NOTES AND CORRESPONDENCE

An Equation of the β -Gyre Rotation of a Cyclone in a Non-uniformly Straining Environmental Flow

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(Manuscript received 1 April 1996, in revised form 9 September 1996)

Abstract

An equation of the β -gyre rotation of a cyclone in a non-uniformly straining environmental flow is derived in a barotropic non-divergent system. The environmental flow, which is generally both zonally and meridionally varying, is assumed to be a quadratic function of coordinates. The total flow is assumed to consist of the environmental flow, a symmetric cyclonic circulation and a flow of β -gyres. The resulting equation says the followings. The β effect forces the β -gyre centres to lie perpendicular to the direction of β , i.e., of the gradient of the Coriolis parameter. The symmetric cyclonic circulation advects the β -gyres counter-clockwise. The deformational component of the environmental flow forces the β -gyres centres to lie along the dilatation axis of the deformation matrix. The rotational component of the environmental flow rotates the β -gyres with an angular velocity equal to half the environmental relative vorticity. The second derivative of the environmental flow affects the β -gyre rotation only through the relative vorticity gradient, and the other components of the second derivative have no influence on the rotation.

1. Introduction

As was shown by Adem and Lezama (1960), a barotropic symmetric cyclone in a uniform environmental flow on an f -plane is translated with exactly the same velocity of the environment.

On a β -plane, however, as was shown by many theoretical and numerical studies (e.g., Chan and Williams, 1987; Willoughby, 1988; Fiorio and Elsberry, 1989; Smith and Ulrich, 1990; Li and Wang, 1994), the advection of the planetary vorticity by a symmetric cyclonic circulation generates asymmetries in the cyclone motion. The asymmetries are dominated by an azimuthal wave-number-one component. This is a pair of a clockwise and a counter-clockwise circulation, respectively, to the east and to the west of the cyclone centre. These β -gyres are advected by the symmetric cyclonic circulation, and oriented in the northeast to southwest direction.

The β -gyres advect the cyclone centre, and the velocity of the cyclone translation becomes different from the environmental velocity at the cyclone centre. As a result, the orientation and intensity of

the β -gyres are of fundamental importance for the cyclone translation.

The β -gyres are also affected by the non-uniformity of the environmental flow. Some aspects of the relation between the β -gyres and environmental non-uniformity have been already revealed (e.g., Smith, 1991; Williams and Chan, 1994; Wang and Li, 1995 and Wang, 1996). However, the influence of the environmental non-uniformity on the β -gyres has not yet been fully clarified. In this note, an equation of the β -gyre rotation of a cyclone is derived in a non-divergent barotropic system on a β -plane. The environmental flow is zonally and meridionally varying and is a quadratic function of coordinates, and therefore non-uniformly straining.

The organization of this note is as follows. In Section 2, the formulation is presented. In Section 3, the case of uniform environment is considered. In Section 4, the effect of the non-uniformity is calculated. In Section 5, concluding remarks are given.

2. Formulation

The governing equation is the two-dimensional non-divergent barotropic momentum equation on a

β -plane,

$$\begin{aligned} \partial u_i / \partial t + u_j \partial_j u_i + \partial_i \phi - f \varepsilon_{ij} u_j + g_i &= 0, \\ \partial_i u_i &= 0, \end{aligned} \quad (2-1)$$

where g_i is an external force, f the Coriolis parameter and ϕ the geopotential. u_i and ∂_i are, respectively, the components of velocity and partial differential operator with the convention of summation, $u_i \partial_j u_i = \sum_{j=1}^2 u_j \partial_j u_i$, etc. The antisymmetric tensor $\varepsilon_{ij} = -\varepsilon_{ji}$ is defined as $\varepsilon_{ij} = \delta_{i1} \delta_{j2} - \delta_{i2} \delta_{j1}$, where δ_{ij} is Kronecker's delta symbol. By the definition,

$$\begin{aligned} \varepsilon_{ij} \varepsilon_{kl} &= \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}, \quad \varepsilon_{ij} \varepsilon_{ik} = \delta_{jk}, \\ \varepsilon_{ij} A_{ij} &= 0 \text{ if } A_{ij} = A_{ji}. \end{aligned} \quad (2-2)$$

The velocity field u_i is decomposed into an environmental flow U_i and an cyclone flow u_i^C ,

$$u_i = U_i + u_i^C, \quad (2-3)$$

where U_i is a solution of (2-1). From (2-1) and (2-3), the cyclone flow u_i^C satisfies that

$$\begin{aligned} \partial u_i^C / \partial t + U_j \partial_j u_i^C + u_j^C \partial_j U_i + u_j^C \partial_j u_i^C \\ + \partial_i \phi^C - f \varepsilon_{ij} u_j^C &= 0, \\ \partial_i u_i^C &= 0. \end{aligned} \quad (2-4)$$

In a coordinate system co-moving with the centre of the cyclone, the above equation is transformed into

$$\begin{aligned} \partial u_i^C / \partial t + (U_j - \bar{u}_j) \partial_j u_i^C + u_j^C \partial_j U_i \\ + u_j^C \partial_j u_i^C + \partial_i \phi^C - f \varepsilon_{ij} u_j^C &= 0, \end{aligned} \quad (2-5)$$

where $\bar{u}_i = dx_i/dt$ and $\bar{x}_i = \bar{x}_i(t)$ is the cyclone centre.

Let (r, θ) be polar coordinates with the origin $\bar{x}_i$. Then, $x_i - \bar{x}_i = r T_i(\theta)$ where $T_1(\theta) = \cos \theta$ and $T_2(\theta) = \sin \theta$. By the definition,

$$\begin{aligned} T_i(\theta) T_i(\theta) &= 1, \quad dT_i(\theta)/d\theta = -\varepsilon_{ij} T_j(\theta), \\ \partial_i &= T_i(\theta) \partial / \partial r - \{\varepsilon_{ij} T_j(\theta) / r\} \partial / \partial \theta, \\ \langle F(r) T_i(\theta) T_j(\theta) \rangle &= \delta_{ij} \langle F(r) / 2 \rangle \end{aligned}$$

where

$$\langle (\dots) \rangle = \int_0^\infty dr r \int_0^{2\pi} d\theta (\dots). \quad (2-6)$$

The environmental flow U_i is assumed to be a quadratic function of the coordinates,

$$\begin{aligned} U_i(x) &= \bar{U}_i + r T_j(\theta) \bar{\partial}_j \bar{U}_i \\ &+ r^2 T_j(\theta) T_k(\theta) \bar{\partial}_j \bar{\partial}_k \bar{U}_i / 2, \end{aligned} \quad (2-7)$$

where $\bar{U}_i = U_i(\bar{x})$ etc. The cyclone flow u_i^C is assumed to consist of two components of wave number

zero and one,

$$\begin{aligned} u_i^C &= -\varepsilon_{ij} \partial_j \psi^C, \quad \psi^C = \psi^S + \psi^G, \\ \psi^S &= \int^r dr S(r, t), \\ \psi^G &= G(r, t) T_i(\alpha) T_i(\theta), \\ S(r, t) > 0, \quad G(r, t) > 0, \\ S(0, t) &= G(0, t) = S(\infty, t) = G(\infty, t) = 0. \end{aligned} \quad (2-8)$$

$u_i^S = -\varepsilon_{ij} \partial_j \psi^S$ represents a cyclonic symmetric circulation, and $u_i^G = -\varepsilon_{ij} \partial_j \psi^G$ is assumed to represent a flow of β -gyres. $\alpha = \alpha(t)$ is the azimuthal angle of the anticyclonic β -gyre centre, i.e., $T_i(\alpha)$ is a unit vector pointing to the anticyclonic β -gyre centre. From (2-8), using (2-6), u_i^S and u_i^G are respectively written in the polar coordinates as

$$\begin{aligned} u_i^S &= -S \varepsilon_{ij} T_j(\theta), \\ u_i^G &= -G' T(\alpha) \cdot T(\theta) \varepsilon_{ij} T_j(\theta) \\ &+ (G/r) T(\alpha) * T(\theta) T_i(\theta), \end{aligned} \quad (2-9)$$

where $G' = \partial G / \partial r$, $T(\alpha) \cdot T(\theta) = T_i(\alpha) T_i(\theta)$ and $T(\alpha) * T(\theta) = \varepsilon_{ij} T_i(\alpha) T_j(\theta)$.

The translation velocity $\bar{u}_i$ of the cyclone centre is different from the steering velocity $\bar{U}_i$. The difference $\bar{u}_i - \bar{U}_i$ is assumed to be caused by the β -gyres and proportional to $u_i^G(\bar{x})$, though other causes may also be present. Since $u_i^G(\bar{x})$ is perpendicular to $T_i(\alpha)$ from (2-8) and (2-9), the difference is perpendicular to $T_i(\alpha)$,

$$\begin{aligned} T_i(\alpha) (\bar{u}_i - \bar{U}_i) &= 0, \\ \varepsilon_{ij} T_i(\alpha) (\bar{u}_j - \bar{U}_j) &\propto \varepsilon_{ij} T_i(\alpha) u_j^G(\bar{x}) > 0. \end{aligned} \quad (2-10)$$

3. Equation of the β -gyre rotation in uniform environment

First, a vector $R_i = R_i(r, \theta, \alpha)$ is defined,

$$R_i(r, \theta, \alpha) = (1/r) T(\alpha) \cdot T(\theta) T_i(\theta). \quad (3-1)$$

Multiplying (3-1) by ∂_i and using (2-2) and (2-6) gives

$$\begin{aligned} \partial_i R_j &= \{\delta_{ij} T(\alpha) \cdot T(\theta) - 3 T(\alpha) \cdot T(\theta) T_i(\theta) T_j(\theta) \\ &+ T_i(\alpha) T_j(\theta)\} / r^2. \end{aligned} \quad (3-2)$$

From (3-2), it is easily seen that R_i is non-divergent,

$$\partial_i R_i = 0. \quad (3-3)$$

Multiplying (2-5) by R_i , integrating over the whole domain, substituting the decompositions (2-7) and (2-8), and using the non-divergence (3-3) and the orthogonality of $T_i(\theta)$ yields

$$\begin{aligned} \langle R_i \partial u_i^C / \partial t \rangle &- (\bar{u}_j - \bar{U}_j) \langle R_i \partial_j u_i^S \rangle \\ &+ \langle R_i u_j^G \partial_j u_i^S \rangle + \langle R_i u_j^S \partial_j u_i^G \rangle \\ &- \bar{f} \langle R_i \varepsilon_{ij} u_j^G \rangle - \langle r R_i \beta_k T_k(\theta) \varepsilon_{ij} u_j^S \rangle \\ &+ \bar{\partial}_k \bar{U}_j \langle r T_k(\theta) R_i \partial_j u_i^G \rangle + \bar{\partial}_j \bar{U}_i \langle R_i u_j^G \rangle \\ &+ (1/2) \bar{\partial}_i \bar{\partial}_k \bar{U}_j \langle r^2 T_i(\theta) T_k(\theta) R_i \partial_j u_i^S \rangle \\ &+ \bar{\partial}_k \bar{\partial}_j \bar{U}_i \langle r T_k(\theta) R_i u_j^S \rangle = 0, \end{aligned} \quad (3-4)$$

where $\bar{f} = f(\bar{x})$ and β_i is the gradient of the Coriolis parameter. In the case of uniform environmental flows, the last four terms in (3-4) disappear. Substituting (2-9) and (3-1) into the first term in (3-4) and using (2-2), (2-6) and the orthogonality of $T_i(\theta)$ gives

$$\begin{aligned} & \langle R_i \partial u^G_i / \partial t \rangle \\ &= \langle r^{-2} T(\alpha) \cdot T(\theta) (\partial / \partial t) \{ G T(\alpha) * T(\theta) \} \rangle \\ &= - \langle G / 2r^2 \rangle d\alpha / dt. \end{aligned} \quad (3-5)$$

Further using (3-2), the second term becomes

$$\begin{aligned} & -(\bar{u}_j - \bar{U}_j) \langle R_i \partial_j u^S_i \rangle \\ &= (\bar{u}_j - \bar{U}_j) \langle u^S_i \partial_j R_i \rangle \\ &= \langle S / 2r^2 \rangle \varepsilon_{ij} T_i(\alpha) (\bar{u}_j - \bar{U}_j). \end{aligned} \quad (3-6)$$

The third and fourth terms become

$$\begin{aligned} & \langle R_i u^G_j \partial_j u^S_i \rangle + \langle R_i u^S_j \partial_j u^G_i \rangle \\ &= - \langle u^S_i u^G_j (\partial_i R_j + \partial_j R_i) \rangle \\ &= \langle (SG / 2r^3) - (SG' / r^2) \rangle \\ &= \int_0^\infty dr (r^{-1/2} S)' (r^{-1/2} G). \end{aligned} \quad (3-7)$$

The fifth and sixth terms become

$$\begin{aligned} & -\bar{f} \langle R_i \varepsilon_{ij} u^G_j \rangle - \langle r R_i \beta_k T_k(\theta) \varepsilon_{ij} u^S_j \rangle \\ &= - \langle S / 2 \rangle \beta_i T_i(\alpha). \end{aligned} \quad (3-8)$$

From (3-5), (3-6), (3-7) and (3-8), the following equation of the β -gyre rotation is obtained:

$$\begin{aligned} d\alpha / dt &= \{ \langle S / r^2 \rangle / \langle G / r^2 \rangle \} \varepsilon_{ij} T_i(\alpha) (\bar{u}_j - \bar{U}_j) \\ &+ \int_0^\infty dr (r^{-1/2} S)' (r^{-1/2} G) / \langle G / 2r^2 \rangle \\ &- \{ \langle S \rangle / \langle G / r^2 \rangle \} \beta_i T_i(\alpha). \end{aligned} \quad (3-9)$$

Since $\bar{u}_i - \bar{U}_i \propto u^G_i(\bar{x})$, both the first and second terms on the RHS in (3-9) are nearly independent of the intensity of the β -gyres. Because of (2-10), the first term is positive, while the second term is expected to be negative, since the maximal point of G usually lies far outside of the maximal radius of S . The first term, which represents the advective effect, rotates the centres of β -gyres counter-clockwise. While, the second term rotates them clockwise. The last term is inversely proportional to the magnitude of G . This term represents the creation of the β -gyres by the symmetric circulation.

If, initially, the β -gyres are small, *i.e.*, $G \simeq 0$ and therefore $\bar{u}_i - \bar{U}_i \simeq 0$, then the last term in (3-9) dominates. This term forces $T_i(\alpha)$ to be perpendicular to β_i , which is toward the north. Consequently, the initial centre of the anticyclonic β -gyre lies to the east of the cyclone centre (the other possibility to the west is physically unrealistic). In the initial

stage, the maximal point of G lies near the maximal radius of S , and hence the second term is small compared with the first.

As the β -gyres grow, the first term rotates their centres counter-clockwise as long as the second term is small compared with the first. As a result, the anticyclonic (cyclonic) center lies in the northeast (southwest) direction. As the β -gyres grow further and the maximal point of G moves outwards, the second term rotates their centres clockwise.

4. Case of a Non-uniformly straining environmental flow

In the case of the non-uniformly straining environmental flow (2-7), from (3-4) there emerge four other terms in the equation of the β -gyre rotation,

$$\begin{aligned} & \overline{\partial_k U_j} \langle r T_k(\theta) R_i \partial_j u^G_i \rangle \\ &+ \overline{\partial_j U_i} \langle R_i u^G_j \rangle \\ &+ (1/2) \overline{\partial_l \partial_k U_j} \langle r^2 T_l(\theta) T_k(\theta) R_i \partial_j u^S_i \rangle \\ &+ \overline{\partial_k \partial_j U_i} \langle r T_k(\theta) R_i u^S_j \rangle. \end{aligned} \quad (4-1)$$

In order to calculate these terms the following equations are used. Firstly,

$$\begin{aligned} \overline{D_{ij} T_i(\theta) T_j(\theta)} &= \bar{\lambda} T_i(2\gamma) T_i(2\theta) \\ &= \bar{\lambda} T_1(2\theta - 2\gamma), \\ \overline{D_{ij} \varepsilon_{ki} T_k(\theta) T_j(\theta)} &= -\bar{\lambda} \varepsilon_{ij} T_i(2\gamma) T_j(2\theta) \\ &= -\bar{\lambda} T_2(2\theta - 2\gamma), \end{aligned} \quad (4-2)$$

where $\overline{D_{ij}} = (\overline{\partial_i U_j} + \overline{\partial_j U_i}) / 2$ is the deformation of U_i at $x_i = \bar{x}_i$, which is a symmetric traceless matrix. $\bar{\lambda} = \{ (\overline{\partial_1 U_2} + \overline{\partial_2 U_1})^2 + 4(\overline{\partial_1 U_1})^2 \}^{1/2} / 2$ is the strain rate of U_i at $x_i = \bar{x}_i$, which is the positive eigenvalue of $\overline{D_{ij}}$. γ is the azimuthal angle of the dilatation axis of $\overline{D_{ij}}$, *i.e.*, $T_i(\gamma)$ and $\varepsilon_{ji} T_j(\gamma)$ are, respectively, the eigenvectors belonging to the eigenvalue $\bar{\lambda}$ and $-\bar{\lambda}$. Secondly,

$$\begin{aligned} \overline{R_{ij} T_i(\theta) T_j(\theta)} &= 0, \\ \overline{R_{ij} \varepsilon_{ki} T_k(\theta) T_j(\theta)} &= -\bar{\omega} / 2, \end{aligned} \quad (4-3)$$

where $\overline{R_{ij}} = (\overline{\partial_i U_j} - \overline{\partial_j U_i}) / 2 = (\bar{\omega} / 2) \varepsilon_{ij}$ is the rotation of U_i at $x_i = \bar{x}_i$, which is an antisymmetric matrix. $\bar{\omega} = \varepsilon_{ij} \overline{\partial_i U_j}$ is the vorticity of U_i at $x_i = \bar{x}_i$. Thirdly,

$$\begin{aligned} & \langle F(r) T_i(\theta) T_j(\theta) T_k(2\theta) \rangle \\ &= \langle F(r) \rangle (\delta_{i1} \delta_{jk} + \delta_{i2} \varepsilon_{jk}) / 4, \end{aligned} \quad (4-4)$$

which is derived from the orthogonality of $T_i(2\theta)$. Finally,

$$\begin{aligned} & \overline{\partial_k D_{ij} \varepsilon_{li} T_l(\theta) T_j(\theta)} \\ &= -\bar{\lambda}_k \varepsilon_{ij} T_i(2\gamma_k) T_j(2\theta), \\ & \overline{\partial_i D_{ij} \varepsilon_{kj} T_k(\theta)} \\ &= -(\delta_{i1} \varepsilon_{jk} + \delta_{i2} \delta_{jk}) \lambda_i T_j(2\gamma_i) T_k(\theta), \end{aligned} \quad (4-5)$$

where $\bar{\lambda}_k$ is the positive eigenvalue of the traceless symmetric matrix $\bar{\partial}_k \bar{D}_{ij}$, and $\bar{\gamma}_k$ is the azimuthal angle of the dilatation axis of $\bar{\partial}_k \bar{D}_{ij}$.

Integrating by parts and using $\bar{\partial}_i \bar{U}_i = 0$, and substituting (2-9), (3-1) and (3-2), and further using (4-2), (4-3) and (4-4), the first two terms of (4-1) become

$$\begin{aligned} & -\bar{\partial}_k \bar{U}_j \langle r T_k(\theta) u^G_i \partial_j R_i \rangle + \bar{\partial}_j \bar{U}_i \langle R_i u^G_j \rangle \\ & = -\langle G/4r^2 \rangle \bar{\lambda} T_2(2\alpha - 2\bar{\gamma}) + \langle G/4r^2 \rangle \bar{\omega}. \quad (4-6) \end{aligned}$$

In the same way, and further using (4-5), the last two terms of (4-1) become

$$\begin{aligned} & - (1/2) \bar{\partial}_l \bar{\partial}_k \bar{U}_j \langle r^2 T_l(\theta) T_k(\theta) u^S_i \partial_j R_i \rangle \\ & + \bar{\partial}_k \bar{\partial}_j \bar{U}_i \langle r T_k(\theta) R_i u^S_j \rangle \\ & = - (1/2) (\bar{\partial}_k \bar{D}_{ji}) \langle S T_k(\theta) T_i(\theta) \varepsilon_{jl} T_l(\theta) T(\alpha) \cdot T(\theta) \rangle \\ & - (3/2) (\bar{\partial}_k \bar{R}_{ji}) \langle S T_k(\theta) T_i(\theta) \varepsilon_{jl} T_l(\theta) T(\alpha) \cdot T(\theta) \rangle \\ & = - \langle S/8 \rangle (\delta_{k1} \varepsilon_{ij} + \delta_{k2} \delta_{ij}) \bar{\lambda}_k T_i(2\bar{\gamma}_k) T_j(\alpha) \\ & - \langle 3S/8 \rangle \bar{\partial}_i \bar{\omega} T_i(\alpha) \\ & = \langle S/8 \rangle \bar{\partial}_i \bar{D}_{ij} \varepsilon_{kj} T_k(\alpha) - \langle 3S/8 \rangle \bar{\partial}_i \bar{\omega} T_i(\alpha) \\ & = - \langle 5S/16 \rangle \bar{\partial}_i \bar{\omega} T_i(\alpha). \quad (4-7) \end{aligned}$$

As a result, from (3-10), (4-6) and (4-7), an equation of the β -gyre rotation in the non-uniformly straining environmental flow is obtained as

$$\begin{aligned} d\alpha/dt & = \{ \langle S/r^2 \rangle / \langle G/r^2 \rangle \} \varepsilon_{ij} T_i(\alpha) (\bar{u}_j - \bar{U}_j) \\ & + \int_0^\infty dr (r^{-1/2} S)' (r^{-1/2} G) / \langle G/2r^2 \rangle \\ & - \{ \langle S \rangle / \langle G/r^2 \rangle \} \beta_i T_i(\alpha) \\ & - (\bar{\lambda}/2) T_2(2\alpha - 2\bar{\gamma}) + \bar{\omega}/2 \\ & - \{ \langle 5S/8 \rangle / \langle G/r^2 \rangle \} \bar{\partial}_i \bar{\omega} T_i(\alpha). \quad (4-8) \end{aligned}$$

The additional two terms due to $\bar{\partial}_i \bar{U}_j$ are exactly independent of the intensity of the β -gyres. The term due to the deformation $\bar{D}_{ij}$ forces $T_i(\alpha)$ to be parallel to $T_i(\bar{\gamma})$, i.e., to lie on the dilatation axis of $\bar{D}_{ij}$. The term due to the rotation $\bar{R}_{ij}$ forces $T_i(\alpha)$ to rotate with an angular velocity $\bar{\omega}/2$. The term due to $\bar{\partial}_i \bar{\partial}_j \bar{U}_k$ depends only on the vorticity gradient $\bar{\partial}_i \bar{\omega}$, and has qualitatively the same effect as the β term.

5. Conclusion

The results of this note are summarized in the Eq. (4-8). The effect of rotational component of the first derivative of the environmental flow is trivial, which means rotation with an angular velocity equal to half the environmental vorticity. On the other

hand, the effect of deformational component of the first derivative is not trivial, which means orientation of the β -gyre centres along the dilatation axis, i.e., orientation of the β -gyres along the contraction axis. This orientation is favourable for the β -gyres to extract energy from the environment (see e.g., Mak and Cai, 1989; Cai, 1992).

The effect of the second derivative of the environmental flow is reduced to that of the relative vorticity gradient. The gradient has qualitatively the same influence on the β -gyre rotation as the planetary vorticity gradient. The effect of the other components of the second derivative happens to vanish. To examine the effects of higher derivatives of the environmental flow and of higher wave number components of asymmetries is left for future study.

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歪んだ一般流の中のサイクロンの β -Gyreの回転の方程式

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歪んだ一般流の中のサイクロンの β -Gyreの回転の方程式を順圧非発散の系で導いた。一般流は、東西および南北方向に変動しているが、座標の2次関数と仮定した。全体の流れは、一般流と軸対称循環と β -Gyreの流れから成ると仮定した。得られた方程式は次の事を示した。 β 効果は β -Gyreの中心をコリオリ因子の勾配に垂直な方向へ向かせる様に働く。軸対称循環は β -Gyreを反時計回りに回す。一般流の変形成分は β -Gyreの中心が変形行列の伸張軸に沿う様に働く。一般流の回転成分は β -Gyreを相対渦度の半分の角速度で回転させる。一般流の二階微分係数は相対渦度の勾配を通じてのみ β -Gyreの回転に寄与し、これは β 効果と定性的に同じ働きをする。