A Balanced Ageostrophic Initialization with a Fixed External Wind Boundary Value for Limited-Area Models¹

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Abstract

The wind in a limited region can be separated into the external and internal winds, which depend upon the vorticity and divergence only outside and inside the region, respectively. This is the basic property of the wind separation.

A given case that the vorticity and divergence are given within the region and the wind is prescribed at the boundary is used to represent a common feature that is often produced in the prediction of a limited-area model and its initialization. If the estimated external wind for a given case satisfies the consistency condition at the boundary, the given case is a consistent case; otherwise, it is an inconsistent case. For an inconsistent case, it is impossible to find a continuous wind that satisfies the wind given at the boundary and yields the vorticity and divergence in agreement with those given within the region. A direct method with two Poisson equations of reconstructing the wind for a given case can only be used if the given case is a consistent case.

The implicit normal-mode initialization (Temperton, 1988) is further illustrated from the wind reconstruction and wind separation. The goal of the initialization is to modify the analyzed ageostrophic wind to become the balanced ageostrophic wind. Only the imbalanced ageostrophic wind causes the fast gravity-inertia waves, and needs to be filtered in the initialization. From one computed example, the balanced ageostrophic wind is about 68 % of the ageostrophic wind, thus it is a good approximation to the ageostrophic wind. It is also found that the rotational component of the ageostrophic wind (3.38 ms^{-1}) is larger than that of the divergent component (1.64 ms^{-1}). The balanced wind is the sum of the geostrophic wind and balanced ageostrophic wind, and it is a good approximation to the observed wind.

It is proved that the fixed wind components at the boundary cannot be used in the initialization because the initialized case in this situation is an inconsistent case. Based on the basic property of the wind separation, it is very natural to use the fixed external wind as the lateral boundary condition in the initialization. The proposed initialization and boundary condition have been tested and the results show that the meteorological noise can be effectively removed at the start of the forecast.

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1. Introduction

In some numerical weather prediction and climate studies, the detailed features we need to know about are in a limited region rather than over the whole globe. These studies involve a problem solved only in a limited region or by a limited-area model. Such a problem requires lateral boundary conditions, and it is often very difficult to solve.

For example, partitioning a wind field into non-divergent and irrotational components for the globe can easily be derived without any lateral boundary condition by solving the Poisson equations,

$$\begin{aligned}\nabla^2\psi &= \zeta = \frac{1}{a \cos \phi} \left(\frac{\partial v}{\partial \lambda} - \frac{\partial u \cos \phi}{\partial \phi} \right), \\ \nabla^2\chi &= \delta = \frac{1}{a \cos \phi} \left(\frac{\partial u}{\partial \lambda} + \frac{\partial v \cos \phi}{\partial \phi} \right),\end{aligned}\quad (1.1)$$

where u and v are the zonal and meridional wind components, respectively, and λ is longitude, ϕ , latitude, ψ , streamfunction, χ , velocity potential, ζ , vorticity, δ , divergence and a , earth radius.

However, in a limited region, the above partitioning problem needs to solve the Poisson equations

$$\begin{aligned}\nabla^2\psi &= \Omega = -\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}, \\ \nabla^2\chi &= D = -\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y},\end{aligned}\quad (1.2)$$

with the following boundary conditions

$$\begin{aligned}\left(\frac{\partial \psi}{\partial n} + \frac{\partial \chi}{\partial s} \right) \Big|_{\Sigma} &= V_t|_{\Sigma}, \\ \left(-\frac{\partial \psi}{\partial s} + \frac{\partial \chi}{\partial n} \right) \Big|_{\Sigma} &= V_n|_{\Sigma},\end{aligned}\quad (1.3)$$

where Σ denotes the boundary of the limited region, and V_t and V_n are tangential and normal components of the wind, respectively, at the boundary, and s and n are distances along and normal to the boundary. In (1.2), U , V , Ω , and D are denoted by

$$U = \frac{u}{m}, \quad V = \frac{v}{m}, \quad \Omega = \frac{\zeta}{m^2}, \quad D = \frac{\delta}{m^2} \quad (1.4)$$

and the vorticity and divergence are expressed by

$$\begin{aligned}\zeta &= m^2 \left[\frac{\partial}{\partial x} \left(\frac{v}{m} \right) - \frac{\partial}{\partial y} \left(\frac{u}{m} \right) \right], \\ \delta &= m^2 \left[\frac{\partial}{\partial x} \left(\frac{u}{m} \right) + \frac{\partial}{\partial y} \left(\frac{v}{m} \right) \right]\end{aligned}\quad (1.5)$$

respectively, and m , the map scale factor.

The boundary conditions (1.3) are neither the Dirichlet nor the Neumann boundary conditions. The above problem has been studied by many authors, especially by Lynch (1988, 1989). Recently, Chen and Kuo (1992b) (hereafter CK92b) referred to the conditions (1.3) as the coupled boundary conditions. The problem of solving two Poisson equations (1.2) with the coupled boundary conditions

(1.3) is more difficult than solving two Poisson equations (1.1) for the globe. This is one example showing the difficulty of solving a boundary value problem for a limited area. The same difficulty also occurs in a limited-area model and its initialization, especially when the streamfunction and velocity potential are used.

In the recent studies of Chen and Kuo (1992a) (hereafter CK92a) and CK92b, the wind in a limited region is separated into the harmonic wind and internal wind. From a global perspective, the harmonic wind in a limited area does not depend on the vorticity and divergence within the region, but represents the effect of the vorticity and divergence outside the region. Due to this property, the harmonic wind is renamed as the external wind in this paper.

It is found from CK92a and CK92b that the difficulty of solving the boundary value problems for a limited region is primarily caused by the external wind. If the wind separation is used and the external wind is dealt with appropriately, the boundary value problem in a limited region can easily be solved. This method has been successfully used for the wind partitioning and reconstruction problems in limited regions, and it can also be used for a limited-area model and its initialization. For the globe, the external wind always vanishes, and no difficulty occurs at the boundary.

The main purpose of this paper is to apply the wind separation and wind reconstruction to the initialization of a limited-area model. However, in order to apply the wind separation and wind reconstruction conveniently to limited-area models and their initializations, some problems discussed in CK92a and CK92b need to be expanded. The basic property of the wind separation is defined in Section 2. This property provides the physical basis for solving the boundary value problem in a limited region. A natural method for obtaining the external wind from the analyzed wind is proposed. In Section 3, we use a concept of a given case to represent a common feature that often occurs in the prediction of a limited-area model and its initialization. The consistent and inconsistent cases are further defined based on the given case. If the direct method of wind reconstruction with two Poisson equations is used for the given case in a limited region, the boundary value must satisfy a certain condition. Recently, many meteorologists use this direct method in a limited region without considering the condition, thus, the necessary requirement of using the direct method for a given case is defined in Section 3.

Non-linear normal mode initialization (NNMI), which was developed by Machenhauer (1977) and Baer (1977), is now widely used. It has played a major part in the improvement of global numerical

weather prediction and continuous data assimilation (Bengtsson, 1991), but not, as yet, for limited-area models and regional data assimilation.

Briere (1982) applied the NNMI to limited-area models and Bourke and McGregor (1983) introduced a vertical mode initialization for limited regions. Temperton (1988) used the physical variables in his implicit normal mode initialization without knowledge of the normal modes themselves. The physical variables derived by Temperton (1988) are further examined for middle latitudes from the wind reconstruction and wind separation in this paper, and it is found that the purpose of the initialization is to modify the analyzed ageostrophic wind to become the balanced ageostrophic wind. The fast gravity-inertia waves in the model are caused by the imbalanced ageostrophic wind. Thus, this initialization is referred to as a balanced ageostrophic initialization. Some detailed structures of the ageostrophic wind can be shown clearly using this method.

The lateral boundary conditions for the streamfunction, ψ , and velocity potential, χ , used by Temperton (1988) to ensure no cross-boundary flow are

$$\psi|_{\Sigma} = 0, \quad \vec{n} \cdot \nabla \chi|_{\Sigma} = 0 \quad (1.6)$$

where Σ is the boundary of the limited-area model and $\vec{n}$ its outward-pointing normal vector. It is obvious that the boundary condition (1.6) cannot satisfy the actual values at the boundary if a limited-area model is nested in a global model. Fixed values (which mean that the values do not change before and after the initialization) of the wind and geopotential height at the boundary are used by Errico and Bates (1988). Briere (1982) also set the wind and geopotential fields to be constant with time at the boundary. It is proved in Section 4 that a fixed wind at the boundary cannot be used for the initialization in a limited region. However, based on the basic property of the wind separation, it is very natural to use a fixed external wind as the lateral boundary condition in the initialization, which is shown in Section 5.

By using the above boundary condition, the balance equations are transformed into the equations of the inner parts, which is discussed in Section 6. Because the Dirichlet boundary value of a variable for the harmonic-sine series expansion is simpler than the Neumann boundary value for the harmonic-cosine one, the harmonic-sine series are used here. In this case, the boundary value of the inner part equations are homogeneous, and they can easily be solved by the double sine series. An iterative procedure for solving the balance equations is discussed in Appendix A.

Some structures of the ageostrophic wind in the initialization are discussed in Section 5 and 7 based

on the computed results. An example of the balanced ageostrophic initialization and its test in a shallow water model are shown in Section 7.

Leith (1980) showed that the quasi-geostrophic initialization was a special case of the NNMI. Because the NNMI takes the form of the balanced ageostrophic initialization, it is easy to prove that the quasi-geostrophic initialization is a special case, which is shown in Appendix B.

2. The basic property of the wind separation in a limited region

a. The wind separation and its basic property

According to CK92a, a function defined on a limited region R bounded by Σ can be divided into the harmonic and inner parts. The streamfunction and velocity potential are written as

$$\psi = \psi_h + \psi_i, \quad \chi = \chi_h + \chi_i \quad (2.1)$$

where the inner parts, ψ_i and χ_i , satisfy

$$\nabla^2 \psi_i = \Omega, \quad \nabla^2 \chi_i = D \quad (2.2)$$

with zero Dirichlet boundary value, and ψ_h and χ_h are the harmonic parts. The internal wind is computed from the inner parts by

$$u_I = -\frac{\partial \psi_i}{\partial y} + \frac{\partial \chi_i}{\partial x}, \quad v_I = \frac{\partial \psi_i}{\partial x} + \frac{\partial \chi_i}{\partial y} \quad (2.3)$$

The global area is divided into two regions: one is the limited region R ; and the other is the area outside the region R , which is referred to as the external region. From a global perspective, the streamfunction and velocity potential in the region R depend upon the vorticity and divergence not only of the region R , but also of the external region. It is seen from (2.2) and (2.3) that no matter how the vorticity and divergence vary in the external region, the inner parts of the streamfunction and velocity potential, as well as the internal wind, do not vary. Thus, the inner parts and internal wind depend only upon the vorticity and divergence within the region R .

The external wind is defined as a difference between the total wind and the internal wind by

$$u_E = U - u_I, \quad v_E = V - v_I. \quad (2.4)$$

Thus, the external wind can be expressed by

$$u_E = -\frac{\partial \psi_h}{\partial y} + \frac{\partial \chi_h}{\partial x}, \quad v_E = \frac{\partial \psi_h}{\partial x} + \frac{\partial \chi_h}{\partial y} \quad (2.5)$$

where the harmonic parts satisfy Laplace equations

$$\nabla^2 \psi_h = 0, \quad \nabla^2 \chi_h = 0 \quad (2.6)$$

with the coupled boundary conditions

$$\begin{aligned} u_E|_{\Sigma} &= \left(-\frac{\partial \psi_h}{\partial y} + \frac{\partial \chi_h}{\partial x} \right) \Big|_{\Sigma}, \\ v_E|_{\Sigma} &= \left(\frac{\partial \psi_h}{\partial x} + \frac{\partial \chi_h}{\partial y} \right) \Big|_{\Sigma}. \end{aligned} \quad (2.7)$$

Fig. 1. The wind components specified at the boundary and the vorticity and divergence given within the region.

Within the region R , the external wind is both non-divergent and irrotational. No matter how the vorticity and divergence vary within the region R , the external wind does not change. Thus, the external wind has no relation to the vorticity and divergence within the region. From a global view, the external wind depends only on the vorticity and divergence in the external region.

Thus, the separation of the wind into the external and internal components has an important physical implication. It represents a separation of the wind into two parts so that each part is related to the vorticity and divergence in only one region. The internal wind is related to the vorticity and divergence only within the limited region, while the external wind depends on those only in the external region. This property may be referred to as the basic property of the wind separation. This property can be used as a physical basis for solving boundary value problems in a limited region.

b. The consistency condition of the external wind

As discussed in CK92b, the two components of the external wind are not only harmonic functions, but also harmonic conjugates. Thus, they are mutually dependent at the boundary. If one is known at the boundary, the other is also determined except for a constant. This is the consistency condition for the external wind at the boundary.

c. A natural method for obtaining the external wind

An external wind can be derived from a given analyzed wind by the following procedures: (1), the vorticity and divergence are computed from a given analyzed wind; (2), the inner parts of the streamfunction and velocity potential are solved from (2.2) based on the computed vorticity and divergence; (3), the internal wind is then computed from the inner parts by (2.3); and finally (4), the external wind is computed from the difference between the internal

and analyzed winds by (2.4). The above procedures are referred to as a natural method for obtaining the external wind.

By using the above procedure, the internal wind is derived from the computed vorticity and divergence within the region. Thus, the external wind derived by (2.4) must be both non-divergent and irrotational in the region. Therefore, the external wind derived by the natural method always satisfies the consistency condition at the boundary automatically. The natural method is very useful for deriving the external wind in a limited region.

d. An inner part adjusted solution

If a wind is both non-divergent and irrotational in a limited region, the two components of the wind must satisfy two Laplace equations; conversely, if the components of a wind satisfies two Laplace equations with Dirichlet boundary values

$$u_E|_{\Sigma}, \quad v_E|_{\Sigma}, \quad (2.8)$$

the wind may not both non-divergent and irrotational. It has been proved in CK92b that, if the direct method with two Laplace equations is used to reconstruct the external wind in a limited region, the necessary and sufficient condition is that the prescribed boundary wind (2.8) satisfies the consistency condition.

If the given boundary value (2.8) does not satisfy the consistency condition, we cannot find a continuous external wind that satisfies the given values at the boundary and is both non-divergent and irrotational within the region. In other words, such an external wind does not exist. However, in this case, the solutions of two Laplace equations with the boundary condition (2.8) can also be uniquely determined; but they are not both non-divergent and irrotational, and must have some vorticity and divergence within the region. Thus, these solutions are referred to as an inner part adjusted solution. In these solutions, the vorticity and divergence, as well as the inner parts of the streamfunction and velocity potential, within the region are adjusted to be compatible with the inconsistent wind at the boundary.

3. Consistent and inconsistent cases, and the requirement of using the direct method for reconstructing wind in a limited region

a. A predicted case and an initialized case

A concept of a given case where the vorticity and divergence are given within the region and the wind is prescribed at the boundary, as shown in Fig. 1, occurs in many situations. For example, if a limited-area model is nested in a global model, the given case shown in Fig. 1 is produced at each time step when the vorticity and divergence within the region and the wind at the boundary are predicted by the

limited-area and global models, respectively. The given case produced in this situation is called a predicted case.

The given case can also be produced in the initialization for a limited region. In the limited-area initializations studied by Briere (1982), Errico and Bates (1988) and others, at the beginning of the initialization, the vorticity and divergence are computed from the analyzed wind. If the wind at the boundary is kept unchanged, as used by Briere (1982) and Errico and Bates (1988), after the initialization process is completed, the vorticity and divergence within the region are modified through solutions of some balance equations; the given case like Fig. 1 is also produced after the initialization. The given case produced in the initialization is called an initialized case.

For either the predicted case or the initialized case, it is necessary to find a continuous wind field that satisfies the wind value specified at the boundary and yields vorticity and divergence in agreement with those given within the region.

What is the condition under which a continuous wind exists for the given case? This question can only be answered based on the properties of the external wind.

b. An estimated external wind at the boundary

For the given case as shown in Fig. 1, the inner parts, ψ_i and χ_i , can be derived from the given vorticity and divergence within the region by solving (2.2), and the internal wind, u_I and v_I , are then computed by (2.3). An estimated external wind at the boundary is derived by

$$u'_E|_\Sigma = U|_\Sigma - u_I|_\Sigma, \quad v'_E|_\Sigma = V|_\Sigma - v_I|_\Sigma. \quad (3.1)$$

Because the computed wind (3.1) at the boundary may not satisfy the consistency condition, we use the term of the estimated external wind instead of the external wind.

c. A consistent case and an inconsistent case

If the estimated external wind at the boundary satisfies the consistency condition, the given case shown in Fig. 1 is referred to as a consistent case. If does not, the given case shown in Fig. 1 is referred to as an inconsistent case.

What does the inconsistent case mean?

As shown in Section 2, in an inconsistent case, we cannot find an external wind which is both non-divergent and irrotational within the region and satisfies an inconsistent estimated external wind at the boundary. Because the wind field in a limited region is the sum of the external and internal winds, an inconsistent case means that it is impossible to find a continuous wind field that satisfies the wind given at the boundary and yields vorticity and divergence in agreement with those given. In other words, such

a continuous wind field does not exist. Thus, the consistent case is a necessary requirement for the predicted case of a limited-area model at each time step and the initialized case of an initialization.

The consistent case can be described in another way in that the given wind at the boundary and the vorticity and divergence within the region are consistent or compatible with each other.

d. The condition of using a direct method with two Poisson equations to reconstruct the wind in a limited region

If we take Laplacian of the equations

$$U = -\frac{\partial\psi}{\partial y} + \frac{\partial\chi}{\partial x}, \quad V = -\frac{\partial\psi}{\partial x} + \frac{\partial\chi}{\partial y} \quad (3.2)$$

and utilize

$$\nabla^2\psi = \Omega, \quad \nabla^2\chi = D, \quad (3.3)$$

we have

$$\nabla^2 U = -\frac{\partial\Omega}{\partial y} + \frac{\partial D}{\partial x}, \quad \nabla^2 V = \frac{\partial\Omega}{\partial x} + \frac{\partial D}{\partial y}. \quad (3.4)$$

The method of solving two Poisson equations (3.4) with the Dirichlet boundary values $U|_\Sigma$ and $V|_\Sigma$ as shown in Fig. 1 is referred to as a direct method with two Poisson equations. It is a generalization of the direct method with two Laplace equations discussed in CK92b. The direct method (3.4) can easily be used to reconstruct the wind over the globe without any boundary condition. However, if this direct method is used to reconstruct the wind for the given case, the boundary value must satisfy a certain condition.

Recently this direct method was used without considering the condition by many meteorologists, for example, Lee and Browning (1994), Juang and Kanamitsu (1994) and others; thus, the necessary requirement for using this method needs to be illustrated. This requirement is not for the solution to be unique, but for guaranteeing that the derived wind yields the divergence and vorticity in agreement with those given. Thus, this requirement results from physics rather than mathematics.

Based on (3.1), the boundary values, $U|_\Sigma$ and $V|_\Sigma$, can be written as

$$U|_\Sigma = u_I|_\Sigma + u'_E|_\Sigma, \quad V|_\Sigma = v_I|_\Sigma + v'_E|_\Sigma. \quad (3.5)$$

The solutions of the Poisson equations (3.4) with the Dirichlet boundary value (3.5) can be considered as the sum of the following two solutions: one is U_1 and V_1 , which are the solutions of the Poisson equations

$$\nabla^2 U_1 = -\frac{\partial\Omega}{\partial y} + \frac{\partial D}{\partial x}, \quad \nabla^2 V_1 = \frac{\partial\Omega}{\partial x} + \frac{\partial D}{\partial y}. \quad (3.6)$$

with the internal wind at the boundary

$$U_1|_\Sigma = u_I|_\Sigma, \quad V_1|_\Sigma = v_I|_\Sigma. \quad (3.7)$$

The other is U_2 and V_2 , which are the solutions of two Laplace equations

$$\nabla^2 U_2 = 0, \quad \nabla^2 V_2 = 0, \quad (3.8)$$

with the estimated external wind at the boundary

$$U_2|_{\Sigma} = u'_E|_{\Sigma}, \quad V_2|_{\Sigma} = v'_E|_{\Sigma}. \quad (3.9)$$

Because the boundary condition of the internal wind (3.7) is directly derived from the vorticity and divergence within the region through (2.2) and (2.3), the solution U_1 and V_1 must be the internal wind and yield the vorticity and divergence in agreement with those given.

If the given case is a consistent case, the estimated external wind at the boundary satisfies the consistency condition. Based on Section 2, the solution U_2 and V_2 must be both non-divergent and irrotational. Thus, in the consistent case, the sum of two solutions, $U_1 + U_2$ and $V_1 + V_2$, satisfies the given boundary condition and yields the vorticity and divergence in agreement with those given.

If the given case is an inconsistent case, the solutions U_2 and V_2 are not both non-divergent and irrotational and have some additional vorticity and divergent within the region. Thus, the sum of the two solutions, $U_1 + U_2$ and $V_1 + V_2$, cannot yield the vorticity and divergence in agreement with those given.

Thus, if the direct method (3.4) and (3.5) is used to reconstruct the wind within a limited region for a given case, a necessary and sufficient condition is that the given case is a consistent case.

If the given case is not a consistent case, as discussed in Section 2, the solutions U_2 and V_2 are the inner part adjusted solution. Thus, the vorticity and divergence yielded by the solution $U_1 + U_2$ and $V_1 + V_2$ of the direct method must be different from those given. The solution $U_1 + U_2$ and $V_1 + V_2$ in this case can also be referred to as an inner part adjusted solution because the vorticity and divergence within the region have been modified and adjusted.

4. Fixed wind cannot be used as the boundary condition for the initialization in a limited region

a. The initialization for a limited-area shallow water model

In a baroclinic model, the linear parts of the model equations can be vertically decomposed into many vertical modes (Bourke and McGregor 1983). For each of the vertical modes, the model equations have the same form as shallow water equations with a different equivalent depth, h_0 . In the present paper, we illustrate our initialization procedure for a limited-area shallow water model.

According to Chen *et al.* (1991), the shallow water model can be written in the form

$$\frac{\partial \Omega}{\partial t} + f_0 D = \Omega_{adv} \quad (4.1)$$

$$\frac{\partial D}{\partial t} - f_0 \Omega + \nabla^2 \phi' = D_{adv} \quad (4.2)$$

$$\frac{\partial \phi'}{\partial t} + \Phi_0 m_0^2 D = \phi_{adv} \quad (4.3)$$

where

$$\Omega_{adv} = -(f' + m^2 \Omega) D - \left(U \frac{\partial(f' + m^2 \Omega)}{\partial x} + V \frac{\partial(f' + m^2 \Omega)}{\partial y} \right) \quad (4.4)$$

$$D_{adv} = (f' + m^2 \Omega) \Omega + V \frac{\partial(f' + m^2 \Omega)}{\partial x} - U \frac{\partial(f' + m^2 \Omega)}{\partial y} - \nabla^2 \left(\frac{m^2}{2} (U^2 + V^2) \right) \quad (4.5)$$

$$\phi_{adv} = -m^2 \left[U \frac{\partial \phi'}{\partial x} + V \frac{\partial \phi'}{\partial y} + \phi' D \right] - \Phi_0 (m^2)' D \quad (4.6)$$

are the terms of the non-linear advection. We assume that

$$f = f_0 + f' \quad (4.7)$$

$$m^2 = (m^2)_0 + (m^2)' = m_0^2 + m^{2'} \quad (4.8)$$

where f_0 and m_0^2 are the mean values of f and m^2 over the integration region, and f' and $m^{2'}$ are deviations from them, respectively. The geopotential height $\Phi = \Phi_0 + \phi'$, where $\Phi_0 = gh_0$ is constant and ϕ' is its deviation. h_0 is the equivalent depth of the shallow water model.

b. The balance conditions for the physical variables

The goal of initialization is to modify the initial state so that the model integration from the initialized state has none or a minimum of fast gravity-inertia modes. For conventional normal mode initialization (Machenhauer 1977; Baer and Tribbia 1977), the normal modes are derived from the linear parts of the model by setting Ω_{adv} , D_{adv} and ϕ_{adv} equal to zero, and then the model equations are transformed into normal mode predictive equations. The normal modes are divided into a set of fast normal modes and a set of slow normal modes. In the initialization process, the slow modes are unchanged but the fast normal modes are modified to satisfy appropriate balance conditions (Juvanon Du Vachat 1988; Daley 1991).

Temperton (1988) showed that the NNMI can be implemented without knowing the normal modes. In his implicit normal mode initialization, the horizontal structure of the normal modes is no longer specified explicitly, and the corresponding calculation of Machenhauer's (1977) scheme in the normal mode space is transformed into the physical space

or the physical variables, which involve the vorticity and divergence.

Because the wind field in a limited region can be reconstructed from the vorticity and divergence, and can be separated into the internal and external winds, the physical variables used by Temperton (1988) are further illustrated from the wind reconstruction and wind separation in this paper. If the wind separation is used, it is very easy to specify the lateral boundary conditions for the initialization.

Let Ω_{adv} and ϕ_{adv} be zero in (4.1) and (4.3), and eliminating D , we have

$$\frac{\partial q}{\partial t} = 0 \quad (4.9)$$

where

$$q = \Omega - \frac{f_0}{c^2} \phi' \quad (4.10)$$

is the linearized potential vorticity, and $c = m_0 \sqrt{\phi_0}$ is the speed of the gravity wave. The solution of the potential vorticity in (4.9) is stationary, and it is a constraint that the Rossby or rotational mode is unaffected by the initialization (Machenhauer, 1977; Bourke and McGregor, 1983).

The predictive equations for the two fast modes of the model equations (4.1)–(4.3) are

$$\frac{\partial \Omega'}{\partial t} - \frac{c^2}{f_0} \nabla^2 D + f_0 D = \Omega'_{adv} \quad (4.11)$$

and

$$\frac{\partial D}{\partial t} - f_0 \Omega' = D_{adv} \quad (4.12)$$

where Ω' is the ageostrophic vorticity denoted by

$$\Omega' = \Omega - \frac{1}{f_0} \nabla^2 \phi' \quad (4.13)$$

In this paper, only the initialization in middle latitudes is discussed. The term Ω'_{adv} is expressed by

$$\Omega'_{adv} = \Omega_{adv} - \frac{1}{f_0} \nabla^2 \phi_{adv} \quad (4.14)$$

and it is referred to as a variation rate of the ageostrophic vorticity caused by advection.

The Machenhauer's condition (1977) requires that the tendencies of the amplitudes of the fast gravity modes vanish, thus,

$$\left. \frac{\partial \Omega'}{\partial t} \right|_{t=0} = 0 \quad \left. \frac{\partial D}{\partial t} \right|_{t=0} = 0 \quad (4.15)$$

(Leith 1980; Errico 1981, 1982a, b).

Based on (4.15), Equations (4.11) and (4.12) become

$$\nabla^2 D - \frac{f_0^2}{c^2} D = -\frac{f_0}{c^2} \Omega'_{adv} \quad (4.16)$$

and

$$\Omega' = -\frac{1}{f_0} D_{adv} \quad (4.17)$$

The low frequency mode (4.10) does not change in the initialization, thus, (4.10) is rewritten as

$$\Omega - \frac{f_0}{c^2} \phi' = \Omega^{(0)} - \frac{f_0}{c^2} \phi'^{(0)} = C(x, y) \quad (4.18)$$

where $\Omega^{(0)}$ and $\phi'^{(0)}$ are the values when the initialization begins. Equations (4.16)–(4.18) are the balance equations for the initialization.

c. The initialized case with fixed wind at the boundary must be an inconsistent case.

In the limited-area initialization studied by Briere (1982), Errico and Bates (1988) and others, the vorticity and divergence, as well as the streamfunction and velocity potential, are used in their balance equations.

If the uninitialized vorticity and divergence are computed from the analyzed wind, which is equivalent to using the natural method, the uninitialized state must be the consistent case.

In the initialization process, the vorticity and divergence within the limited region are modified through solutions of the balance equations (4.16)–(4.18). If the fixed wind at the boundary is used as done by Briere (1982) and Errico and Bates (1988), the consistency condition will not be satisfied after the vorticity and divergence within the region are modified in the initialization. Thus, their initialized results must be the inconsistent case.

In an inconsistent case, it is impossible to find a continuous wind field that satisfies the fixed wind at the boundary and yields the vorticity and divergence in agreement with the initialized ones. Thus, the fixed wind at the boundary cannot be used in the initialization.

5. Fixed external wind boundary condition for the initialization in a limited region

Because only the vorticity and divergence within the region are modified in the initialization, based on the basic property of the wind separation, the external wind does not vary in the region including at the boundary. Thus, it is very natural to use fixed external wind as the boundary condition for the initialization in the limited region.

a. An ageostrophic streamfunction

The ageostrophic wind is expressed by

$$U' = U - U_g, \quad V' = V - V_g \quad (5.1)$$

where the geostrophic wind is denoted by

$$U_g = \frac{u_g}{m} = -\frac{1}{f_0} \frac{\partial \phi'}{\partial y}, \quad V_g = \frac{v_g}{m} = \frac{1}{f_0} \frac{\partial \phi'}{\partial x}. \quad (5.2)$$

The geostrophic streamfunction is defined by $\psi_g = \phi'/f_0$, and the ageostrophic streamfunction is the difference between the streamfunction and geostrophic streamfunction, *i.e.*,

$$\psi' = \psi - \psi_g. \quad (5.3)$$

Thus, the streamfunction is the sum of the geostrophic and ageostrophic streamfunctions. The geostrophic streamfunction differs from the geopotential height only a constant factor, and it can be used instead of the geopotential height.

The vorticity, Ω , is the sum of a geostrophic vorticity expressed by

$$\Omega_g = \frac{\zeta_g}{m^2} = \frac{\partial V_g}{\partial x} - \frac{\partial U_g}{\partial y} = \frac{1}{f_0} \nabla^2 \phi' \quad (5.4)$$

and an ageostrophic vorticity denoted by

$$\Omega' = \Omega - \Omega_g \quad (5.5)$$

The geostrophic and ageostrophic streamfunctions satisfy

$$\nabla^2 \psi_g = \Omega_g \quad (5.6)$$

and

$$\nabla^2 \psi' = \Omega' \quad (5.7)$$

respectively. Thus, from (5.5), we have

$$\nabla^2 \psi = \nabla^2 \psi_g + \nabla^2 \psi' \quad (5.8)$$

The ageostrophic wind is expressed by

$$U' = -\frac{\partial \psi'}{\partial y} + \frac{\partial \chi}{\partial x}, \quad V' = \frac{\partial \psi'}{\partial x} + \frac{\partial \chi}{\partial y} \quad (5.9)$$

b. Fixed external wind boundary condition

If a set of the analyzed variables

$$U^{(0)}(x, y), \quad V^{(0)}(x, y), \quad \Phi^{(0)}(x, y) \quad (5.10)$$

is given in a limited region R [$0 \leq x \leq L_x$, $0 \leq y \leq L_y$] bounded by Σ , the external wind components, $u_E^{(0)}$ and $v_E^{(0)}$, are obtained by applying the natural method to the wind in (5.10), and they automatically satisfy the consistency condition at the boundary.

The geostrophic streamfunction can be written as the sum of the inner and harmonic parts,

$$\psi_g^{(0)} = \psi_{gi}^{(0)} + \psi_{gh}^{(0)}, \quad (5.11)$$

where the harmonic part, $\psi_{gh}^{(0)}$, satisfies the Laplace equation

$$\nabla^2 \psi_{gh}^{(0)} = 0 \quad (5.12)$$

with the Dirichlet boundary value, $\psi_g^{(0)}|_{\Sigma} = \phi'^{(0)}|_{\Sigma}/f_0$, and the inner part is defined by

$$\psi_{gi}^{(0)} = \psi_g^{(0)} - \psi_{gh}^{(0)}, \quad (5.13)$$

and it also satisfies the Poisson equation

$$\nabla^2 \psi_{gi}^{(0)} = \Omega_g^{(0)} = \frac{1}{f_0} \nabla^2 \phi'^{(0)} \quad (5.14)$$

with zero Dirichlet boundary value.

The variables in (5.10) can be rewritten as

$$\begin{aligned} U^{(0)}(x, y) &= u_E^{(0)}(x, y) + u_I^{(0)}(x, y) \\ V^{(0)}(x, y) &= v_E^{(0)}(x, y) + v_I^{(0)}(x, y) \\ \psi_g^{(0)}(x, y) &= \psi_{gh}^{(0)}(x, y) + \psi_{gi}^{(0)}(x, y). \end{aligned} \quad (5.15)$$

The external wind, $u_E^{(0)}$ and $v_E^{(0)}$, in a limited region is both non-divergent and irrotational, and it must not vary if the vorticity and divergence are modified within the region in the initialization.

A similar method is also used for the geostrophic streamfunction, and it is assumed that only the geostrophic vorticity within the region is modified. This means that only the inner part of the geostrophic streamfunction is modified but its harmonic part does not vary in the initialization.

That the boundary values of $u_E^{(0)}|_{\Sigma}$, $v_E^{(0)}|_{\Sigma}$ and $\psi_{gh}^{(0)}|_{\Sigma}$ are kept unchanged is referred to as a fixed external wind at the boundary during the initialization. Because $u_E^{(0)}$, $v_E^{(0)}$ and $\psi_{gh}^{(0)}$ are all harmonic functions, and if their values at the boundary are kept unchanged, their values within the region also do not vary.

From the above, it is seen that only the inner parts of the streamfunction, velocity potential and geopotential height are modified in the initialization. Thus, the balance equations (4.16)–(4.18) need to be transformed into the equations for the inner parts. The solutions derived from these inner part balance equations can be expressed by

$$\begin{aligned} U^{(I)}(x, y) &= u_E^{(0)}(x, y) + u_I^{(I)}(x, y) \\ V^{(I)}(x, y) &= v_E^{(0)}(x, y) + v_I^{(I)}(x, y) \\ \psi_g^{(I)}(x, y) &= \psi_{gh}^{(0)}(x, y) + \psi_{gi}^{(I)}(x, y). \end{aligned} \quad (5.16)$$

where the internal wind, $u_I^{(I)}$ and $v_I^{(I)}$, at the boundary changes due to the variation of the vorticity and divergence within the region. Thus, the wind, U and V , at boundary is no longer fixed in the initialization. The initialized geopotential height is obtained by

$$\phi'^{(I)} = f_0 \psi_g^{(I)}. \quad (5.17)$$

c. Some features of ageostrophic wind and the boundary condition for the initialization

We use the analysis at 0000 UTC 30 March 1979 based on data from ECMWF (European Center of Medium-range Weather Forecasts) as an example. The mesh size is 47×37 and the grid spacing is 120 km. The analyzed 500 hPa geopotential height and the U and V components of the wind field are

Fig. 2. Observed values on the 500 hPa level at 0000 UTC 30 March 1979. (a) Geopotential height (the contour interval is 40 gpm); (b) $U = u/m$ -component; (c) $V = v/m$ -component (the contour interval of the wind components is 5 m s^{-1}); (d) the inner part of the streamfunction (the contour interval is $30 \times 10^5 \text{ m}^2 \text{ s}^{-1}$); (e) the inner part of the geostrophic streamfunction (same as (d)).

shown in Fig. 2a–2c. The inner parts of the streamfunction and geostrophic streamfunction are shown in Fig. 2d and 2e, respectively.

The inner parts of the ageostrophic streamfunction and velocity potential are shown in Fig. 3a and 3b. Figure 3c is the internal ageostrophic wind. In order to express the relative intensity of the

ageostrophic streamfunction and velocity potential, the rotational and divergent parts of the internal ageostrophic wind are also shown in Fig. 3a and 3b, respectively. Comparing Fig. 3a and 3b, it can be seen that the rotational component of the internal ageostrophic wind is larger than its divergent component. The mean speed of the rotational compo-

Fig. 3. (a) The distribution of the inner part of the ageostrophic streamfunction (the contour interval is $50 \times 10^4 \text{ m}^2 \text{ s}^{-1}$) and the rotational component of the internal ageostrophic wind; (b) the inner part of the velocity potential (same as (a)) and the divergent component of the internal ageostrophic wind; (c) the internal ageostrophic wind vector and its speed (same as Fig. 2b).

Fig. 4. The boundary conditions: (a) The harmonic u-component, u_{sh} ; (b) the harmonic v-component, v_{sh} (same as Fig. 2b); (c) the harmonic part of the geopotential height.

nent of the internal ageostrophic wind for the whole limited region is 3.38 m s^{-1} and that of divergent component is 1.64 m s^{-1} . They are of the same order, but the speed of the rotational component is roughly twice that of the divergent component. Meteorologists often pay more attention to the di-

vergent component of the ageostrophic wind than its rotational component. As shown in Fig. 2, the rotational component of the ageostrophic wind is larger than its divergent component. Thus, the rotational component is also important.

Comparing Fig. 2a with Fig. 3a, it can be seen that an anticyclonic ageostrophic circulation center (about 116°E, 48°N) occurs in the low (positive geostrophic vorticity) center and relatively larger ageostrophic streamfunction is located along a trough line near the east coast. The ageostrophic vorticity in such a distribution is probably due to the curvature of the air trajectory in the gradient wind. However, some other distribution may not be explained by the gradient wind. As a whole, the geostrophic vorticity is not a good approximation to the wind vorticity, and the ageostrophic vorticity cannot be neglected.

The components of the external wind, u_E and v_E , are shown in Fig. 4a and 4b, respectively. As discussed above, these external winds do not change during initialization. The harmonic part of the geopotential height is derived from the Laplace equation with the Dirichlet boundary value, and it is shown in Fig. 4c. It also does not vary in the initialization.

6. A balanced ageostrophic initialization in a limited region

a. The balance equations for the inner parts

Utilizing $D = \nabla^2 \chi$, (5.6), and (5.7), the balance equations (4.16)–(4.18) can be rewritten as

$$\nabla^2(\nabla^2 \chi) - \frac{f_0^2}{c^2} \nabla^2 \chi = -\frac{f_0}{c^2} \Omega'_{adv} \quad (6.1)$$

$$\nabla^2 \psi' = -\frac{1}{f_0} D_{adv} \quad (6.2)$$

$$\nabla^2 \psi_g - \frac{f_0^2}{c^2} \psi_g + \nabla^2 \psi' = \Omega^{(0)} - \frac{f_0}{c^2} \phi'^{(0)} = C(x, y) \quad (6.3)$$

The divergence is separated into the inner and harmonic parts as

$$D = D_i + D_h = \nabla^2 \chi_i \quad (6.4)$$

where the harmonic part satisfies

$$\nabla^2 D_h = 0 \quad (6.5)$$

with the Dirichlet boundary condition, $D_h|_{\Sigma} = D|_{\Sigma}$. Equation (6.1) can be rewritten as

$$\nabla^2 D - \frac{f_0^2}{c^2} \nabla^2 \chi_i = -\frac{f_0}{c^2} \Omega'_{adv} \quad (6.6)$$

Utilizing (6.4) and (6.5), (6.6) can be rewritten as

$$\nabla^2 \left(D_i - \frac{f_0^2}{c^2} \chi_i \right) = -\frac{f_0}{c^2} \Omega'_{adv} \quad (6.7)$$

where the boundary value of the variable, $D_i - (f_0/c)^2 \chi_i$, is zero.

Because the boundary conditions of the inner parts are all homogeneous, the equations can be solved by the finite Fourier sine transform. Let F denote a finite Fourier sine transform operator in the two-dimensional space, *i.e.*,

$$\begin{aligned} F[f_i(x, y)] &= F_{I, mn} \\ &= \frac{4}{L_x L_y} \int_0^{L_x} \int_0^{L_y} f_i(x, y) \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} dx dy \end{aligned} \quad (6.8)$$

The double Fourier sine series of f_i is

$$\begin{aligned} f_i(x, y) &= F^{-1}[F_{I, mn}] \\ &= \sum_{m=1}^M \sum_{n=1}^N F_I(m, n) \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \end{aligned} \quad (6.9)$$

Here F^{-1} is the inverse Fourier sine transform operator from wave to physical space.

Let us introduce variables, $\psi'_{adv, i}$ and $\chi_{adv, i}$, which satisfy

$$\nabla^2 \psi'_{adv, i} = \Omega'_{adv}, \quad \nabla^2 \chi_{adv, i} = D_{adv} \quad (6.10)$$

with zero Dirichlet boundary values; and the variables, $\psi'_{adv, i}$ and $\chi_{adv, i}$, are referred to as the variation rates of the inner parts of the ageostrophic streamfunction and velocity potential due to advection, respectively. Thus, the solutions of (6.10) are

$$\begin{aligned} \psi'_{adv, i} &= -F^{-1} \left[\Omega'_{adv, mn} / \left(\frac{1}{L_{mn}^2} \right) \right], \\ \chi_{adv, i} &= -F^{-1} \left[D_{adv, mn} / \left(\frac{1}{L_{mn}^2} \right) \right] \end{aligned} \quad (6.11)$$

where

$$\Omega'_{adv, mn} = F[\Omega'_{adv}], \quad D_{adv, mn} = F[D_{adv}]$$

and L_{mn} is expressed by

$$\frac{1}{L_{mn}^2} = \frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} \quad (6.12)$$

Utilizing (6.4), the solution of (6.7) is written in the form

$$\nabla^2 \chi_i - \frac{f_0^2}{c^2} \chi_i = -\frac{f_0}{c^2} \psi'_{adv, i} + D_h \quad (6.13)$$

For the same reason, the solution of the inner part of the ageostrophic streamfunction from (6.2) is written as

$$\psi'_i = -\frac{1}{f_0} \chi_{adv, i} \quad (6.14)$$

Equation (6.3) is rewritten as

$$\nabla^2 \psi_{gi} - \frac{f_0^2}{c^2} \psi_{gi} + \nabla^2 \psi'_i = \Omega^{(0)} - \frac{f_0}{c^2} \phi'^{(0)} + \frac{f_0^2}{c^2} \psi_{gh} \quad (6.15)$$

The Equations (6.13)–(6.15) are the balance equations for the inner parts of the three variables χ_i , ψ'_i and ψ_{gi} .

Fig. 5. The changes of $\delta V_I^{(\nu)}$ with iteration.

c. The concept of a balanced ageostrophic initialization

The inner parts of the velocity potential and ageostrophic streamfunction that satisfy (6.13) and (6.14) are referred to as the balanced inner parts of the velocity potential and ageostrophic streamfunction, respectively. The initialization requires solving (6.13)–(6.15) to obtain these balanced inner parts. An internal ageostrophic wind computed from these balanced inner parts is referred to as a balanced internal ageostrophic wind. The initialization in a limited region is to modify analyzed ageostrophic wind to obtain the balanced ageostrophic wind. Thus, it is referred to as a balanced ageostrophic initialization.

From (6.15), it is seen that the inner part of the geostrophic streamfunction is also modified in the initialization due to the variation of ψ' . From the solutions of (6.13)–(6.15), the inner parts, $\chi_i^{(0)}$, $\psi_i^{(0)}$ and $\psi_{gi}^{(0)}$, are modified to become the balanced or initialized values $\chi_i^{(I)}$, $\psi_i^{(I)}$ and $\psi_{gi}^{(I)}$. Because the boundary of the inner part equations are homogeneous, the balance equations can be solved by the double sine series. The balance equations (6.13)–(6.15) are solved by the iterative method shown in Appendix A.

At this point, it is appropriate to justify the linearization about a state of rest, and the constant Coriolis parameter and map scale factor in the NNMI. If a state of rest and constant Coriolis parameter and map scale factor are assumed in the linear parts of the equations, the terms for a nonzero mean flow field and the variable deviations of the Coriolis parameter and of the map scale factor will be moved to the right hand side of the non-linear forcing terms. The inertia-gravity modes are modified based on the solution of the non-linear balance equations. If these solutions converge, the

above assumptions can hardly affect the final converged solutions. Bijlsma (1989, 1991) has studied the NNMI including the beta terms in the linearized model equations. His results show the equivalence between the f-plane approach and the inclusion of beta terms for initialization.

7. An example of the balanced ageostrophic initialization

a. The convergence of the iterative method

The iterative procedure shown in Appendix A for solving the balance equations (6.13)–(6.15) converges very fast. Because the external wind does not vary in the initialization, we use the variation of the internal wind at ν -th iteration

$$\begin{aligned} u_I^{(\nu)} &= -\frac{\partial \psi_{gi}^{(\nu)}}{\partial y} + \frac{\partial \chi_i^{(\nu)}}{\partial x} - \frac{\partial \psi_i'^{(\nu)}}{\partial y}, \\ v_I^{(\nu)} &= \frac{\partial \psi_{gi}^{(\nu)}}{\partial x} + \frac{\partial \chi_i^{(\nu)}}{\partial y} + \frac{\partial \psi_i'^{(\nu)}}{\partial x} \end{aligned} \quad (7.1)$$

to measure the convergence of the solution. The mean variation of the root-mean-square of the speed of the internal wind vector between two successive iterations, $\delta V_I^{(\nu)}$, is computed by

$$\begin{aligned} \delta V_I^{(\nu)} &= \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N \\ &\times \sqrt{(u_I^{(\nu)} - u_I^{(\nu-1)})_{i,j}^2 + (v_I^{(\nu)} - v_I^{(\nu-1)})_{i,j}^2}. \end{aligned} \quad (7.2)$$

During the iterations, the values of $\delta V_I^{(\nu)}$ decrease rapidly with increasing number of iterations as shown in Fig. 5. It is seen from Fig. 5 that after only 3 or 4 iterations, the changes in the internal wind become negligible.

b. The initialized results, the balanced ageostrophic wind and the balanced wind

The balanced ageostrophic streamfunction, velocity potential and geostrophic streamfunction after the fourth iteration are shown in Figs. 6a, 6b and 6c, respectively. The initialized two wind components reconstructed from the balanced geostrophic and ageostrophic streamfunction and velocity potential are shown Figs. 6d and 6e, and the geopotential height at the 500 hPa level computed from the geostrophic streamfunction is shown in Fig. 6f. The mean absolute differences before and after initialization are 1.19 m s^{-1} in U component, 1.26 m s^{-1} in V component, and 5.49 m in the geopotential height. Thus, the changes of these variables produced by initialization are acceptably small.

The ageostrophic wind can be considered as the sum of an imbalanced part and a balanced one. Only the imbalanced part can induce the fast gravity-inertia waves and needs to be filtered in the initialization. The balanced ageostrophic wind remains behind after the initialization.

Fig. 6. The initialized results: (a) the inner part of the ageostrophic streamfunction (same as Fig. 3a); (b) the inner part of the velocity potential, (the contour interval is $100 \times 10^3 \text{ m}^2 \text{ s}^{-1}$); (c) the inner part of the geostrophic streamfunction (same as Fig. 2d); (d) the initialized $U = u/m$ -component, (same as Fig. 2b); (e) the initialized $V = v/m$ -component (same as Fig. 2b); (f) the initialized geopotential field (same as Fig. 2a).

The mean value of the imbalanced part of the ageostrophic wind, $\bar{v}_{imb-ag}$, is calculated by

$$\bar{v}_{imb-ag} = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N \sqrt{(U^{(0)} - U^{(I)})_{i,j}^2 + (V^{(0)} - V^{(I)})_{i,j}^2} \quad (7.3)$$

where $U^{(0)}$, $V^{(0)}$ and $U^{(I)}$, $V^{(I)}$ are two components of the ageostrophic wind before and after the initialization, respectively. The computed mean value of $\bar{v}_{imb-ag}$ over the limited region is 1.82 m s^{-1} . The mean value of the ageostrophic wind, $\bar{v}_{ag}$, over the region is expressed by

$$\bar{v}_{ag} = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N \sqrt{(U^{(0)} - U_g^{(0)})_{i,j}^2 + (V^{(0)} - V_g^{(0)})_{i,j}^2} \quad (7.4)$$

and its computed result is 5.50 m s^{-1} .

It is seen from the above that the mean value of the imbalanced part, $\bar{v}_{imb-ag}$, is about 33 % of the mean ageostrophic wind, while its balanced part is then about 67 %. Thus, the balanced ageostrophic wind is a good approximation to the ageostrophic wind.

The sum of the geostrophic and balanced ageostrophic winds is referred to as a balanced wind, which is expressed by

$$U_b = U_g^{(I)} + U^{(I)}, \quad V_b = V_g^{(I)} + V^{(I)}. \quad (7.5)$$

The difference between the observed wind and balanced wind is referred to as an imbalanced wind, and the mean value of the imbalanced wind, $\bar{v}_{imb}$, in the limited region is expressed by

$$\bar{v}_{imb} = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N \sqrt{(U^{(0)} - U_b)_{i,j}^2 + (V^{(0)} - V_b)_{i,j}^2}. \quad (7.6)$$

The computed value of $\bar{v}_{imb}$ is 1.84 m s^{-1} , which is nearly the same as the imbalanced part of the ageostrophic wind, $\bar{v}_{imb-ag}$.

However, the mean difference between the observed and geostrophic winds is expressed by the mean ageostrophic wind shown by $\bar{v}_{ag}$ in (7.4). The value of the mean ageostrophic wind is 5.50 m s^{-1} , and it is about three times of the imbalanced wind, $\bar{v}_{imb}$. Thus, the balanced wind is a very good approximation to the observed wind, and it is much more accurate than the geostrophic wind.

c. The initialized results tested by a shallow water model

In this subsection, we will compare the predicted results of a shallow water model integrated from the initial values with and without the balanced ageostrophic initializations. In order to minimize the initial tendency of the ageostrophic vorticity and divergence at the start of the forecast (or initial model shock), it is essential to use the same form of the model for the balanced ageostrophic initialization as for the forecast production, which has been mentioned by Bengtsson (1991). If the same form is

used, the 'spin up' time is greatly reduced or eliminated entirely.

The forecast model used here for the initialization test is the harmonic-Fourier spectral shallow water model described by Chen *et al.* (1991). In this model, the form of the equations is the same as those used in the balanced ageostrophic initialization, and the model is also solved by the same harmonic-sine spectral method. Thus, when the model is integrated from the balanced ageostrophic initialized data, the initial tendency of the ageostrophic vorticity and divergence are automatically zero at the start of the forecast.

By using the semi-implicit time integration scheme, the time step, Δt , is 10 minutes for the grid spacing of 120 km. To start the model integration, we perform three steps: a forward step of $(\Delta t)/2$, a centered one of Δt , then a centered one of $2\Delta t$, which is the basic time step of our scheme.

Because of the high-frequency gravity-inertia waves, the effect of initialization on the forecast can be seen by plotting the variables at a single point ($I = 20$, $J = 16$). The solid line in Fig. 7a shows the ageostrophic vorticity for the uninitial-ized run. The dashed line, for the initialized run, is much smoother, and the initial shock is absent. The same situation is also shown by the divergence and geopotential height in Figs. 7b and 7c. From Fig. 7 it can be seen that the balanced ageostrophic initialization can effectively remove the meteorological noise at the beginning of integration of a limited-area model. Figure 7 also shows that if the analyzed data are used directly as the initial condition without initialization, the imbalanced ageostrophic vorticity and divergence will cause the forecast to be contaminated by spurious, high-frequency gravity-wave oscillations that are not observed in the real atmosphere. Because we do not use any explicit dissipation mechanism in the present shallow water model, these oscillations die away very slowly.

8. Conclusions

From the above, the following conclusions can be reached.

1) The wind in a limited region can be separated into the external wind and internal wind. The internal wind depends upon the vorticity and divergence only within the region, while the external wind depends on those only in the external region. This is the basic property of the wind separation.

The natural method can be used to derive the external wind, and the derived external wind satisfies the consistency condition automatically.

2) The concept of a given case that the vorticity and divergence are given within the region and wind is prescribed at the boundary represents a common feature of the predicted case and initialized case. If the estimated external wind for a given case satisfies

Fig. 7. The first 12 h forecast of an individual grid point ($I = 20$, $J = 16$) using an uninitialized (solid curves) and the quasi-geostrophic initialized (dashed curve) data: (a) the ageostrophic vorticity (in 10^{-5} s^{-1}); (b) the divergence (in 10^{-5} s^{-1}); (c) the geopotential height (in m).

the consistency condition at the boundary, the given case is called the consistent case; otherwise, it is an inconsistent case. In an inconsistent case, it is impossible to find a continuous wind that satisfies the wind given at the boundary and yields the vorticity and divergence in agreement with those given within the region. A direct method with two Poisson equations for reconstructing the wind within the region can only be used if the given case is a consistent case.

3) The implicit normal mode initialization (Temperon 1988) in middle latitudes is further illustrated from the wind reconstruction and separation. The purpose of the initialization is to modify the analyzed ageostrophic wind to be the balanced ageostrophic wind. Only the imbalanced part of the ageostrophic wind can produce the fast gravity-inertia wave and needs to be filtered out in the initialization.

Based on the computed results at the 500 hPa level for one example, the mean value of the rotational component of the ageostrophic wind over the region is 3.38 m s^{-1} , and it is larger than that of the divergent component, which is 1.64 m s^{-1} .

The mean computed results over the limited region also show that the mean ageostrophic wind is 5.50 m s^{-1} , of which the imbalanced part is about 33 % and the balanced about 67 %. The balanced ageostrophic wind is a good approximation to the ageostrophic wind.

The balanced wind is the sum of the geostrophic wind and the balanced ageostrophic wind. The computed mean imbalanced wind is about 1.84 m s^{-1} and it is only one third of the mean ageostrophic wind. The balanced wind is a good approximation to the observed wind, and it is more accurate than the geostrophic wind.

4) The fixed wind components at the boundary cannot be used as the boundary conditions for the initialization because the initialized case in this situation must be an inconsistent case. Based on the property of the wind separation, it is natural to use the fixed external wind as the lateral boundary condition of the initialization in a limited region.

Under this boundary condition, only the inner parts of the ageostrophic and geostrophic streamfunction and velocity potential are modified through the solutions of the balance equations. Because the boundary conditions of these inner parts are all homogeneous, these balance equations are solved by the double Fourier sine series efficiently and accurately.

The balanced ageostrophic initialization was tested in a harmonic-Fourier spectral shallow water model. The results show that this initialization removes the meteorological noise very effectively at the start of the forecast.

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Appendix A

An iterative method for solving the balance equations of the inner parts

a. The iterative procedure for solving the balance equations

The balance equations (6.13)–(6.15) are solved in an iterative procedure as follows. For the ν -th iteration, the equations (6.13)–(6.15) can be written in the form

$$\nabla^2 \chi_i^{(\nu)} - \frac{1}{L_0^2} \chi_i^{(\nu)} = -\frac{f_0}{c^2} \psi_{adv,i}'^{(\nu-1)} + D_h^{(0)} \quad (\text{A.1})$$

$$\psi_i'^{(\nu)} = -\frac{1}{f_0} \chi_{adv,i}^{(\nu-1)} \quad (\text{A.2})$$

$$\nabla^2 \psi_{gi}^{(\nu)} - \frac{1}{L_0^2} \psi_{gi}^{(\nu)} = F^{(\nu)} \quad (\text{A.3})$$

where

$$\begin{aligned} F^{(\nu)} &= \Omega^{(0)} - \frac{f_0}{c^2} \phi^{(0)} - \nabla^2 \psi_i'^{(\nu)} + \frac{1}{L_0^2} \psi_{gh}^{(0)} \\ &= \Omega^{(0)} - \frac{1}{L_0^2} \psi_{gi}^{(0)} - \nabla^2 \psi_i'^{(\nu)} \end{aligned} \quad (\text{A.4})$$

and $L_0 = c/f_0$ is the deformation radius. The variable, $\nabla^2 \psi_{gi}^{(\nu)} - (1/L_0)^2 \psi_{gi}^{(\nu)}$, in the left-hand side of (A.3) is the geostrophic potential vorticity. Thus, the potential vorticity in the left-hand side of (6.15) is the sum of the geostrophic potential vorticity and the ageostrophic vorticity. Utilizing (A.2), we have

$$\nabla^2 \psi_i'^{(\nu)} = -\frac{1}{f_0} D_{adv}^{(\nu-1)}$$

The ageostrophic vorticity in (A.4) can be replaced by the divergence advection, then (A.4) can be rewritten as

$$F^{(\nu)} = \Omega^{(0)} - \frac{1}{L_0^2} \psi_{gi}^{(0)} + \frac{1}{f_0} D_{adv}^{(\nu-1)}, \quad (\text{A.5})$$

The solutions of (A.1) and (A.3) can be written in the form

$$\begin{aligned} \chi_i^{(\nu)} &= \\ &-F^{-1} \left[\left(-\frac{f_0}{c^2} \psi_{adv,i,mn}'^{(\nu-1)} + D_{h,mn}^{(0)} \right) / \left(\frac{1}{L_0^2} + \frac{1}{L_{mn}^2} \right) \right] \end{aligned} \quad (\text{A.6})$$

and

$$\psi_{gi}^{(\nu)} = -F^{-1} \left[F_{mn}^{(\nu)} / \left(\frac{1}{L_0^2} + \frac{1}{L_{mn}^2} \right) \right] \quad (\text{A.7})$$

respectively. The variables, $\psi_{adv,i}'^{(\nu-1)}$ and $\chi_{adv,i}^{(\nu-1)}$, are computed by

$$\begin{aligned} \psi_{adv,i}'^{(\nu-1)} &= -F^{-1} \left[\Omega_{adv,mn}'^{(\nu-1)} / \left(\frac{1}{L_{mn}^2} \right) \right], \\ \chi_{adv,i}^{(\nu-1)} &= -F^{-1} \left[D_{adv,mn}^{(\nu-1)} / \left(\frac{1}{L_{mn}^2} \right) \right] \end{aligned} \quad (\text{A.8})$$

where

$$\Omega_{adv,mn}'^{(\nu-1)} = F[\Omega_{adv}'^{(\nu-1)}], \quad D_{adv,mn}^{(\nu-1)} = F[D_{adv}^{(\nu-1)}].$$

In each iterative step, the non-linear terms, $\Omega_{adv}'^{(\nu-1)}$ and $D_{adv}^{(\nu-1)}$, are calculated by using the values of $U^{(\nu-1)}$, $V^{(\nu-1)}$, $D^{(\nu-1)}$, and $\Omega^{(\nu-1)}$ in the previous step. The beginning step is $\nu = 1$.

b. Wind field reconstruction during iteration

In the ν -th iteration, the internal ageostrophic wind is computed by

$$\begin{aligned} u_I'^{(\nu)} &= \frac{\partial \chi_i^{(\nu)}}{\partial x} - \frac{\partial \psi_i'^{(\nu)}}{\partial y}, \\ v_I'^{(\nu)} &= \frac{\partial \chi_i^{(\nu)}}{\partial y} + \frac{\partial \psi_i'^{(\nu)}}{\partial x} \end{aligned} \quad (\text{A.9})$$

and the internal geostrophic wind is

$$u_{gI}^{(\nu)} = -\frac{\partial \psi_{gi}^{(\nu)}}{\partial y}, \quad v_{gI}^{(\nu)} = \frac{\partial \psi_{gi}^{(\nu)}}{\partial x}. \quad (\text{A.10})$$

The total wind can be obtained by

$$\begin{aligned} U^{(\nu)} &= u_E^{(0)} + u_{gI}^{(\nu)} + u_I'^{(\nu)}, \\ V^{(\nu)} &= v_E^{(0)} + v_{gI}^{(\nu)} + v_I'^{(\nu)} \end{aligned} \quad (\text{A.11})$$

and the divergence and total vorticity in the limited region including its boundary are expressed by

$$\begin{aligned} D^{(\nu)} &= \nabla^2 \chi_i^{(\nu)} + D|_{\Sigma}^{(0)}, \\ \Omega^{(\nu)} &= \nabla^2 \psi_{gi}^{(\nu)} + \nabla^2 \psi_i'^{(\nu)} + \Omega|_{\Sigma}^{(0)} \end{aligned} \quad (\text{A.12})$$

Because the boundary values are zero if the inner parts are expressed by the double sine series, the boundary values of $\nabla^2 \chi_i$ and $\nabla^2 \psi_i$ are zero. The boundary conditions of the vorticity and divergence, Ω and D , need to be added for computing their partial derivatives in the non-linear advection terms, Ω_{adv}' and D_{adv} . These boundary conditions are also unchanged in the initialization.

The above procedure is repeated until the balance equations (A.1)–(A.3) are satisfied. If this iterative procedure converges, the inner parts of the ageostrophic streamfunction and the velocity potential will converge to the balanced inner parts of the ageostrophic streamfunction and velocity potential. At the same time, the ageostrophic wind converges

to the balanced ageostrophic wind. Although the harmonic parts do not vary during initialization, they play roles in the non-linear advection terms which affect initialization.

At each iteration, the non-linear advection terms, Ω_{adv} , D_{adv} and ϕ_{adv} , are computed by the transform method (Orszag 1970; Eliassen *et al.*, 1970). In this approach, the product of variables are performed by transforming these variables to the gridpoint values, multiplying them together at each gridpoint, and transforming the product back to wave space.

Appendix B

The relation to the quasi-geostrophic initialization

Leith (1980) showed that the quasi-geostrophic initialization was a special case of the NNMI. Because the NNMI takes the form of the balanced ageostrophic initialization, this form is very easily related to the quasi-geostrophic initialization.

If we let the ageostrophic vorticity always be zero, the balance equation (6.2) can be omitted, and (6.3) is rewritten as

$$\nabla^2 \psi_g - \frac{f_0^2}{c^2} \psi_g = \Omega^{(0)} - \frac{f_0}{c^2} \phi^{(0)} = C(x, y) \quad (\text{B.1})$$

Equation (6.1) is kept unchanged and it is the velocity potential form of the ω -equation for the shallow water model. In the quasi-geostrophic initialization (Phillips 1960; Daley 1991), the initialized state is solved from (6.1) and (B.1). Its results are described by only two variables, the geostrophic streamfunction and velocity potential, and the wind is reconstructed by

$$U = \frac{\partial \chi}{\partial x} - \frac{\partial \psi_g}{\partial y} \quad V = \frac{\partial \chi}{\partial y} + \frac{\partial \psi_g}{\partial x}. \quad (\text{B.2})$$

Thus, the quasi-geostrophic initialization is a special case of the balanced ageostrophic initialization.

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領域モデルのための外成風境界値を固定した バランス非地衡風イニシャリゼーション

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限られた領域の風は、領域のそれぞれ外部と内部の渦度と発散だけに依存する、外成風と内成風に分離できる。これは風の分離における基本的な特性である。

領域モデルによる予報とイニシャリゼーションにしばしば現れる共通の特徴を説明するために、領域の内部で過度と発散が与えられ、境界で風が設定されている場合が取り上げられる。この場合に算出される外部風が、境界で一致条件を満たせば一致する場合と呼び、そうでなければ不一致な場合と呼ぶ。不一致な場合には、境界で与えられた風と等しく、かつ内部で与えられた過度と発散に一致する、連続な風の間を見出すことは不可能である。一致する場合であれば、風を再構成するために二つのポアソン方程式を用いる直接法を用いることができる。

インプット・ノーマル・モード・イニシャリゼーション (Temperton, 1988) が、風の再構成と分離の観点からさらに明らかにされる。イニシャリゼーションの目的は、解析された非地衡風を変形して、バランスした非地衡風にすることである。バランスしない非地衡風だけが速い慣性重力波を引き起こし、イニシャリゼーションで除去される必要がある。一つの計算例では、バランスした非地衡風は非地衡風全体の約 68 % であり、したがって非地衡風のよい近似になっている。非地衡風の回転成分 (3.38 m s^{-1}) が発散成分 (1.64 m s^{-1}) より大きいことも示される。バランスした風は地衡風とバランスした非地衡風の和であり、観測される風のよい近似になっている。

イニシャリゼーションにおいて境界で風を固定するのは不一致な場合になるため、それはできないことを証明する。風の分離の基本的な特性に基づけば、イニシャリゼーションの側面境界条件として、固定された外部風を用いるのがたいへん自然である。提案されたイニシャリゼーションと境界条件がテストされ、結果は、予報の初期値から気象学的なノイズが効果的に除去され得ることを示している。