NOTES AND CORRESPONDENCE

Validation of the Method of Detecting the Long-term Trend of Extremely Heavy Rainfall by Using the Monte Carlo Method

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1. Introduction

The long-term trend of extremely heavy rainfall is a matter of concern, particularly in association with the global warming due to increasing greenhouse effects (IPCC, 1992). While some numerical simulations are performed for increasing activity of very intense precipitation accompanied with the global warming (*e.g.* Gordon *et al.*, 1992), any detection of the tendency in the observational data could not be made successfully, until Iwashima and Yamamoto (1993) proposed a new method (abbreviated as the IY-method hereafter). The main purpose of the present paper is to examine the validity of the IY-method with the use of the Monte Carlo method.

Statistical treatment of extreme events has been traditionally performed utilising the so-called Gumbel model (Brooks & Carruthers, 1953; Gumbel, 1958; Tabony, 1983). Noticing an apparent time change of the estimated parameters of the model, some attempts were made to find a long-term trend of heavy rainfall activity (*e.g.* Perry & Howells, 1982). Such an attempt could not, however, produce any reliable results, because of the too small size of the treated sample to estimate the parameters accurately. Therefore, a new method has been proposed which does not use any conventional model in statistics (Iwashima and Yamamoto, 1993).

The IY method is based upon the following working hypothesis. The time of record-breaking intense rainfall should be, in principle, random at stations if there is no long-term trend in severity of heavy rainfall. Then, the time at many stations in an observational network of some extent should be distributed uniformly over the data-available period. On the other hand, the frequency of the occurrence time should necessarily increase with the lapse of time, if some increasing trend exists in the extreme

intensity of precipitation. Therefore, the presence of any long-term trend can be examined by inspecting whether the frequency of the record-breaking extreme events would be statistically uniform or not over a time duration.

Iwashima and Yamamoto (1993) applied the IY method to the data of extremely large daily precipitation for 100 years at 55 stations in Japan. They obtained the result that the frequency distribution of the appearance time of the first, the second and the third ranks of maximum daily precipitation increased, with statistical significance, in the course of time. This implies that there should exist some long-term increasing trend of significant strength in the past 100 years over Japan.

Although Iwashima and Yamamoto (1993) could detect successfully an existence of a trend with use of the IY method, the validity of this method is not necessarily self-evident. No discussion was given of the potential ability of this method to estimate quantitatively any long-term trend. The purpose of the present paper is to answer the following questions concerning the IY method:

- (1) Can the validity of this method be determined using the Monte Carlo method?
- (2) Can this method estimate quantitatively the rate of long-term trend?
- (3) How does the performance of this method decline under a decreasing number of data stations?

2. Method of statistical simulation

The characteristics of pseudo-random numbers generated by a computer are similar to those of real random numbers in many aspects, and the differences between them are so small as to be neglected in many experiments (Fushimi, 1989). Therefore,

Fig. 1. Fictitious time series of pseudo-random numbers produced by the third-order linear congruential generator with use of Skellams constants recommended by Pollard (1977). A series without a long-term trend is shown in the upper panel and that with a long-term trend of $k = 20\%$ in the lower panel.

the pseudo-random number is also frequently called a random number, and we follow this usage hereafter.

Sequences of random numbers W_n ($n = 1, 2, \dots$) are produced by the third order linear congruential generator:

$$W_n = k_1 W_{n-1} + k_2 W_{n-2} + k_3 W_{n-3} \pmod{p} \quad (1)$$

To the constants p , k_1 , k_2 and k_3 , the following numerical values recommended by Skellam are given (Pollard, 1977):

$$\begin{aligned} p &= 997783; \quad k_1 = 360137; \\ k_2 &= 519815; \quad k_3 = 616087 \end{aligned} \quad (2)$$

The random number W_n is transformed to new random number V_n as follows:

$$V_n = W_n \times 100 + 200 \quad (3)$$

Fig. 2. Scatter-diagram of maximum value and its appearance time in a time series at each of 500 stations. Crowdedness of the maximum appearance clearly increases with the lapse of time for the case of long-term increasing with $k = 10\%$ in the lower panel, while no appreciable change in crowdedness appears in the case of no trend in the upper panel.

This is made only for such a reason that the magnitude of V_n is near the actual value of largest daily precipitation in units of millimeters.

With use of V_n ($n = 1, 2, \dots, 50000$), we construct fictitiously a time series of annual data for 100 years at 500 stations, which are distributed in each of 10 networks composed of 50 stations. Thus, the series of random number can be specified as $S_{i,j,0}(t)$, which designates the data at the time $t (= 1, \dots, 100)$, at the i -th station ($i = 1, \dots, 50$) of the j -th network ($j = 1, \dots, 10$). The third subscript signifies the long-term trend mentioned just below, and the subscript of zero means no trend. The 100-year mean and the standard deviation of $S_{i,j,0}(t)$ fall in range of 183–220 and 50–64, respectively.

It is easily seen that all the time series of $S_{i,j,0}(t)$ have no systematic trend for 100 years. A series with a linear trend is designated by $S_{i,j,k}(t)$ with a

Fig. 3. Frequency distribution of appearance time of the maximum for the same data as in Fig. 2. The case without long-term trend is shown in the left panel, and a case with an increasing trend of $k = 10\%$ in right panel. The shadings indicate the 95 % confidence intervals.

non-zero value of k . This series is defined by the following formula:

$$S_{i,j,k}(t) = S_{i,j,0}(t) + c_{i,j,k} \times (t - 50), \quad (4)$$

where

$$c_{i,j,k} \equiv \sigma_{i,j} \times (k/100)/100 \quad (5)$$

and $\sigma_{i,j}$ is the standard deviation at the i -th station of the j -th network estimated from the data for 100 years, and k is constant for the rate of linear tendency measured as a percentage. Figure 1 shows examples of $S_{i,j,0}(t)$ and $S_{i,j,20}(t)$ for $t = 1, \dots, 100$. The difference of the long-term tendency between the two series is effectively concealed by very large year-to-year fluctuations.

3. Results

3.1 Validation of the IY method

The maximum value and its appearance time during 100 years are picked out for each of the time series $S_{i,j,k}(t)$ at the 500 stations. Scatter-diagrams of the maximum and its appearance time are given in Fig. 2 for two cases with no trend ($k = 0$) and with a trend of $k = 10\%$. In Fig. 2, time changes of the data does not show any systematic change in the case of $k = 0$ (upper panel), but an obvious increase in the case of $k = 10\%$ (lower panel). These situations suggest strongly the validity of the method.

With use of the data at 50 stations within the j -th network, the frequency $F_j(\tau)$ of the maximum appearance time can easily be counted for the τ -th class of 20 year width ($\tau = 1, 2, 3, 4$ and 5). The

Fig. 4. Dependency of the chi-square value on the long-term trend. The values of chi-square are computed for the first rank of the maximum at the 50 stations. Heavy and light shades show the range of $A_{chi-2} \pm \sigma_{chi-2}$ and $A_{chi-2} \pm 2\sigma_{chi-2}$, respectively. See the text concerning determination of the chi-square value.

computations of $F_j(\tau)$ for each of the 10 networks ($j = 1, 2, \dots, 10$) give the average $A_F(\tau)$ and the standard deviation $\sigma_F(\tau)$ of the frequency for each class. These make it possible to estimate a statistical significance under an assumption of Gaussian distribution of the computed $F_j(\tau)$.

Two histograms in Fig. 3 show frequency distributions $A_F(\tau) \pm 2\sigma_F(\tau)$ for the case of no systematic trend ($k = 0$) (left panel) and for the case of increasing trend of $k = 10\%$ (right panel), respectively. It is clearly seen that the frequency in question evidently increases with the lapse of time for the case of $k = 10\%$, while there is no significant deviation from uniform distribution for the case of $k = 0$. Much more significant increases can be seen for the case of k greater than 10 %, although not shown here. Such an obvious difference in the frequency distribution between the cases $k = 0$ and $k \geq 10\%$ illustrates the validity of the IY method.

3.2 Estimate of long-term trends

The chi-square value under a condition of uniform frequency distribution is used for quantitative estimate of long-term trends of extreme events. The time of maximum value occurrence over 100 years is picked out for each station in each network. These occurrence times for 50 stations in each network are distributed into 5 classes of 20-year width and then the chi-square value χ_j^2 for the j -th network is computed by the following formula:

$$\chi_j^2 = \sum_{p=1}^5 (n_p - e_p)^2 / e_p, \quad (6)$$

Fig. 5. Same as Fig. 4, except for adoption of the first, the second and the third ranks of the maximum in data sampling.

where n_p is the observed frequency of occurrence and e_p is the expected frequency for uniform distribution of occurrence.

χ_j^2 should essentially be a stochastic quantity. The average A_{chi-2} and the standard deviation σ_{chi-2} of chi-square, which are computed from 10 values of χ_j^2 for each network ($j = 1, 2, \dots, 10$), give some useful implications. Figure 4 shows a dependency of the chi-square value defined above on long-term trend in question for the case of 5 classes of time intervals, namely four degrees of freedom, with values of $A_{chi-2} \pm \sigma_{chi-2}$ and $A_{chi-2} \pm 2\sigma_{chi-2}$. It is well known that the probability is about 68 % for the occurrence between $A_{chi-2} - \sigma_{chi-2}$ and $A_{chi-2} + \sigma_{chi-2}$, and about 95 % for the occurrence between $A_{chi-2} - 2\sigma_{chi-2}$ and $A_{chi-2} + 2\sigma_{chi-2}$, respectively, if the distribution is of Gaussian type.

Figure 4 shows that the chi-square value increases, in general, with increasing long-term trend. With 95 % probability, a chi-square value equal to 15 corresponds approximately to a trend between $k = 5\%$ and 23% , and chi-square equal to 20 to a trend between $k = 7\%$ and 28% , respectively. On the other hand, it is seen that any trend cannot be detected meaningfully for case of chi-square value less than 10.

Figure 5 shows the results similar to Fig. 4, except adoption of the second and the third ranks of the maximum together with the first one. Comparing with Fig. 4, it is clearly seen that an estimation of long-term trend in Fig. 5 can be performed more accurately. It is concluded that detection of a long-term trend of extreme events can be made by using the chi-square value with greater accuracy for larger sample size.

Fig. 6. Same as Fig. 5, except for two cases of station number of 10 and 50, and illustrating only the range of $A_{chi-2} \pm 2\sigma_{chi-2}$.

3.3 Sample size effect on performance of the IY method

The results presented above are obtained for networks consisting of 50 stations. It is interesting to see how the results would change for a network of a smaller number of stations. Figure 6 illustrates the dependency of the chi-square value upon the long-term trend for two cases of station number of 10 and 50. The dependency is illustrated with 95 % confidence interval for samples involving the first, the second and third ranks of maximum. The value of chi-square for a network composed of 10 stations is clearly much less sensitive to the change of trend than the case of 50 stations. This implies that the data from a network composed of more stations can give a more accurate estimate of the trend.

Another issue is to see how is the effect of the lowest rank of extreme event adopted in a data sample. Figure 7 shows a dependency of the chi-square value upon the lowest rank to be involved in the data sample for the case of trend = 20 % and station number = 50. The width of the 95 % confidence interval is much larger for the first and the second ranks than those for the third, fourth, and so on. The width of the confidence interval for the third rank is different, but so small as to be disregarded, from that for the tenth rank. The smaller is the width of the confidence interval, the larger is the accuracy in the estimate of the long-term trend. This implies that the accuracy of the long-term trend estimate is almost the same, irrespective of choice of the lowest rank, except for the first and the second lowest rank.

4. Concluding Remarks

Validation by a Monte Carlo method using pseudo-random numbers is successfully performed

Fig. 7. Dependency of the chi-square value upon the lowest rank of the maximum adopted in the data sample. The 95% confidence interval is shown for 50 stations and a long-term trend of $k = 20\%$.

for the IY method of detecting any long-term trend of such extreme events as heavy rainfall. It is shown that a long-term trend can be quantitatively estimated by employing the chi-square value for uniform distribution of frequency as a measure. The accuracy of the estimate is improved with increasing size of data sample, while the choice of the lowest rank adopted in a data sample has no essential effect, except in both the cases of the first and the second rank. Further examination in the near future is required to survey whether other measures would

have more usefulness as a trend estimate than the chi-square measure used in the present work

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References

- Brooks, C.E.P. and N. Carruther, 1953: *Handbook of Statistical Methods in Meteorology*. Her Majesty's Stationary Office, 412 pp.
- Fushimi, M., 1989: *Random Number*, Univ. Tokyo Press., 164 pp. (In Japanese).
- Gordon, H.B., P.H. Whetton, A.B. Pittock, A.M. Flower and M.R. Haylock, 1992: Simulated changes in daily rainfall intensity due to the enhanced greenhouse effect: implications for extreme rainfall events. *Climate Dynamics*, **8**, 83–102.
- Gumbel, E.J., 1958: *Statistics of Extremes*. Columbia Univ. Press., 375 pp.
- IPCC, 1992: *Climate Change 1992—The Supplementary Report to the IPCC Scientific Assessment*. Cambridge Univ. Press., 200 pp.
- Iwashima, T. and R. Yamamoto, 1993: A statistical analysis of the extreme events: Long-term trend of heavy daily precipitation. *J. Meteor. Soc., Japan*, **71**, 637–640.
- Perry, A.H. and K.A. Howells, 1982: Are large falls of rain in Wales becoming more frequent? *Weather*, **37**, 240–243.
- Pollard, J.H., 1977: *A Handbook of Numerical and Statistical Techniques*. Cambridge Univ. Press, 349 pp.
- Tabony, R.C., 1983: Extreme value analysis in meteorology. *Meteor. Mag.*, **112**, 77–98.

極端に強い降雨の長期傾向の検出方法の有効性確認に対する モンテカルロ法の適用

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集中豪雨等極端に激しい現象の長期傾向の有無を調べるために、最大記録の現象の発現時期の頻度分布に着目した統計方法(IY法)が岩嶋と山元(1993)により提案されたが、この論文では、IY方法の正当性が擬似乱数を用いたモンテカルロ法により検討された。その結果、IY法の正当性が示されると共に、長期傾向の量的算定精度についても議論された。