NOTES AND CORRESPONDENCE

Experimental Observations of 3-D Lagrangian Motion in a Steady Baroclinic Wave

By T. Tajima, T. Nakamura and K. Kurokawa

Toyama National College of Technology, 13 Hongo-machi, Toyama City 939, Japan

(Manuscript received 4 September 1996, in revised form 30 October 1996)

Abstract

Chaotic Lagrangian motion in a steady baroclinic wave, which is highly relevant to meteorological and oceanographical problems, was investigated by Sugata and Yoden (1994) by tracing a marked fluid particle for a long time in a numerical solution of their model. To test their result, we have conducted experiments on steady baroclinic waves in a differentially-heated rotating fluid annulus by tracking a 3-D trajectory of one neutrally buoyant tracer particle suspended in the fluid for a long time. We show here four trajectories that have been analyzed up to now. In the wavenumber-5 wave, the observed trajectories include the preferred routes of region transitions expected from the numerical investigation. This supports the Lagrangian view of the heat transport presented by Sugata and Yoden: The jet absorbs a large number of hot fluid particles from the outer boundary layer and releases them in the inner boundary layer, while the fluid particles nearly conserve their temperature in the meandering jet. Furthermore, it is of great interest that there was found one event of region transition which does not take place in the numerical simulation. In the wavenumber-4 wave, however, the tracer particle was observed to remain trapped within the jet for a long time. This leads to the orthodox view of the heat transport: The jet absorbs a large amount of heat from the outer boundary and releases it into the inner boundary so that heating and cooling of the fluid particles take place during every cycle of the meander of the jet.

1. Introduction

Studies of chaotic Lagrangian motion of a fluid particle have been well known to be of fundamental importance in many branches of fluid dynamics, extensively so in the field of meteorology and oceanography (*cf.* Flierl, 1981; Pierrehumbert, 1991; Moses and Steinberg, 1986). Sugata and Yoden (1994), hereafter referred to as SY, investigated the Lagrangian motion in a steady baroclinic wave of wavenumber-5 by tracing a marked fluid particle for a long time in a numerical solution of their model (Sugata and Yoden, 1993). As was schematically shown in our previous papers (Tajima and Nakamura, 1995; Tajima *et al.*, 1995a, 1995b), the steady baroclinic wave has been well known to have the following basic structure: the upper (UJ, eastward)- and lower (LJ, westward)-level jets, the high (HV)- and low (LV)-pressure vortices, and the inner (IB)-, outer (OB)- and lower (LB)-boundary

layers. Based on this structure, Sugata and Yoden calculated the residence time of the fluid particle in each region and the transition matrix which expresses frequencies of transitions between the regions. Among many features of their trajectory, it is of great interest that several cyclic routes of region transitions are remarkably preferred in the chaotic motion and play a dominant role in mass and heat transport between the warm and cold sides. Then they obtained the Lagrangian view of the heat transport: The jet absorbs a large number of hot fluid particles from the outer boundary layer and releases them in the inner boundary layer, while the fluid particles nearly conserve their temperatures in the meandering jet. In the orthodox view, on the other hand, it is a large amount of heat that the jet absorbs from the outer boundary and releases into the inner boundary. Then heating and cooling of the fluid particle take place in every cycle of the meander of the jet.

To test the result of SY, we have conducted exper-

Fig. 1. Schematic of the apparatus having two independent turntables and the monitor system. Here, **a** is videocamera, **b** scaffold, **c** ultraviolet light projector, **d** annular tank, **e** belt, **f** motor, **g** carbon brush, **h** large turntable, **i** small turntable, **j** transmitter. The videopictures taken by the videocamera **A** and **B** are transmitted to the receiver **A** and **B**, respectively.

iments on steady baroclinic waves in a differentially-heated rotating fluid annulus by tracking a 3-D trajectory of one neutrally buoyant tracer particle in the fluid. In this paper we show four trajectories that have been analyzed up to now.

In the wavenumber-5 wave, the observed trajectories include the preferred routes of region transitions expected from the numerical investigation. This supports the Lagrangian view of the heat transport presented by Sugata and Yoden. Furthermore, it is of great interest that there was also found one event of region transition which does not take place among the total 455 transitions counted in the numerical simulation. In the wavenumber-4 wave, however, the tracer particle was observed to remain trapped within the jet for a long time (more than one hour). This leads to the orthodox view.

In the low-pressure vortex, the tracer particle was observed to spiral up in the core region and down in the transition zone (which is sometimes called a separatrix layer in the papers concerning a chaotic advection (*cf.* Weiss and Knobloch, 1989)).

We have had only a few observed trajectories and their total durations are very short so that we could not get statistically reliable data of the residence times and transition matrix of the fluid particle.

2. The apparatus and experimental procedures

2.1 Overall view of apparatus

A schematic of the present apparatus (Tajima *et al.*, 1995a, 1995b) is illustrated in Fig. 1. The apparatus has two concentric turntables which can be independently rotated. An annular glass tank (Fig. 2; see Tajima *et al.* (1995a) for the detailed explana-

Fig. 2. Schematic of the set-up of the annular glass tank. The upper part of the figure shows the top view, the lower a cross section along the line **AA** in the top view. The symbols α , β and γ are test, warm, and cold baths, respectively. The dashed circle in the top view describes the inner boundary of the bottom of the test bath. Here, the unit of length is mm.

tion of its thermal control system.) is placed at the center of a small upper turntable which is rotated at an angular velocity Ω . Measuring instruments, such as videocameras and an ultraviolet light projector are mounted on the large lower turntable. Two videocameras are used here. Videocamera **A** is set

above the tank to observe the horizontal position of the tracer particle and videocamera B is fixed at the side of the tank to observe the vertical position. The ultraviolet light projector is set above the videocamera B. The views of these videocameras cover about a quarter of the test fluid below the ultraviolet light projector. Pictures of the videocameras A and B are simultaneously transmitted through the receivers to the video mixer which makes the horizontal and vertical pictures appear on the left and right halves of one screen, respectively. The pictures coming from the video mixer are recorded on video tape and simultaneously displayed on the monitor TV.

In our previous papers, we used a cylindrical glass as the inner wall of the test bath in the tank. Then a dead angle appears around the inner wall when the fluid is observed from above the tank by the videocamera A. To get rid of it, we use here a slightly tapered aluminum wall instead of the cylindrical glass one.

2.2 Tracer particles

Neutrally buoyant tracer particles are made from acrylic resin-type fluorescent color paint spray (produced by the Japan Paint Co.). When this paint is dried and gets hard, its density is found to be quite near to that of the working fluid (water in the present experiments). The dried paint is ground into particles with a size of approximately less than 0.2 mm. To confirm the buoyancy of the particles for more than 30 minutes, they are chosen from those that keep suspended in the static water of depth 5 cm and temperature 20°C for more than 30 minutes. When illuminated by ultraviolet light, the particle emits a point-light in the dark background.

2.3 Experimental condition

In the present paper, all experiments were conducted under the following conditions: $d = 5$ cm, $T_W = 28^\circ\text{C}$, $T_C = 12^\circ\text{C}$, room temperature $T_R = 20^\circ\text{C}$ and $\Omega = 0.83$ rad s^{-1} . Then the thermal Rossby and Taylor numbers are estimated to be $\Theta = 1.25$ and $T_a = 2.5 \times 10^6$, respectively. Either steady wavenumber-4 or 5 baroclinic waves appear mainly under this condition. The wavenumber-4 and 5 waves drift eastward at angular velocities of $\omega = 0.0228$ rad s^{-1} and 0.0230 rad s^{-1} , respectively, in the frame fixed to the annulus. Which mode we obtain in an experiment depends on how the test fluid arrives at this condition (Tamaki and Ukaji, 1985). Here, however, we did not use manipulation to choose the mode in the experiments on tracking a tracer particle.

SY treated only the wavenumber-5 wave in their numerical model. The size of the tank in the model is very close to that in the present experiments and the physical parameters are $d = 8$ cm, $\Delta T (= T_W - T_C) = 3^\circ\text{C}$ and $\Omega = 0.60$ rad s^{-1} . Then, the thermal Rossby and Taylor numbers of

the model ($\Theta = 0.655$; $T_a = 9.15 \times 10^6$) are rather close to those of the present experiments.

2.4 Experimental procedure

The experiment starts with a few tracer particles being injected into the working fluid. After a transient period of about 20 minutes from the onset of the rotation of the tank, a steady flow of wavenumber-4 or 5 is expected to appear and we begin to search for a suspended particle in the fluid by rotating the large turntable for the videocameras to catch it while watching the monitor TV. Once the particle is found, tracing the particle is carried out by manipulating the rotation speed in accordance with the motion of the particle so as to keep the point-light on the TV screen.

The physical trajectory of the suspended particle inside the tank is derived from the recorded videopictures. We use a personal computer on whose screen a pointer is displayed and the coordinates of its top are registered in a computer disk when the button of the mouse is clicked. To count seconds, blip sounds are after-recorded periodically at every second on the videopictures. Thus the spot of the point-light on the videopictures is analyzed into digital data of its coordinates measured on the TV screen by putting the top of the pointer to the spot while manipulating a computer mouse and clicking it manually in response to the blips. This procedure is done twice to analyze the horizontal and vertical videopictures. Finally we need a calculation formula to calibrate the physical positions of the particle from the digital data. We selected several fixed points inside the fluid and derived the required formula by comparing their physical positions inside the tank and coordinates measured on the TV screen.

3. Observed trajectories

We will present four observed trajectories. In the following figures trajectories are described in the co-rotating frame of the drifting waves (Flierl, 1981). The beginning of the trajectory is marked by a black dot and the end by a circle.

The first trajectory was observed in a wavenumber-4 wave, which is described in Fig. 3. It shows that the particle on the trajectory remains trapped within the lower-level, westward jet for the entire duration of 3600s. As described in (b), the particle ascends going to the inner side and descends to the outer side. This motion is clearly seen in (e) where the trajectory is projected onto the meridional plane and, furthermore, the periodic time-variation of height is shown in (f). The meridional cross sections (c) and (d) show that the particle appears to be trapped within a narrow tube.

The second and third observed trajectories were found in the wavenumber-5 wave and are shown in

Fig. 3. Trajectory observed in the lower-level jet of the wavenumber-4 wave: (a) and (b) are the overall horizontal projection of the trajectory formed in the total duration 3600 s and its bird's-eye view, respectively; (c) and (d) are the meridional cross sections along the lines indicated by A and B in (a), respectively; (e) shows the trajectory projected onto the meridional plane; (f) shows the time-dependence of height.

Figs. 4 and 5, respectively. The overall horizontal projections of these are given in (a) and their divided sections in (c) and (e) of Figs. 4 and in (b)–(e) of Fig. 5 to see detailed movements of the particle. Their time-variations of height are shown in (g). The symbols of the regions that the tracer particle passes are shown on the abscissa of the graph (g), from which the routes of the trajectories can be realized.

Compared with the first trajectory, the particle shows frequent transitions among the regions. In the numerical investigation (SY) of the wavenumber-5 wave, the frequencies of the transitions were counted to thus make a transition matrix. All the transitions that happened in the second and third trajectories are found to be relatively preferred ones in that transition matrix, except for the one from

the outer-boundary to the lower-jet that happens at 817s in the third trajectory. This exception does not take place among the total 455 transitions counted in the numerical investigation.

At the final stage (e) of the third trajectory, the particle is transported from the low- to high-level jet through the upward-flow in the outer-boundary layer. This transition between the two jets through the outer-boundary layer is included in the preferred cyclic routes of region transitions shown by SY (see Fig. 10 in SY).

From these trajectories we can see clearly the behavior of the particle in the low-pressure vortex. It appears most clearly in the duration 350–680 s of Fig. 4. In that time interval the particle remains trapped within the low-pressure vortex. As found

Fig. 4. The first trajectory observed in the wavenumber-5 wave: The thin vertical lines are projections of the trajectory onto the upper or lower boundary with an interval of five seconds. (a) and (b) are the overall horizontal projection of the trajectory formed in the total duration 1128s and its bird's-eye view, respectively; (c) and (d) describe the divided trajectory of duration 0–345 s and its bird's-eye view, respectively. Here the trajectory in the upper-level jet is drawn by a bold line; (e) and (f) describe the divided trajectory of duration 345–1128 s and its bird's-eye view, respectively. Here the trajectory in the outer-boundary layer is drawn by a bold line; (e) shows the time-dependence of height. Here the regions in which the tracer particle resides are shown above the upper abscissa by their abbreviations defined in Section 1.

Fig. 5. The second trajectory observed in the wavenumber-5 wave: (a) is the overall horizontal projection of the trajectory formed in the total duration 1975 s; (b), (c), (d) and (e) are the divided trajectories of duration 0–510 s, 510–1200 s, 1200–1460 s and 1460–1975 s, respectively. Here the trajectories in the outer-boundary are drawn by bold lines. In (e) a thin line indicates a trajectory in the lower-level jet; (f) describes the bird's-eye view of (e), where a bold line indicates a trajectory in the outer-boundary layer; (g) shows the time-dependence of height.

Fig. 6. Trajectory observed in the low-pressure vortex: (a) and (b) are the overall horizontal projection of the trajectory formed in the total duration 1139s and its bird's-eye view, respectively; (c) shows the time-dependence of height.

in (e), the height becomes greater as the particle goes close to the inner side in this vortex and lower to the outer side, but decreases gradually with every turn. As was found in SY and experimentally in our previous paper (Tajima *et al.*, 1995a, 1995b), the structure of vortices is composed of a core region, which is rather well isolated, and next to the core, a transition zone (sometimes called a separatrix layer in the papers concerning a chaotic advection (*cf.* Weiss and Knobloch, 1989)) where fluid particles are frequently transported to and from its outside, but rarely to the core. The residence time in the transition zone must be rather shorter than in the core. In the second and third trajectories all the residence times in the low-pressure vortex are less than 330s and much shorter than the time over 1100s in which the particle keeps trapped in the core in the fourth trajectory that will be given later. Hence it is safe to assume that the particle remains trapped within the transition zone when trapped in the low-pressure vortex in the second and third trajectories.

As to the behavior of the particle in the high-pressure vortex, we find four residences in the high-pressure vortex in Figs. 4 and 5. In all these residences the particle is found to revolve anticyclonically around the outside of the core, which means that it remains trapped within the transition zone. Though the tendency of the time-variation of height is uncertain in the duration 0–170 s of the second trajectory and 1266–1308 s of the third one, the particle clearly shows the descending motion during the other two occasions. Though we did not gain evidence for the ascending motion of the particle, as shown in Fig. 11 of SY, the numerical investigation

expects that the particle spirals up and down in the transition zone of the high-pressure vortex.

Figure 6 shows the fourth trajectory which describes the particle trapped within the core of the low-pressure vortex for more than 1100 s. Unfortunately the wavenumber of the flow can not be discriminated from this trajectory. As found from (c), the particle spirals up cyclonically in the core and reaches near the top-surface of the fluid. After that, it transfers to the inner-boundary layer and finally descends rapidly to the lower-boundary layer. SY showed a trajectory having the same route as this in Fig. 11 of their paper. As was seen in the second and third trajectories, on the other hand, the particle spirals down in the transition zone of the low-pressure vortex. As found from (c), furthermore, the variation of height in one period is quite smaller than in the transition zone. These features reflect the advection anticipated in the core. Up to now, on the other hand, we have not found a trajectory including a stay in the core of the high-pressure vortex.

4. Concluding remarks

In all four trajectories that have been analyzed so far, the total duration of one tracing was one hour at most and the number of the observed trajectories was one or two for each wavenumber. Therefore, we could not here estimate reliable data of the residence times and transition matrix. These data are of fundamental importance in the studies of the mixing and transport properties and characterize the Lagrangian motion of each wavenumber's wave. To get statistically reliable data, the total duration should

be much longer (more than 10 hours) in one tracing and many trajectories should be observed for each fixed wavenumber. The result of such experiments will be presented in a forthcoming paper.

When the second and third trajectories in the wavenumber-5 wave are compared with the first one in the wavenumber-4 wave, we should notice that the residence time in each region is very short (less than 7 minutes) in the former wave, while the particle keeps trapped within the lower-level jet for a very long time (more than one hour) in the latter. This shows one of the examples of the wavenumber-dependence of the Lagrangian motions and it is very important to find that these features support quite different views of the heat transport, namely SY's Lagrangian view and the orthodox one which were explained in the introduction and depend on the wavenumber. Therefore, we hope that numerical investigations such as SY will be carried out to investigate the wavenumber-dependence of the Lagrangian motion.

Acknowledgments

This work is funded partially by a Grant in aid of the Japanese Government Ministry of Education, Science, Sports and Culture, No. 07640582. The authors wish to thank Dr. S. Sugata and Prof. S. Yoden for stimulating us to develop a technique for observing 3-D Lagrangian trajectories and for helpful discussions concerning our preliminary result. They also wish to thank Profs. R. Kimura and K. Kawahira for valuable comments and encouragement.

References

- Flierl, G.R., 1981: Particle motions in large-amplitude wave fields. *Geophys. Astrophys. Fluid Dyn.*, **18**, 39–74.
- Mosses, E. and V. Steinberg, 1986: Flow patterns and nonlinear behavior of traveling waves in a convective binary fluid. *Phys. Rev. A*, **34**, 693–696.
- Pierrehumbert, R.T., 1991: Large-scale horizontal mixing in planetary atmospheres. *Phys. Fluids A*, **3**, 1250–1260.
- Sugata, S. and S. Yoden, 1993: A numerical study on regime transitions of the rotating annulus flow with a semi-spectral model. *J. Meteor. Soc. Japan*, **71**, 491–501.
- Sugata, S. and S. Yoden, 1994: Chaotic Lagrangian motion and heat transport in a steady, baroclinic annulus wave. *J. Meteor. Soc. Japan*, **72**, 569–587.
- Tajima, T. and T. Nakamura, 1995: Methods of visualization for observing directly an internal structure of steady baroclinic waves in rotating annulus experiments. *J. Visual. and Information Soc. Japan*, **15**, 40–46.
- Tajima, T., T. Nakamura and T. Kuroda, 1995a: Laboratory experiments of Lagrangian motions in a steady baroclinic wave -Internal structures of vortices- *J. Meteor. Soc. Japan*, **73**, 37–46.
- Tajima, T., T. Nakamura and T. Sakata, 1995b: Experimental observations of internal structures of vortices in steady baroclinic waves. *J. Atmos. Sci.*
- Tamaki, K. and K. Ukaji, 1985: Radial heat transport and azimuthally temperature fields in a differentially heated rotating fluid annulus undergoing amplitude vacillation. *J. Meteor. Soc. Japan*, **63**, 168–179.
- Weiss, J.B. and E. Knobloch, 1989: Mass transport and mixing by modulated traveling waves. *Phys. Rev. A*, **40**, 2579–2589.

定常傾圧波動における3次元ラグランジュ運動の実験的観測

田島俊彦・中村敏郎・黒川浩志

(国立富山工業高等専門学校)

気象学や海洋学の問題に深く関係している定常傾圧波動におけるカオスのラグランジュ運動について、菅田・余田 (1994) は波動を再現する数値模型で流体粒子を長時間追跡して調べた。彼らの結果を実験的に確かめるため、半径方向に温度差が与えられた回転流体で再現される定常傾圧波動内に浮遊させたトレーサー粒子の3次元軌道を長時間観測する実験をした。ここではこれまでに得られた4つの軌道を紹介する。波数5の波動では、数値模型から期待される起こり易い領域間遷移を含む経路が含まれている。この事は、菅田・余田によって提唱された熱輸送のラグランジュ的観点を支持している。即ちジェット流は外側の境界層から沢山の暖かい流体粒子を吸収し、それらを内側の境界層に放出する。そのとき、流体粒子の温度は蛇行するジェット内においてほとんど一定である。さらに、数値シミュレーションで起こってない流体粒子の領域間遷移が一例みつかったのは大変興味深い。しかしながら、波数4の波動では流体粒子が長時間ジェット流内に閉じ込められているのが観測された。これは、熱輸送について一般に認められた観点に導く。即ち、ジェット内の流体粒子は外側の境界層から熱を受取り、それを内側の境界層に放出する。従って、流体粒子の温度は蛇行するジェット内において変化する。