer coefficient for the sensible heat flux, and \( T_s (= T_1) \) the surface soil temperature. Equation (25) states that no liquid water flux exists above the ground surface. Equation (26) states that the vapor flux at the ground surface equals the rate of evaporation determined by the bulk formula, while Eq. (27) indicates the heat removed from the ground surface. For each flux term, a positive value indicates an upward flux.

The soil albedo \( a \) is determined as a function of the surface volumetric soil water content. Figure 4 shows the relationship between the albedo and the soil water content of a 2 cm surface layer, estimated by the observation of Narita sand and loam (Kondo et al., 1992).

The boundary conditions are given at the bottom of the pan and at a depth of 0.3 m for the Narita Sand as

\[ Q_{\text{b,liq}} = \begin{cases} K(\theta) & \text{at } z=0.3 \text{ m for Field observation} \\ 0 & \text{at } z=0.1 \text{ m for Pan experiment} \end{cases} \]  
(28)

\[ Q_{b,eap} = 0, \]  
(29)

\[ Q_s = 0, \]  
(30)

where no flux exists at the bottom of the pan, and only a liquid-water flux induced by the force of gravity is assumed at \( z=0.3 \text{ m} \) in the field observations.

3. Field observations and pan experiments

3.1 Field observations

Meteorological and hydrological observations were carried out at the Chiba Experiment Station of the Institute of Industrial Science, University of Tokyo, in order to estimate the evaporation from a bare soil. A field of bare soil consisting of a fine sand called Narita sand extended over an area of about 30 m \( \times \) 30 m, with the outside of the soil field covered by grass of several tens of centimeters in height.

Evaporation from the field was estimated by the
Bowen ratio method over the period from 24 July to 3 October 1990. Solar radiation, soil albedo, atmospheric radiation, and wind velocity were measured at a height of 0.8 m. Air temperature and humidity were measured by a ventilated psychrometer at heights of 0.15, 0.3, and 0.6 m. The ground heat flux was estimated from the soil temperature profile at 8 points distributed from the ground to a depth of 0.4 m. The soil water potential and volumetric soil water content were observed by a tensiometer and the use of the oven drying method (Asanuma, 1991).

In the present model, the bulk transfer coefficients, $C_h$ and $C_e$ are assumed from the observed data of sensible heat flux, as $C_h = C_e = 3.0 \times 10^{-3}$, where the reference height is 0.6 m.

Figure 5 shows the observed surface heat budget over the period from 27 August to 1 September 1990. The latent heat flux $L_e$ was large on 27 Aug., but decreased as the soil dried.

Figure 6 displays the latent heat flux observed by the Bowen ratio method and that calculated by the present model over the period from 27 August to 1 September, while Fig. 7 is the same as Fig. 6 except for the period from 8 to 12 September. The observed and calculated latent heat flux differ slightly during the night. The reason has not been clarified yet, but the accuracy of the observations is about $\pm 20 \text{ Wm}^{-2}$, which is about the same as the difference found in Figs. 6 and 7.

Figure 8 shows the observed (points) and calculated (lines) volumetric soil water content over the period from 27 August to 1 September, while Figs. 9a and b are the observed and calculated soil temperatures for the same period. It can be seen that the present model generally simulates the heat and water transport within the soil.

### 3.2 The pan experiment

An evaporation experiment was carried out at the Geophysical Institute of Tohoku University over the period from 30 May to 5 June 1989, using soil filled pans with a depth of about 0.1 m and a diameter of
0.3 m. The evaporation was determined by weighing the pan, and the soil temperature and volumetric soil water content were measured. The details of this experiment were presented in Kondo et al. (1992).

The bulk transfer coefficient $C_e$ (Eq. 26) for a limited small surface depends on the scale of the surface. According to Kondo et al. (1992), $C_e$ was expressed for the pan as

$$C_e = 1.26 \frac{Sc^{-2/3}}{Re^{-1/2}} (5.7 \times 10^4 < Re < 4.2 \times 10^4),$$

where $Sc (\equiv \nu / D_{act})$ is the Schmidt number, $Re (\equiv \frac{\nu u}{\nu})$ the Reynolds number, $\nu$ the kinematic viscosity of air, $u$ the wind speed, and $x$ the pan size. And $C_h = C_e$ is assumed here.

In Fig. 10, the observed and calculated latent heat flux are shown by the open circles and solid line, respectively, exhibiting a strong agreement between the two. Figure 11 shows the volumetric soil water content, both calculated (lines) and measured (symbols), during the pan experiment. In Fig. 11, the calculated soil water content at 1-2 cm is always more than those observed. Supposedly, the difference is caused by the uncertainty of hydraulic conductivity estimated in the present study at the surface soil layer.

4. The relative humidity profile in the soil pores

Most recent soil models usually assume that a local equilibrium exists between the soil water and vapor in the soil pore, but the present study suggests that the water vapor found close to the ground surface is not always in equilibrium during the drying stage. The equilibrium humidity can be calculated by Eq. (1), and is expressed as the relative humidity (the ratio of humidity to the saturation value over a plane water surface).

Figures 12 (a) and (b) show the soil layer relative humidity in the 'large pores' calculated by the present model, where (a) is for Narita sand and (b) is for loam. With soil drying, the relative humidity in the top soil layer (0-1 cm deep) decreases. For example, the relative humidity is approximately 30% in the top soil layer on the evening of 5 June 1989 (Fig. 12 b), while simultaneously the volumetric soil water content is 0.05 (Fig. 11). The equilibrium humidity estimated by Eqs. (1) and (14) for the soil ($\theta = 0.05$) is about 80 %, which is considerably larger than the present calculated value.

The reason for the humidity difference should be that the humidity is in a non-equilibrium state in the soil 'large pores' at the ground surface, resulting from the water vapor being successively removed to the atmosphere during the drying stage. Similarly to the vegetated canopy, even if the vapor in the stomata (which is analogous to the small pore) is
humidity and the equilibrium value.

Another reason may be that the relative humidity at the ground surface cannot be estimated by the average matric potential of the top soil layer (0-1 cm) because of the large potential gradient near the ground surface. In future, detailed measurements should be made of the water vapor profile in the top soil layer to determine the difference between actual humidity and the equilibrium value.

5. Conclusions

Recent soil models used in numerical simulations are fundamentally based on the model developed by Philip (1957), with a local equilibrium between the liquid and vapor phases being assumed not only in the deep soil but also at the ground surface. In the present study, however, each soil layer has been treated as a 'porous canopy' with the humidity at each depth calculated by the vapor diffusion equation.

A new formulation for vaporization in the soil 'large pores' was presented, and the resistance to the vaporization within the soil was experimentally estimated. A multi-layer soil model was then constructed, taking into consideration the vaporization and diffusion of water vapor in the soil. The evaporation was calculated with the vapor diffusion equation, together with the ground surface humidity and the humidity profile in the soil.

The model was tested by making use of observational and pan experiment data. During the drying stage of the soil, the humidity within the 0-1 cm soil layer decreased in the daytime, and was found to be less than that of an equilibrium state with the soil water. This result suggests that the ground surface humidity is controlled by not only the ground soil water content but also by other conditions such as air humidity, wind velocity, and the vapor diffusion within the surface soil.

Further investigation concerning the actual vapor behavior at the ground, the vaporization in the soil, and the resistance $R(\theta)$ for various soil types should be conducted to determine the water exchange of dry soil fields.

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References


土壌間隙内部の蒸発過程を考慮した裸地面蒸発のモデル化

近藤純正
(東北大学理学部)

三枝信子
(筑波大学生物科学系)

土壌間隙内部における水の気化と水蒸気輸送を考慮した裸地面蒸発の多層モデルを開発した。このモデルでは、土壌中の任意の深さで起きる水の気化を表現するため、気化に対する抵抗を表すパラメータを土壌含水率の関数として導入した。また、土壌を多孔性のキャノピーと考え、間隙中において水蒸気拡散方程式を解くことにより土壌中の水蒸気輸送量を評価した。この土壌多層モデルにより、野外観測と野外実験により得られた乾燥期間中の土壌面からの蒸発量、地温、含水率分布の時間変化をよく再現することができる。

近年広く用いられている土壌の数値計算モデルでは、土壌間隙内部で常に水と水蒸気が平衡状態に達していると仮定される場合が多い。しかし地表面に開いた間隙中では、蒸発期間中、地中から大気へ水蒸気が常に連続的に運ばれるため、間隙中の湿度は土壌内部の閉じた間隙で実現する平衡相対湿気度とは異なっていることがモデル計算から予想された。