Super- and Sub-Harmonic Responses of Tropical Kelvin Waves to the Heating with a Seasonal Modulation

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(Manuscript received 30 May 1995, in revised form 5 December 1995)

Abstract

By applying a seasonally modulated condensation heating on tropical Kelvin waves with a certain form of damping, the parametric instability and super(sub)harmonic response of the Kelvin waves are investigated. It is suggested that the excitation of intraseasonal oscillation and tropospheric QBO may be related to the superharmonic and subharmonic resonances with the seasonal cycle, respectively. The main features of the spatial structures and propagation of these resonant modes were shown and compared with those of intraseasonal oscillation and tropospheric QBO to examine this point of view.

1. Introduction

Perhaps it is trivial to know that as a result of annual forcing of solar heating, the atmosphere varies in a dominant annual cycle. Nevertheless, besides this annual cycle what cycles may else be induced directly or indirectly by such an annual forcing is still of significance, for we know very well that the response of a non-linear system like atmosphere to a periodic forcing turns out to quite complex; cycles of response longer or shorter than that of forcing often appear and are termed subharmonics and superharmonics, respectively. In spite of the generality of such a question, the reason why our interest is evoked is that we are wondering whether these super(sub)harmonic responses may account for or, at least, contribute to such oscillations as ISO (intraseasonal oscillation) and tropospheric QBO (quasi-biennial oscillation) which are known to be two long-period oscillations particularly observed in the tropical atmosphere.

With the external annual forcing by solar heating, the atmosphere has no static basic state or even a steady basic flow. The basic state, as the first-order response, should be also with an annual cycle, which can be demonstrated by such things as monsoons and the annual migration of the ITCZ. As a result of linearization around the basic state, our problem then, to some extent, is equivalent to that of impacts of a seasonally varying basic state on the atmospheric waves.

For the purpose of simplicity, we will only consider the Kelvin wave case to propose a conceptual model for ISO and tropospheric QBO. It may be an approximation for the situation of a tropical atmosphere. With the existence of strong damping and diabatic heating, aspects of the free Kelvin wave are greatly modified. Any attempts to explain the features of oscillations with a frequency as low as ISO or QBO by a free Kelvin wave will turn out to be in vain. We will accordingly place the emphases on the roles of damping and heating in turn in the following discussion. Even so, the key point is the problem of so-called 'parametric instability' (Drazin and Reid 1981) due to the inclusion of a seasonal variation in the heating. Similar problems related to the subharmonic resonance of gravity waves to the diurnal cycle can be found in Orlanski (1973), Fels (1974) and the effect of diurnal cycle on intraseasonal wave-CISK in Zhao and Weare (1994).

We shall first consider the effects of damping in Section 2, then introduce a seasonally modulated condensation heating and discuss the super(sub)harmonic response to it in Section 3. Sections 4 and 5 are devoted to the roles of super(sub)harmonic oscillations in the excitations of ISO and tropospheric QBO, respectively.

2. The effects of damping on Kelvin waves

As the preliminary approach, we should know the role of damping first with the absence of heating. Due to the vigorous cumulus convection in the tropical troposphere, there is a vertical mixing of momentum, which will create a strong damping of Kelvin waves. This dissipative process must be included...
in our model. However, to represent this effect, we will not use the most convenient form of so-called ‘Rayleigh friction’, as is usually used, because we believe that by using it a very important mechanism for the selection process of vertical modes was neglected. It seems natural that those modes with shallower vertical structures or larger vertical wave-numbers possess relatively sharper vertical shear of wind and thus have the trend to damp more rapidly. We assume that the cumulus friction can be parameterized into the form $c_\alpha \frac{\partial^2 u}{\partial p^2}$, analogized with that of turbulence. As for radiative effects, we only utilize the ‘Newtonian cooling’. The equations in pressure coordinates are

\begin{align}
\frac{\partial u}{\partial t} &= -\frac{\partial \phi}{\partial x} + \alpha \frac{\partial^2 u}{\partial p^2}; \\
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial p} &= 0, \\
\frac{\partial \phi}{\partial t} + \sigma \omega &= -\beta \frac{\partial \phi}{\partial p},
\end{align}

where $u, \omega, \phi$ are the perturbations of zonal velocity, vertical velocity and geopotential, $\sigma$ the static stability is assumed to be constant, and the constants $\alpha, \beta$ are the damping coefficient of friction and radiative damping.

The above equations (1)–(3) may be combined to form a single equation in $u$ as

\[
\frac{\partial^2 u_{pp}}{\partial t^2} - \frac{\partial}{\partial t} \left[ \frac{\partial^2 u_{pp}}{\partial p^2} + \beta \frac{\partial u_{pp}}{\partial t} \right] - \alpha \frac{\partial^2 u_{pp}}{\partial p^2} + \sigma \frac{\partial^2 u}{\partial x^2} = 0.
\]

With the vertical diffusion taken into account, we should adopt an adherent boundary condition for $u$ at the surface. However, such a boundary condition seems too strong a limitation to the free atmosphere and also results in a difficulty in identifying the normal modes. Here, we just assume stress-free boundary conditions for $u$ at both surface and the top of the atmosphere to make problem easier to deal with. In this case, the effect of vertical diffusion on every single vertical mode is equivalent to that of Rayleigh friction. There are also boundary conditions for $\omega$; that is, $\omega$ should be zero at the upper and the lower boundary. We can thus write boundary conditions as

\[
u_{pp}, \omega \big|_{p=p_s, \Omega} = 0,
\]

where $p_s$ denotes the mean surface pressure. The normal modes for (4) as well as the original Eqs. (1)–(3) then may be chosen as

\[
\cos \left( \frac{n\pi}{p_s} \right) \exp(\imath \omega t + \imath kx); \ n = 1, 2 \ldots
\]

which, substituted into (4), gives the dispersion relation

\[
\omega = \pm \sqrt{\frac{k^2}{E_n^2} - \frac{1}{4} (\alpha E_n^2 - \beta)^2 + \frac{1}{2} (\alpha E_n^2 + \beta)i},
\]

where $E_n = n\pi/p_s$, $\omega$ is the frequency and $k$ the zonal wave-number.

For this modification of free Kelvin waves, we still expect the geostrophic relation and meridional trapping condition to be satisfied, i.e.

\[
\frac{\beta_0 y u}{-\frac{\partial \phi}{\partial y}} = \frac{\beta}{\omega} < 0; \ \text{when} \ y \rightarrow \pm \infty.
\]

From (1) we have

\[
i \omega u = -\imath k \beta_0 \frac{\beta}{\omega + \alpha E_n^2 u},
\]

where $u, \phi$ are functions of $y$ only. (6) combined with (8) yields:

\[
\frac{du}{u} = \frac{\imath k \beta_0}{\omega + \alpha E_n^2} dy.
\]

Thus

\[
u(y) \sim \exp \left[ \frac{E_n^2 \beta_0}{2k\sigma} \left( \pm \sqrt{\frac{k^2}{E_n^2 - \frac{1}{4} (\alpha E_n^2 - \beta)^2} + \frac{1}{2} (\alpha E_n^2 - \beta)i} \right) \right].
\]

In order to satisfy the demand of the trapping condition (7), the only possible case is when minus is taken in (9). So the waves are still eastward-propagating waves, even if there is strong damping.

Consequently, about the effects of damping on Kelvin wave, we can conclude as follows. In addition to a exponential decay $\exp[-1/2(\alpha E_n^2 + \beta)t]$ of the wave amplitude, which is simply the sum of the effects of friction and cooling, there are also two more effects due to the difference between the damping rates of friction and cooling.

1) The phase speed is slowed down and the waves become dispersive. In fact, from the dispersion relation the zonal phase speed is

\[
c_n = \frac{\text{Re}(\omega)}{k} = \sqrt{\frac{\sigma}{E_n^2} - \left( \frac{\alpha E_n^2 - \beta}{4k^2} \right)^2},
\]

which, compared with that of Kelvin wave (i.e. $\sqrt{\sigma/E_n^2}$), leads to our conclusion. Moreover, as was mentioned above, this effect gives strong preference to the modes with shallower vertical structure or a larger number of $n$. What is worth mentioning here is that our conclusion about the slower phase speed effect is something different from that stated by Chang (1977). He arrived at the same conclusion by just using a Rayleigh friction and letting cooling have the same damping rate as that of friction.

2) From the meridional structure described by $u(y)$, it is obvious that damping will result in a poleward or equatorward propagating to Kelvin waves, details about which will be discussed in the next section.
Kelvin waves with both forcing and damping

3.1 Heating

According to the analyses above, with the existence of strong damping resulting from the cumulus friction and radiative damping, all of the Kelvin wave modes are stable, and thus can never be observed as some kind of stationary oscillation. For the forcing and maintenance of these modes, a persistent energy supply is necessary. As is well known, cumulus heating is considered to be responsible for this energy supply. In order to include it, Yamasaki (1969), Hayashi (1970) and Lindzen (1974) have introduced the mechanism of so-called 'wave-CISK' in the studies of tropical waves. A basic assumption of this theory is that cumulus heating can be parametrized to be proportional to the vertical wind at the top of boundary layer of large-scale motion induced by the waves.

On the other hand, a basic fact in the tropical atmosphere is that, due to the annually meridional migration of the sun around the equator, the ITCZ keeps on moving annually southward and northward while the two trade winds or monsoons are strengthened and weakened alternatively. Based on such an annually varying basic state or background, waves associated with low-frequency oscillation will be influenced on considerably and gain a persistent energy supply from it. Our problem is, therefore, how this mechanism works. In the present study, we will just focus our attention on the impact of this seasonally varying basic state on the cumulus heating, and, through which, its impact on Kelvin waves will be investigated. This means we will neglect the effects of the advection due to the annual variation of the basic flow of trade winds or monsoons.

To characterize this seasonally varying basic state of the tropical troposphere, we use a zonally averaged difference between the meridional winds in lower troposphere at 5°S (v1) and 5°N (v2) as an index of meridional moist convergence in ITCZ. It exhibits a prevailing annual cycle. The climatological monthly mean fields are computed from the observational data of 1989 through 1993 (GANAL).

3. Kelvin waves with both forcing and damping

Fig. 1. The zonally averaged difference between the meridional winds in lower troposphere at 5°S (v1) and 5°N (v2) as an index of meridional moist convergence in ITCZ. It exhibits a prevailing annual cycle. The climatological monthly mean fields are computed from the observational data of 1989 through 1993 (GANAL).

Fig. 2. The zonally averaged precipitation (w/m²) over the equator, which is an output of a CCSR/NIES GCM experiment. Seasonal variations (both annual cycle and semiannual cycle) are obvious.
The coefficient $\eta(p)$ is attached with a periodic modulation of one half year or one year, which is due to the seasonal variations of the amount of moisture available in the lower troposphere. $\epsilon$ is the amplitude of this modulation, and $w$ is the vertical velocity. Here, we demand $\eta(p) \to 0$, as $p$ goes to 0, to prevent the convective heating in the stratosphere.

The basic equations are

$$\frac{\partial u}{\partial t} - \frac{\partial \phi}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0,$$  \hspace{1cm} (10)

$$\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial p} = 0,$$  \hspace{1cm} (11)

and

$$\frac{\partial \partial \phi}{\partial t \partial p} + \sigma \omega = - \frac{R}{\rho c_p} \frac{\partial}{\partial p} \eta(p) \frac{\partial \phi}{\partial p}.$$  \hspace{1cm} (12)

For mathematical simplicity, we only consider the following case of

$$\frac{\eta(p)}{p} = \text{const.}$$

or

$$\eta(p) = \eta_0 \frac{c_p}{R^p},$$

where $\eta_0$ is a non-dimensional constant. Obviously, $\eta(p) \to 0$ as $p \to 0$. And also with the bound by surface on vertical velocity, such a heating profile approximately represents the realistic situation of cumulus heating, which has its strongest impact on the middle troposphere. Therefore, (12) can be rewritten as

$$\frac{\partial \partial \phi}{\partial t \partial p} + \sigma \left(1 - \eta_0 - \eta_0 \epsilon \cos \frac{2\pi T}{t} \right) \omega = - \beta \frac{\partial \phi}{\partial p}.$$  \hspace{1cm} (13)

This suggests a wave-convection feedback somehow resulting in a reduced value of the effective static stability. Such an effective static stability can oscillate over a relatively great amplitude (compared to its mean value) with the seasons if convective heating is comparable to the static energy (i.e. when $\eta_0 \sim 1$). So as a problem of parametric instability the present one may be typical enough.

### 3.2 Mathieu equation model

As was done in the previous section, we can combine Eqs. (10), (11) and (13) into a single equation of $u$:

$$\frac{\partial^2 u_{pp}}{\partial t^2} - \alpha \frac{\partial}{\partial t} \frac{\partial^2 u_{pp}}{\partial p^2} + \beta \frac{\partial u_{pp}}{\partial t} - \alpha \beta \frac{\partial^2 u_{pp}}{\partial p^2} + \sigma \left(1 - \eta_0 - \alpha \epsilon \cos \frac{2\pi T}{t} \right) \frac{\partial^2 u}{\partial x^2} = 0$$  \hspace{1cm} (14)

and the same boundary conditions as (5) are demanded to be satisfied. The convenient mathematical structure (contains the partial derivatives with respect to $p$ of 0th, 2th and 4th only) enjoyed by this equation enable us to choose the normal modes as

$$V(t) \cos \left(\frac{n\pi}{k} - p\right) \exp(ikx); \hspace{0.5cm} n = 1, 2, \ldots$$

The other two boundary conditions for $\omega$ and $\phi$ can still be satisfied easily by the way stated in the previous section. By substituting it in (14) and introducing a transform

$$V(t) = U(t) \exp \left[ -\frac{1}{2} \left(\alpha E_n^2 + \beta \right) t \right]$$

(14) can be reduced to the famous Mathieu equation

$$\frac{d^2 U}{dt^2} + (a - 2q \cos 2\pi \eta_0) U = 0.$$  \hspace{1cm} (15)

Therein, a procedure of non-dimensionalization has been taken by letting $t_0 = \pi t/T$, and $a, q$ denote $T^2/k^2 \sigma(1 - \eta_0) / E_n^2 - (\alpha E_n^2 - \beta^2)/4$ and $T^2 k^2 \sigma \eta_0 \epsilon / 2n^2 E_n^2$, respectively. By the way, we’d like to point out here that many forcing-response problems of a physical or a dynamical system are eventually found to be governed by the Mathieu equation. Our subsequent discussions will be also based on this model.

Since the Mathieu equation covers a wide range of phenomena, many studies have been devoted to the properties of its solution, to which one can refer to Abramowitz and Stegun (1964). The reason why the Mathieu system draws a wide attention is that it possesses properties of super(sub)harmonic resonance (also be termed by parametric instability). Such properties are usually of dynamical interest. Herein, we do not want to emphasize the purely mathematical aspects too much, We just give a brief
Figure 4 is the parametric diagram of the solutions for the Mathieu equation. The diagram is symmetric in $q$ and $-q$. The $a$-axis corresponds to the stable free oscillation, the curves of marginal stability correspond to solution $U(t)$ of period $2T$ or $T$, i.e. of the frequency of the forced oscillation or half that frequency. The unstable solutions (in the shadow domains) oscillate with these periods, whereas their amplitudes increase exponentially. It can be seen that parametric resonance between frequencies of free and forced oscillations occurs for small values of $q$ when $a$ is close to an integer. For larger $q$, the domains of resonance become much wider. In the latter case, the resonant frequencies may be much different from the eigenfrequencies of free oscillations. The resonant frequency sequence is

$$w_l = l\frac{\omega_0}{2}, \quad l = 1, 2, 3, \cdots$$

where $\omega_0 = 2\pi/T$. Among them, $\omega_0/2$ is subharmonic and those with $l > 2$ are superharmonics. The stable solutions (in the white domains) are oscillatory but not periodic for $q \neq 0$, which are called quasi-periodic oscillation (be composed of such frequencies, the ratio of one to another is irrational number).

Because of the effect of strong damping, there is exponential decay to the wave amplitudes, so only those modes with unstable solutions of $U(t)$ can develop. Therefore, the meaningful solutions of Mathieu equation for our problem are unstable ones only. The asymptotic form (when $t$ is large enough) of these unstable solutions is

$$U(t) \sim \exp[\lambda(a, q)t + i\omega_0 t]$$

The calculation of unstable growth rate $\lambda(a, q)$ is somewhat complex. It has something to do with the so-called Floquet theory, about which one can refer to any advanced textbook on ordinary differential equations. In this paper, $\lambda(a, q)$ is calculated according to this theory by using a numerical method. Details of this calculation are not included in here.

3.3 Discussions

The unstable modes can then be written as

$$u \sim \exp \left[ \lambda(a, q) - \frac{1}{2}\alpha E_n^2 + \beta \right] t$$

$$\cdot \cos \left( \frac{n\pi}{P_u} \right) \exp(i\omega_0 t + ikx)$$

In order to overcome damping, $\lambda(a, q)$ is required to satisfy the relation

$$\lambda(a, q) \geq \frac{1}{2}(\alpha E_n^2 + \beta).$$

(16)

On the other hand, the meridional structures and the direction of phase propagation can be determined by the geostrophic relation and meridionally trapping condition, i.e. (6) and (7). A procedure such as was followed in the previous section will give the meridional structure of unstable modes as below

$$Y(y) \sim \exp \left[ \frac{2\beta k}{\lambda \omega_0} k + ik\beta_0(2\lambda + \alpha E_n^2 - \beta) \right] y^2 ,$$

so the unstable modes can finally be written as

$$u(x, y, p, t) \sim \exp \left[ \lambda(a, q) - \frac{1}{2}(\alpha E_n^2 + \beta) \right] t \cdot \cos \left( \frac{n\pi}{P_u} \right) \exp \left[ \frac{2\beta k}{\lambda \omega_0} k + ik\beta_0(2\lambda + \alpha E_n^2 - \beta) \right] y^2 \exp(i\omega_0 t + ikx).$$

The demand of trapping condition (7) requests $k < 0$, which indicates that these modes are eastward-propagating waves. Their phase speeds are $-\omega_1/k$ and, in the meanwhile, since for the unstable modes the relation (16) holds, it is easy to see the factor $2\lambda + \alpha E_n^2 - \beta$ in (17) remains positive. Therefore, there exist constantly poleward-propagating components for the unstable modes described by
There have been also a large number of theoretical studies focusing on the mechanism of the origin, eastward propagation and the spatial structures of ISO (e.g., wave-CISK: Lau and Peng 1987; Takahashi 1987 etc., WISHE; Emanuel 1987). Here, we will just propose our viewpoint on the mechanism of ISO and check whether the main features of ISO can be described well or not by our model. As a matter of course, for a low-frequency oscillation like ISO, it can only be some stationary response to forcing rather than a transient oscillation related to free modes due to the existence of a strong damping effect. Hence, in interpreting the origin of the period of 30-60 days, the eigenfrequency is regarded to be of secondary importance, and in our present framework ISO is naturally owed to the superharmonic response to the seasonal cycle of tropical atmosphere, although this is different from the result of a GCM simulation conducted by Hayashi and Golder (1993) which argues that ISO can be simulated even in the absence of the annual cycle. Within the broad range from 30 to 60 days, superharmonics of semiannual cycle (related to the migrating of the ITCZ across the equator twice a year in the eastern hemisphere) i.e.

\[
\frac{2\pi}{2T}, \quad T = \text{half one year}; \quad l = 6, 7, \ldots, 12
\]

and superharmonics of one year cycle (related to the seasonal variation of the ITCZ which always remains north to the equator in the western hemisphere) i.e.

\[
\frac{2\pi}{2T}, \quad T = \text{one year}; \quad l = 12, 13, \ldots, 24
\]

are contained. (However, for the problem of ISO, the semiannual cycle of seasonal variation seems more important than the annual cycle, since the latter is too far from the periods of ISO and the problem may be atypical.) It is easy to imagine that as these superharmonics are so near to each other that they become indistinguishable in any spectral analysis with limited precision and thus appear to be continuous in the power spectrum.

Since there are so many superharmonics within the broad range from 30 to 60 days, it seems too tedious to undergo an over-all investigation on every one of them. Here, instead of giving out all the possible unstable modes, we will just check those modes with the main feature of the spatial structures of ISO to see whether they are or not in the unstable domains defined by (16). This can be conducted without difficulty because of the broad ranges of superharmonics within 30-60 days. Hereafter, we denote the modes by a pair of integers \((m, n)\), where \(m\) is the zonal wavenumber while \(n\) is the order of vertical modes.

We now set the necessary parameters in this analysis. The static stability \(\alpha\) is chose to be

\[
\exp \left[ i\omega t + \frac{k\beta_{0}(2\lambda + \alpha E_{\lambda}^{2} - \beta)}{(2\lambda + \alpha E_{\lambda}^{2} - \beta)^{2} + 4\omega^{2}y^{2}} \right].
\]

Obviously, this poleward propagation results from the effect of either damping or the unstable growth. The poleward phase speed is

\[
c_{y} = \left( \frac{\partial y}{\partial t} \right)_{\theta} = -\frac{\partial \theta}{\partial t} \frac{\partial \theta}{\partial y} = \frac{\omega_{l}(2\lambda + \alpha E_{\lambda}^{2} - \beta)^{2} + 4\omega^{3}}{2k\beta_{0}(2\lambda + \alpha E_{\lambda}^{2} - \beta)y},
\]

where \(\theta\) denotes the phase. The poleward phase speed is \(y\)-dependent, and it becomes infinite on the equator and tends to zero when \(y\) is great enough. The Rossby transform radius is also enlarged to

\[
L_{0} = \frac{\omega_{l}}{\beta_{0}[k]} + \frac{(2\lambda + \alpha E_{\lambda}^{2} - \beta)^{2}}{4\beta_{0}\omega_{l}[k]}.
\]

Within the tropical region bound by \(L_{0}\), we may expect to find the signals of these super(sub)harmonic oscillations. Although features of these modes have some similarities to that of classical Kelvin waves, there is an essential difference between them. That is, the former is a stationary response to a periodic forcing and the frequencies of such kinds of oscillation are decided by (or locked to) the super(sub)harmonics of the period of forcing rather than eigenfrequencies which is the case of classical Kelvin wave. Even if the spatial structures of these two kinds of oscillations are the same, their frequencies and phase speeds may be quite different from each other.

4. Superharmonics and intraseasonal oscillation

In their pioneering work, Madden and Julian (1971, 1972) first detected the 30-60 day oscillation or intraseasonal oscillation in the tropical troposphere. Since then many researchers have been devoted to studying its structure and propagation. Based on these works, the key features of the intraseasonal oscillation may be synthesized as follows: (i) There is a predominance of low-frequency oscillations in the broad range from 30 to 60 days; (ii) The oscillations have predominant zonal scales of wavenumbers 1, 2 and 3. Winds and geopotentials in the upper troposphere are out of phase with those in lower troposphere; (iii) The oscillations propagate eastward along the equator at a slow phase speed of about 10 m/s; (iv) Strong convection is confined to the equatorial regions of the Indian Ocean and western Pacific sector, while the wind pattern appears to propagate around the globe; (v) There is a marked northward propagation of the disturbance over India and East Asia during the northern summer monsoon season and, to a lesser extent, southward penetration over north Australia during the northern winter.
4 × 10^{-6} \text{ kg}^{-2} \text{m}^4 \text{s}^{-2}, \text{ which is the mean value of troposphere. The coefficient for friction } \alpha \text{ may be chose in such way that the damping time scale of vertical modes with } n = 1 \text{ is let to be 1 week and that for radiative damping } \beta \text{ is 3 weeks. In the eastern hemisphere } T = \text{ half one year}, \text{ the cumulus heating is quite strong, so parameter } \eta_0 \text{ is set to be 0.9 and the seasonal contrast is not so dominant, so we can let } \epsilon = 0.3. \text{ The results of calculation shows that only mode } (1,1) \text{ is an unstable mode of ISO time scale. The growth rate is } 1/6 \text{ days, greater than that of damping which is } 1/12 \text{ days. It has a period of about 45 days and a phase speed of } 10 \text{ m/s for eastward propagation. This mode exhibits the main spatial structure of ISO, i.e. zonal wavenumber 1 and the out-of-phase vertical structure between the winds in lower and upper troposphere; the latter seems simply due to the cosine structures of vertical normal modes. The low phase speeds, which coincide with that of observation, are the result of both the damping effect in the way we stated in Section 2 and the reduction of the static stability by convection. Because of the differences among the phase speeds of unstable modes, our model ISO will surely have a dispersive character.}

As was mentioned in the previous section, unstable modes keep on propagating poleward, and it is then natural for us to suppose that this poleward propagation may account for the similar property of ISO. The meridional phase speed at 15°N or 15°S of the model ISO is 0.8 m/s while that observed in the summer monsoon region is about 1.3 m/s. We also suppose that the realistic phase speeds observed are the result of Doppler-shift by the meridional velocities of monsoons and thus there is a marked northward propagation of the disturbance during the northern summer monsoon season and southward penetration during northern winter. However, further study on this topic is difficult and beyond our present framework.

Finally, we have to face a problem of great importance. That is, the mechanism of scale selection. Our model does not exclude the possibility of the unstable increase of high-order modes (those with large } m \text{ or } n), \text{ whereas the observed ISO usually has a structure of low-order modes } (m = 1, 2, 3; n = 1, 2). \text{ In fact, it seems natural that those modes with shallower vertical structure or shorter zonal scales possess relatively sharper wind shear, so it is easier for the diffusion of momentum due to cumulus convection and thus the trend to damp more rapidly. As the result, only those high-order modes with higher frequencies can be unstable (this instability might be mainly due to the inclusion of the wave-convection feedback rather than parametric instability), while for those high-order modes with the low-frequency ISO, exponential increase rates } \lambda \text{ can no longer cancel out the greater damping rates, so low-frequency oscillation of ISO of high-order mode will hardly, if at all, be found.}

5. Subharmonic and tropospheric QBO

The quasi-biennial oscillation (QBO) of the zonal wind observed in the tropical stratosphere is one of the predominant phenomena for the explanation of which, Holton and Lindzen (1968, 1972) developed a theory which proposed a mechanism of the interaction between the waves of the eastward-moving Kelvin waves and the westward-moving Rossby-gravity waves from the troposphere and the zonal mean flow in the stratosphere. On the other hand, QBO is also observed in the tropical troposphere (Trenberth, 1975; Ebdon, 1975). Relations between the QBO in the stratosphere and that in troposphere was studied by some researchers, e.g. Yasunari (1989), Xu (1992), but it is not so far quite certain what these relations are.

While the QBO in the stratosphere has no zonal structure, the tropospheric QBO has a zonal structure of wavenumber one or two with eastward propagation and an upward-propagating vertical structure. It also appears to be phase-locked to the seasonal cycle (Yasunari (1989)). These features of the tropospheric QBO strongly suggest some connection between it and the subharmonic response (corresponding to } 2T \text{) of Kelvin modes to forcing with an annual cycle which we dealt with above. So we suppose that this subharmonic response with a period of two years may be responsible for the origin of the tropospheric QBO. We have noticed the work of Brier (1978) which also suggested such a link, but no further analytical investigation was given there.

We consider now } T = \text{ one year}, \text{ which is mainly the case for the eastern hemisphere. The cumulus convection there is not so active and the seasonal contrast of it is relatively dominant and we thus set } \eta_0 = 0.7 \text{ and } \epsilon = 0.4. \text{ The damping time scale of modes with } n = 1 \text{ now is close to be } 1.5 \text{ weeks, for the cumulus friction is also not so strong as that in western hemisphere. The rest of parameters remain the same as those in the previous section. Calculation shows that with the parameters above, mode } (1,2) \text{ becomes the unstable mode corresponding to the subharmonic resonance of two years, and its growth rate is } 1/3.6 \text{ days while that of damping is } 1/4.8 \text{ days. It was clearly demonstrated that besides the agreement of the periods of about two years, the features of zonal structure, eastward propagation, and the phase-lock to the annual cycle are very well simulated by our model.}

However, mode } (1,2) \text{ has a 'second-baroclinic structure that is characterized by lower-level and upper-level vertical velocities with opposite signs, which is contrary to that of the observed tropospheric QBO. This defect could be alleviated by incorporating the effect of air-sea interactions, al-
through it is beyond the scope of the present paper to examine this possibility. On the other hand, the boundary condition for $a$ seems too strict a limitation to permit vertical propagation of modes with relatively shorter vertical wavelength. By giving up such a boundary condition, we can write the vertical modes as $\exp(ihp)$; $h$ here is the vertical wavenumber. The rest of the analysis almost remains the same and a vertical propagation (both upward and downward) mode can be obtained. However, it's still impossible for our model to give a reasonable interpretation for the upward-only propagation of the tropospheric QBO.

6. Summary and conclusions

By introducing the concepts of super(sub)-harmonic resonance (or parametric instability), we have investigated the responses of tropical Kelvin waves to the cumulus heating with a seasonal modulation and suggested some possible mechanism for the excitation of the intraseasonal oscillation and tropospheric QBO.

Here, what we want to lay stress on about ISO is that in our present model two kinds of instabilities are included, that is, the wave-convection positive feedback and parametric instability. Both of them contribute to the growth rate of unstable modes, however, the present wave-convection feedback can not result in an oscillatory instability by itself, while parametric instability is always oscillatory and then mobile. With this kind of instability, unstable modes can be moving ones. And also we think with a suitable form of friction taking the discrepancies of different modes into account, the mechanism of scale selection of ISO can be included.

Except for the vertical structure and upward propagation, features of troposphere QBO were exhibited quite well by the subharmonic resonance modes. This at least constrained the authors to attach an importance to the two-year subharmonic of the annual cycle in the studies of QBO in troposphere, no matter what manner the impact of the annual cycle may actually be. Even if no coupling between air and sea, like Jin et al. (1994) and Tziperman et al. (1994), is considered, the subharmonic response of the atmosphere itself to the seasonal cycle may still exist.

Acknowledgments

The authors wish to thank Prof. Y. Hayashi and S. Miyahara for reading the manuscript and offering very helpful suggestions. Zhao, one of the authors, is grateful to Prof. Y.H. Ding in National Climate Center of China for his invaluable encouragement through recent years.

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加熱の季節変化に対する熱帯ケルビン波の高(低)調波応答

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ダンピングを持つケルビン波に季節変化している凝結加熱を加えて、ケルビン波のパラメトリック不安定または高(低)調波応答を調べた。季節内振動と対流圈QBOの励起はこの季節変化に対する高(低)調波応答と関係しているかもしれない。この見方を確認するために、共振モードの空間構造と伝播についての主な特徴を示し、実際の対流圈QBO及び季節内振動と比較する。