Adjoint Data Assimilation in Coupled Atmosphere-Ocean Models: Determining Initial Conditions in a Simple Equatorial Model

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Abstract

The general problem of retrieving the initial conditions in coupled atmosphere-ocean models by the adjoint data assimilation method was formulated. For a simple coupled equatorial model, where the atmosphere and the ocean were each represented by a linear shallow water model, retrieval of three oceanic initial conditions (the sea level height [SLH] and the two horizontal current components) was tested with identical twin experiments. Wind and SLH data, generated from a 90-day unstable local-growth simulation of a warm event, were assimilated to test the effects of (i) data type and sparsity, (ii) initial guess, and (iii) noisy data on retrieving the oceanic initial conditions.

SLH data were found to be more efficient in retrieving the oceanic initial conditions than the wind data, and the initial SLH field was more accurately retrieved than the initial currents. The retrieval of the initial current fields was sensitive to the temporal density of data, especially with wind data, where once a day would be the minimum density needed. As the initial guess of the oceanic state could contain errors in magnitude and phase (*i.e.* location) of the warm event anomaly, data assimilation was found to readily correct the error in the magnitude of the initial guess, but not the large phase error. Assimilation of noisy data showed that the retrieval of the initial conditions was far more sensitive to noise in the SLH data, than in the wind data.

1. Introduction

Adjoint data assimilation, a variational data assimilation method pioneered by Sasaki (1958, 1970), has found increasing applications in meteorology and oceanography (Daley, 1991; Errico, 1997). The adjoint approach is now widely used for estimating model parameters, initial conditions, boundary conditions, and sensitivity of model output with respect to input. It has even been linked recently to the artificial neural network method (Hsieh and Tang, 1998).

Coupled atmosphere-ocean models are important for understanding the climate system as well as for forecasting climate variability such as the El Niño phenomenon (Zebiak and Cane, 1987; Ji *et al.*, 1994). Verifying model parameters and determining initial conditions by data assimilation are very relevant to improving the prediction skills of coupled models. Lu and Hsieh (1997) have investigated the potential of the adjoint method in determining the

parameters in coupled models.

In this study, we derived the general formulation for optimally estimating the initial conditions in coupled atmosphere-ocean models by adjoint data assimilation. To illustrate the general method with a specific model, we chose the Philander, Yamagata, Pacanowski (1984) simple coupled model (hereafter called the PYP model), famous for its simulation of the initial growth of small perturbations in the equatorial atmosphere-ocean system (Yamagata, 1985). Identical twin experiments were performed with the PYP model to tackle the following three problems in determining the initial conditions of simple coupled models (henceforth, simple models) by the adjoint method: (1) The nature of information transfer between the atmosphere and the ocean, (2) sufficiency of information from the equatorial wave guide area, especially from the TOGA TAO array, and (3) the sensitivity of the estimated initial conditions to the initial guess and to noise in the data.

General lack of data has been an obstacle for initializing the coupled models by assimilating data from the tropical atmosphere and ocean. Fortunately, recent expansions of the observing system in the tropical Pacific by the TOGA experiment have

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enriched the existing data sets and allowed exploration of data assimilation in the simple models. Carton *et al.* (1996) have recently examined the relative importance of the main elements of the present observing system—sea level height data (SLH) provided by TOPEX/POSEIDON, the subsurface temperature data (mainly by the Volunteer Observing Ship XBTs), and the subsurface temperature and current data by the TOGA TAO array moorings—in establishing the ocean state of an ocean general circulation model. Kamachi and O'Brien (1994) tried to assimilate drifting buoy trajectory into an equatorial Pacific Ocean model to determine its mean upper layer thickness. Chen *et al.* (1995) nudged surface wind data into the Zebiak and Cane (1987) model to overcome the well-known “spring barrier” in El Niño prediction.

In Section 2, we formulate a general procedure for optimally initializing a coupled model using the adjoint data assimilation method, and apply it to the PYP model. Results from identical twin experiments with the PYP model are given in Section 3, with a summary and discussion in Section 4. This paper is the second paper in a trilogy on adjoint data assimilation in coupled atmosphere-ocean models: Lu and Hsieh (1997) studied the retrieval of model (damping and coupling) parameters, followed by this paper examining the retrieval of model initial conditions, to be followed by Lu and Hsieh (1998) with the combined retrieval of model parameters and initial conditions.

2. Variational initialization in coupled model systems using the adjoint method

2.1 Method

Let $\mathbf{x}$ be the vector of state variables in the atmospheric model, with $\mathbf{X}$ the atmospheric model operator. Let $\mathbf{y}$ and $\mathbf{Y}$ denote respectively the state vector and the operator of the ocean model. The coupled model with unknown initial conditions $\mathbf{x}_0$ and $\mathbf{y}_0$ is described by:

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= \mathbf{X}(\mathbf{x}, \mathbf{p}_x, \mathbf{y}, t), \\ \mathbf{x}(0) &= \mathbf{x}_0, \\ \frac{d\mathbf{y}}{dt} &= \mathbf{Y}(\mathbf{y}, \mathbf{p}_y, \mathbf{x}, t), \\ \mathbf{y}(0) &= \mathbf{y}_0,\end{aligned}\quad (2.1)$$

where the vectors $\mathbf{p}_x$ and $\mathbf{p}_y$ contain model parameters (*e.g.* friction parameters, coupling parameters, *etc.*). The atmosphere-ocean coupling is considered in (2.1) by defining the operators $\mathbf{X}$ and $\mathbf{Y}$ to be compound functions of their counterpart state variables $\mathbf{y}$ and $\mathbf{x}$. The variational initialization problem involves minimizing a “cost function”

$$J(\mathbf{x}_0, \mathbf{y}_0) = \int_0^T D dt,$$

with

$$D = [\mathbf{x} - \mathbf{x}_{\text{obs}}]^{\text{tr}} \mathbf{W}_x(t) [\mathbf{x} - \mathbf{x}_{\text{obs}}] + [\mathbf{y} - \mathbf{y}_{\text{obs}}]^{\text{tr}} \mathbf{W}_y(t) [\mathbf{y} - \mathbf{y}_{\text{obs}}], \quad (2.2)$$

where the subscript ‘obs’ denotes the observations, superscript ‘tr,’ the transpose, T , the assimilation period, $\mathbf{W}_x(t)$ and $\mathbf{W}_y(t)$, the weighting matrices (usually computed from the inverse of the covariance matrices of the observational errors), and $D(\mathbf{x}, \mathbf{y}, t)$, the distance between the model state and the observed state at time t .

To find the optimal initial conditions iteratively by the descent method, we need the gradient of J with respect to $\mathbf{x}_0$ and $\mathbf{y}_0$. This can be done by the adjoint method, *i.e.*, by introducing two Lagrange multiplier vectors $\mathbf{x}^*(t)$ and $\mathbf{y}^*(t)$ and constraining J with (2.1) to form a Lagrange function L :

$$L(\mathbf{x}, \mathbf{y}, \mathbf{x}^*, \mathbf{y}^*) = \int_0^T \left\{ D + \mathbf{x}^*(t)^{\text{tr}} \left[\frac{d\mathbf{x}}{dt} - \mathbf{X} \right] + \mathbf{y}^*(t)^{\text{tr}} \left[\frac{d\mathbf{y}}{dt} - \mathbf{Y} \right] \right\} dt. \quad (2.3)$$

After integration by parts, L becomes

$$L(\mathbf{x}, \mathbf{y}, \mathbf{x}^*, \mathbf{y}^*) = [\mathbf{x}^*(t)^{\text{tr}} \mathbf{x} + \mathbf{y}^*(t)^{\text{tr}} \mathbf{y}]_{t=0}^{t=T} + \int_0^T \left\{ D - \left[\frac{d\mathbf{x}^*(t)^{\text{tr}}}{dt} \mathbf{x} + \mathbf{x}^*(t)^{\text{tr}} \mathbf{X} \right] - \left[\frac{d\mathbf{y}^*(t)^{\text{tr}}}{dt} \mathbf{y} + \mathbf{y}^*(t)^{\text{tr}} \mathbf{Y} \right] \right\} dt. \quad (2.4)$$

The first order variation of L is:

$$\begin{aligned}\delta L &= [\mathbf{x}^*(t)^{\text{tr}} \delta \mathbf{x} + \mathbf{y}^*(t)^{\text{tr}} \delta \mathbf{y}]_{t=0}^{t=T} \\ &+ \int_0^T \left\{ \nabla_x D^{\text{tr}} \delta \mathbf{x} + \nabla_y D^{\text{tr}} \delta \mathbf{y} \right. \\ &- \left[\left(\frac{d\mathbf{x}^*}{dt} \right)^{\text{tr}} \delta \mathbf{x} + \left(\frac{\partial \mathbf{X}}{\partial \mathbf{x}} \right)^{\text{tr}} \mathbf{x}^*(t) \delta \mathbf{x} \right. \\ &\quad \left. \left. + \left(\frac{\partial \mathbf{X}}{\partial \mathbf{y}} \right)^{\text{tr}} \mathbf{x}^*(t) \delta \mathbf{y} \right] \right. \\ &- \left[\left(\frac{d\mathbf{y}^*}{dt} \right)^{\text{tr}} \delta \mathbf{y} + \left(\frac{\partial \mathbf{Y}}{\partial \mathbf{y}} \right)^{\text{tr}} \mathbf{y}^*(t) \delta \mathbf{y} \right. \\ &\quad \left. \left. + \left(\frac{\partial \mathbf{Y}}{\partial \mathbf{x}} \right)^{\text{tr}} \mathbf{y}^*(t) \delta \mathbf{x} \right] \right\} dt,\end{aligned}\quad (2.5)$$

where variation with respect to the Lagrange multipliers has been omitted as this will simply give back the original model equations. By letting the initial conditions $\mathbf{x}^*(T)$, $\mathbf{y}^*(T)$, $\delta L/\delta \mathbf{x}$ and $\delta L/\delta \mathbf{y}$ be zero (where division by a vector means division by an

element of the vector), we can obtain the adjoint model:

$$\begin{aligned} -\frac{d\mathbf{x}^*}{dt} &= \left(\frac{\partial \mathbf{X}}{\partial \mathbf{x}}\right)^{\text{tr}} \mathbf{x}^*(t) + \left(\frac{\partial \mathbf{Y}}{\partial \mathbf{x}}\right)^{\text{tr}} \mathbf{y}^*(t) - \nabla_{\mathbf{x}} D \\ \mathbf{x}^*(T) &= 0, \\ -\frac{d\mathbf{y}^*}{dt} &= \left(\frac{\partial \mathbf{Y}}{\partial \mathbf{y}}\right)^{\text{tr}} \mathbf{y}^*(t) + \left(\frac{\partial \mathbf{X}}{\partial \mathbf{y}}\right)^{\text{tr}} \mathbf{x}^*(t) - \nabla_{\mathbf{y}} D \\ \mathbf{y}^*(T) &= 0, \end{aligned} \quad (2.6)$$

Since $L = J$ at the stationary points of L , the formulae for computing the gradient of J with respect to $\mathbf{x}_0$ and $\mathbf{y}_0$ are obtained from (2.5) :

$$\begin{aligned} \frac{\delta J}{\delta \mathbf{x}_0} &= -\mathbf{x}^*(0) \\ \frac{\delta J}{\delta \mathbf{y}_0} &= -\mathbf{y}^*(0) \end{aligned} \quad (2.7)$$

In summary, the general computational procedure for estimating the coupled model initial conditions by the adjoint method is:

- (i) specify first guesses for $\mathbf{x}_0$ and $\mathbf{y}_0$;
- (ii) integrate the coupled model from $t = 0$ to $t = T$;
- (iii) compute J within $[0, T]$ using the forward model trajectory and data;
- (iv) compute the gradient (2.7) by a backward integration of the adjoint model (2.6) from $t = T$ to $t = 0$;
- (v) use a descent algorithm to find a new estimate of $\mathbf{x}_0$ and $\mathbf{y}_0$;
- (vi) if the convergence criteria are satisfied, terminate the procedure, otherwise redo the procedure from (ii) using the new guesses for $\mathbf{x}_0$ and $\mathbf{y}_0$.

2.2 Experiments

We next illustrate the above procedure with the PYP model. The atmosphere and the ocean, each represented by a single-layer linear shallow water model, had the 2 horizontal velocity components and the height perturbation denoted respectively by u_a , v_a , and h_a for the atmosphere, and u_o , v_o , and h_o for the ocean (see Appendix for details). They were coupled in a simple way where the changes in the thermocline depth in the wind-driven ocean affected the sea-surface temperature which, in turn, heated (or cooled) the atmosphere, with the coupling intensity determined by two coefficients, α and γ . The atmosphere-ocean motions were damped by Rayleigh friction and Newtonian cooling determined by the coefficients A and B for the atmosphere, and a and b for the ocean, respectively. This model has

been used to explore the unstable local-growth of the atmosphere-ocean coupled system in the tropical Pacific, though it is not entirely realistic since no mechanism modulates the unstable growth of the coupled system. A weaker coupling parameter set (Table 1) than that used by PYP was chosen to run this model for 90 days, to address the adjoint initialization problems for simple models at a seasonal time scale. The three initial oceanic conditions (h_0 , u_0 , and v_0) in the control run were obtained after allowing the ocean to first relax (in the absence of atmospheric forcing and coupling) for 10 days from an initial sea level height anomaly of Gaussian distribution centered around the equator in the middle of the ocean, *i.e.*

$$h = 0.5 \exp [-(x^2 + y^2)/(500 \text{ km})^2] \text{ cm}, \quad (2.8)$$

where the perturbation amplitude of 0.5 cm relative to an equivalent depth of 20 cm would correspond to a 2.5 m displacement of the thermocline from a mean thermocline depth of 100 m. The atmosphere and the ocean were then coupled once per day for another 10 days. The atmospheric and oceanic states at the end of this 10-day run were saved as initial conditions for a 90-day control run. Since the tropical atmosphere adjusts rapidly to the ocean (Gill, 1980), most operational simple models (*e.g.* Zebiak and Cane, 1987) have an equilibrium atmosphere. Hence only the oceanic initial conditions will be retrieved by adjoint data assimilation. The initial SLH (after the models were coupled for 10 days) is shown in Fig. 1a, the evolution of the SLH of the 90-day control run along the equator is shown in Fig. 1b, (and the current field is similar to Fig. 3 of PYP). The usefulness of the wind and SLH data were investigated in terms of efficiency of information transfer between the atmosphere and the ocean in retrieving the three initial oceanic conditions. After appropriately scaling the 6 model variables to be of the order 1 (see Appendix), the cost function J is taken to be

$$J = \int_0^T \int_S \frac{1}{2} \{ (u_a - u_a^{\text{obs}})^2 + (v_a - v_a^{\text{obs}})^2 + (h_o - h_o^{\text{obs}})^2 \} ds dt, \quad (2.9)$$

where the integrations are over the spatial domain S and the time period T , with the "observed" values indicated by the superscript 'obs.' The effects of data sparsity on retrieving the initial conditions were investigated by varying the data window, *i.e.*, data were made available once every 1, 10, 30 or 90 days (but not at $t = 0$), where 1 day is the TAO array temporal resolution of observations, 10 days, the time step of the Zebiak and Cane model, 30 days, the monthly scale, and 90 days the sparsest window of this study.

The continuous adjoint model is derived in the Appendix. The adjoint of the discretized coupled

Table 1. Parameters used in the coupled atmosphere-ocean model.

Parameter	Values
A	$2.3 \times 10^{-6} \text{ s}^{-1} [(5 \text{ days})^{-1}]$
B	$7.7 \times 10^{-7} \text{ s}^{-1} [(15 \text{ days})^{-1}]$
$c_a = (gd_a)^{\frac{1}{2}}$	66 m s^{-1}
g_a	40 cm s^{-2}
a	$1.2 \times 10^{-7} \text{ s}^{-1} [(100 \text{ days})^{-1}]$
b	$1.2 \times 10^{-7} \text{ s}^{-1} [(100 \text{ days})^{-1}]$
$c_o = (gd_o)^{\frac{1}{2}}$	1.4 m s^{-1}
g_o	2.0 cm s^{-2}
α	$2.5 \times 10^{-3} \text{ s}^{-1}$
γ	$6.5 \times 10^{-8} \text{ s}^{-1}$

model was coded directly from the GFDL C-grid shallow water code and was validated by the tangent linear model (see Lu and Hsieh, 1997). All experiments were performed using a limited-memory quasi-Newton method in the NAG FORTRAN Library routine, E04DGF. The root mean square error (RMSE) and the correlation coefficient between retrieved initial conditions and the true initial conditions were used to score the quality of the estimations for the initial conditions. The relative merit of using RMSE or correlation coefficient for model verification has been discussed by Murphy and Epstein (1989).

The retrieval of initial conditions involved over 10^3 times more unknowns than the retrieval of 6 model parameters in Lu and Hsieh (1997). However, the problem was not much harder except that the NAG routine took roughly an order of magnitude more iterations to converge than in the former study.

3. Results

3.1 Information transfer between the atmosphere and the ocean

With first guesses taken to be zero, the role of different data types and the effects of their temporal sparsity in determining the initial conditions of our system were investigated by conducting experiments separately assimilating the wind and the SLH data. Though the adjoint model clearly allows information transfer between the atmosphere and the ocean [see (A.3–A.4) in the Appendix], the efficiency of information transfer is associated with the usefulness of the particular type of data assimilated and with the model physics. These experiments will therefore give a basic account on the usefulness of the wind and the SLH data for our system. The quality of retrieval of the initial conditions was scored for the whole model domain as well as for two sub-areas, where area 1 was the model domain between 8°S and 8°N , representative of the TAO array area, and area 2, the rest of the model domain.

First, we studied the situation when data were spatially plentiful, but temporally sparse. Fig. 2a

and 2b depict the RMSE and the correlation of the retrieved h_0 field with respect to the true h_0 field by assimilating the wind or the SLH data, available at every grid point but once every 1, 10, 30 or 90 days. The retrieval of h_0 was insensitive to the data sparsity, *i.e.*, the RMSE and the correlation did not worsen remarkably with the increasing temporal sparsity of the wind or SLH data except for the experiment where the SLH data were available only at day 90. The quality of the retrieved h_0 field over area 1 was about the same as that over area 2 with the RMSE slightly smaller in area 2. When assimilating the SLH data, the RMSE is typically two orders of magnitude smaller than that when assimilating the wind data. However, the correlation was about the same for assimilating either data sets; it was very high (0.92–0.99) even for the case where the wind data (u_a and v_a) or the SLH data (h_o) were fewer in number than the unknown initial conditions (h_0 , u_0 , and v_0), *i.e.* when the wind or SLH data were available only at day 90.

One tends to think that good agreement would mean low RMSE and high correlation. However, it is quite common to have high RMSE occurring in conjunction with high correlation, and vice versa. Since RMSE is not normalized with respect to the magnitude, a region of strong signals would tend to have a high RMSE even when the agreement is quite good. In contrast, if two anomaly fields are well aligned but have very different magnitudes, correlation values will be high despite the disagreement in the magnitude. The characteristics of the h_0 retrieval for different experiments might be interpreted as follows: Since the strength of the anomalies was much greater in area 1 than in area 2, (see *e.g.*, the SLH in Fig. 1), the RMSE and the correlation were both likely to be larger in area 1 than in area 2, as found in Fig. 2. Hence, the smaller RMSE obtained by assimilating the SLH data than by assimilating the wind data suggested that the SLH data were more useful than the wind data in retrieving h_0 .

In contrast to the retrieval of h_0 , the retrieval of initial current fields was indeed sensitive to data sparsity. For u_0 , the retrieval was more sensitive to the sparsity of the SLH data than the wind data (Figs. 2c and 2d). Degradation of the RMSE and the correlation occurred simultaneously when the wind data window was enlarged to 30 days, while the RMSE started to grow when the SLH data window was longer than one day. Interestingly, the correlation did not immediately follow this RMSE degradation; instead, its degradation occurred when the SLH data window was enlarged to 30 days. Although the u_0 retrieval was more sensitive to the SLH sparsity, the retrieval quality by assimilating the SLH data was better than that by assimilating the wind data, *e.g.* when the wind data was available every 30 days, the correlation in area 2 was

Fig. 1. (a) Spatial structure of the thermocline depth perturbation h_0 after integrating the coupled model for 10 days using the parameters in Table 1. This will be taken as the ocean's initial state for the 90-day control run and other experiments. (b) Temporal and spatial structure of thermocline perturbation along the equator. Shaded regions indicate elevated thermocline. The depth perturbation is against the equivalent depth d_0 (of 20 cm). Since in this model, the SLH is related to the thermocline depth perturbation by a constant factor, we will also refer to the h_0 field as the SLH field.

less than 0.5 (Fig. 2d), much lower than the corresponding value with SLH data. Note that the RMSE in area 1 of the u_0 retrieval was larger than that in area 2, while the correlation was smaller in area 2. The larger RMSE in area 1 was associated with the stronger variations of the zonal current in the area, while the smaller correlation in area 2 might be caused by the more complicated structures of the current field induced by the boundaries.

The v_0 retrieval was even more sensitive to data sparsity than the u_0 retrieval, *e.g.* the degradation of the retrieval quality occurred when both the wind and the SLH data window were enlarged to longer than one day (Figs. 2e and 2f). Like the u_0 retrieval, the RMSE grew with the increasing temporal sparsity of the SLH data, while degradation of the correlation was not so obvious. But the correlation degradation was more obvious than the RMSE degradation when the wind data were assimilated. When the SLH data, or the wind data were assimilated once every 30 days, the SLH data were only half as numerous as the wind data (h_0 versus u_a and v_a), but the RMSE values for assimilating either data set were comparable, while the correlation by assimilating the SLH data was about 60 % higher than that by assimilating the wind data. It seemed that sparse SLH data contained sufficient information to obtain

the correct pattern of the retrieved fields, and the increasing sparsity mainly produced larger RMSE, whereas the wind sparsity mainly caused the pattern of retrieved fields to deteriorate. This again indicated that the SLH data were more useful in retrieving the current fields. The RMSE in area 1 was slightly larger than that in area 2, while the correlation in area 1 was slightly higher than in area 2.

We can therefore conclude that by assimilating wind only, the wind data can be transferred to the ocean part to determine the oceanic initial conditions. However, temporally denser wind data are required to achieve an overall retrieval quality for the three oceanic initial conditions (h_0 , u_0 , and v_0) similar to that obtained by assimilating SLH data. Of the three initial conditions, h_0 was less sensitive to the temporal sparsity of the wind data and the SLH data than u_0 and v_0 . Without prior information about the first-guesses, temporal resolution of 10 days for the wind or 30 days for the SLH are required for a good retrieval of the oceanic initial conditions in our system.

Next, experiments were conducted to examine the effects of combined temporal and spatial sparsity of the wind and SLH data on retrieving the initial ocean conditions. For these experiments, wind and

Fig. 2. The (a) RMSE and (b) correlation skill in retrieving the initial conditions h_0 , the (c) RMSE and (d) correlation skill in retrieving u_0 , and the (e) RMSE and (f) correlation skill in retrieving v_0 , for a 90-day assimilation run with wind or SLH available every 1, 10, 30, or 90 days at every grid point. For one day of integration, the ocean model took 8 time steps, while the atmospheric model took 90 steps. The labels w and h denote, respectively, experiments where wind or SLH data were available at every grid point in the model. The label a1 & 2 means that the RMSE or correlation was calculated over the entire model domain, while a1 or a2 means that the values were only over area 1 (the zonal band 8°S–8°N, *i.e.* the TAO array area) or area 2 (the rest of the model domain). The RMSE is plotted with a logarithmic vertical axis, where a longer bar represents a smaller error. The RMSE has no units because the model variables have been nondimensionalized (see Appendix).

Fig. 3. Same as Fig. 2 except for assimilating data at every 2nd grid in the meridional direction and every 15th grid point in the zonal direction in area 1 (TAO array).

SLH data were sampled from area 1 with the TAO array spatial resolution of 2° latitude by 15° longitude and once every 1, 10, 30 or 90 days. Zero first guesses were again used. The wind data in area 1 were found to be relatively more important than the SLH data in area 1 in determining h_0 (Figs. 3a and 3b), *i.e.*, without any wind data in area 2,

the quality of the h_0 retrieval was about the same as that in corresponding experiments (Figs. 2a and 2b) where wind data were available in area 2; the growth of the RMSE and the decrease of the correlation were minor, and no substantial degradation of the retrieval occurred in area 2. However, without the SLH data in area 2, the RMSE degradation

in the h_0 retrieval was remarkable (by 1–2 orders of magnitude larger, with degradation in both area 1 and area 2). Though the RMSEs were comparable between the experiments assimilating the wind data in area 1 and the SLH data in area 1, the correlation for the experiments assimilating the SLH data showed more apparent degradation, caused mainly by the degradation of the h_0 retrieval in area 2.

Without the wind or the SLH data in area 2, the information provided was generally insufficient for retrieving the current fields. Compared with the experiments assimilating the wind everywhere in space, the RMSE of the u_0 retrieval increased by about one order of magnitude (*cf.* Figs. 2c and 3c), and the correlation was lower than 0.5 when the window was longer than one day (*cf.* Figs. 2d and 3d). However, the low overall correlation was caused by the low correlation in area 2, as the correlation for area 1 was insensitive to the increasing wind sparsity and was acceptable (about 0.6). Compared with the experiments assimilating the SLH data everywhere in space, the RMSEs for assimilating the SLH once 1 or 10 days were about two orders of magnitude larger, while the correlations were very low for all the experiments except when having the SLH data once a day. This indicated that without some information from area 2, the available information was insufficient for retrieving the zonal current field. Without wind data or SLH data in area 2, the information insufficiency was even more remarkable for v_0 retrieval (Figs. 3e and 3f). The retrieval failed because the correlations were generally lower than 0.4, except for the experiment where the SLH data were available once a day.

We concluded that the initial sea surface height (h_0 of our model) was more easily estimated and less sensitive to the assimilated data type (wind and SLH), and the temporal and spatial sparsity of data, than the initial currents (u_0 and v_0). With SLH data available only in the narrow equatorial area 1, the retrieval of initial conditions degraded substantially relative to the case where SLH data were available at every grid. In contrast, with wind data in area 1 only, there was no major retrieval degradation relative to the case where wind data were available at every grid; hence wind data in area 2 appeared much less important in affecting the retrieval quality, than wind data in area 1. Observations (wind or SLH) in area 1 was enough for estimating h_0 , but not for the current fields, especially in area 2.

3.2 Impact of initial guess on the initialization

ENSO is generally believed to be a quasi-periodic phenomenon with the time scale set by the transit times of equatorial Kelvin or Rossby waves. Its predictability depends on our being able to identify where the coupled system is on the ENSO cycle. An ocean spun-up by wind stress may contain errors in

the intensity and phase of the cycle as represented by the oceanic anomaly. Using such a spun-up ocean as the first-guess for the adjoint data assimilation could affect the convergence of the minimization procedure. To study this, we conducted four experiments where the first-guesses were from a spun-up ocean with defects: (a) The oceanic anomaly has the correct phase (*i.e.* location) but incorrect intensity. For generating the first guesses, the magnitude of the initial h in (2.8) was first systematically scaled up by 10 % from the control. The first-guess for case 1 was then generated in the same way as the true initial conditions, but by using the perturbed h . For case 2, 10 % random noise was added to the “true” initial conditions from the control run. (b) The oceanic anomaly has incorrect phase: The first-guesses were generated by relaxing the perturbed h with the same Gaussian form as (2.8) but in case 3 located initially further to the left (centered at grid 50 along the equator) relative to the anomaly in the control run (centered at grid 100 along the equator). In case 4, the anomaly was located further to the right (centered at grid 150 along the equator). Instead of describing a warm event developing in the central equatorial Pacific, case 3 and case 4 described a warm event developing in the western equatorial Pacific, and in the eastern equatorial Pacific, respectively.

To compare with the results from previous studies using zero initial guess, experiments in this section were also conducted with data available once every 1, 10, 30, and 90 days only in area 1 with TAO array spatial resolution. With either the wind data or the SLH data, retrievals of the three initial states from case 1 were very good, showing little sensitivity to the increasing temporal sparsity. Fig. 4 shows the RMSE and the correlation for the retrievals with the wind data or the SLH data available only at day 90. Though the data were far fewer than the unknowns for these two experiments, the RMSEs were very small and the correlations were very high, especially for the current fields, *e.g.*, with only the wind data, the RMSE was of the order of 10^{-4} for u_0 and 10^{-5} for v_0 , much smaller than those found earlier by zero initial guess (*cf.* Figs. 4a, 3c and 3e). The correlations for the current fields were also improved in this case (Figs. 4b, 3d and 3f). The quality of the retrievals for area 2 was also very good (not shown).

Case 2 was found to be more difficult to retrieve than case 1 as the RMSE and the correlation skill in retrieving the initial conditions from noisy initial guess deteriorated with increasing temporal sparsity of the wind or the SLH, in agreement with our understanding that the detrimental effect of noise in the initial guess would be greater when fewer data were available. However, even with data only at day 90, the retrievals were still better than those obtained by zero initial guess, especially for the cur-

Fig. 4. (a) RMSE and (b) correlation from retrieving three initial conditions from case 1 and case 2 by assimilating wind or SLH data at day 90 in area 1 (TAO array), and from case 3 and case 4 by assimilating once every day both wind and SLH data in area 1 (TAO array).

rent field retrieval (*cf.* Figs. 4 and 3). With data only in area 1, retrievals for area 2 were as good as in area 1 (not shown). Since our model is linear, the first guesses of the in-phase case should have the same spatial distributions as the true initial conditions. This might explain why the retrievals for case 1 needed few data and the information propagated readily from the later time to the starting time to determine the initial conditions. More information might be required to establish the phase of the initial ocean because phase restructuring implies energy redistribution. As the noise in the initial guess is harmful to retrieving the initial conditions, the noise level could be an important factor in the retrievals.

The results from the two out-of-phase (*i.e.* anomaly in wrong location) cases supported the above arguments. Having only the wind data or the SLH data in area 1 was generally insufficient for retrieving the initial conditions in both case 3 and case 4. With both wind and SLH data, the retrievals were still very poor. From Fig. 4 where both wind and SLH data were assimilated once every day, we observed the same characteristics of information insufficiency found in the experiments with zero first guess or with very sparse data, *i.e.*, in retrieving the h_0 , the RMSEs were large and the correlations were high, while the retrievals for current fields were poor. Compared with Fig. 3, the retrievals for case 3 and case 4 were poorer than those assimilating wind or SLH data with zero initial guess. Hence the efficiency of the adjoint method, and the information sufficiency, depend on the initial guess in the phase (*i.e.* location) of the warm water anomaly.

3.3 Impact of noisy data on initialization

Noise in the data is detrimental to a successful retrieval of initial conditions, as it tends to cause ill-conditioning of the problem (Thacker and Long,

1988). We conducted experiments to test the effect of assimilating noisy wind and SLH data in area 1 (with the TAO array spatial resolution). Normally distributed random noise with zero mean and standard deviation r (which was 10 % of the maximum value of each state variable at day 90 of the control run) was added at every time step. The noisy initial conditions in the last section (case 2) were used as first guess.

There was serious degradation in the retrieval quality when noisy SLH data were assimilated. In contrast, there was only minor degradation when noisy wind data were assimilated. Ironically, when noisy wind data were assimilated, the retrieval results were better than when both noisy wind and SLH data were assimilated, despite the availability of more data in the latter situation — *e.g.* when assimilating TAO array data once per day, the correlation skill in retrieving h_0 was almost 1.0 with noisy wind data, 0.18 with both noisy wind and SLH data, and basically zero with only noisy SLH data. Hence noisy SLH data were most harmful to retrievals of the initial oceanic conditions.

4. Summary and discussion

In this paper, the adjoint data assimilation method was extended to optimally initialize atmosphere-ocean coupled models. This general method was illustrated with the Philander *et al.* (1984) simple equatorial coupled model (with the atmosphere and the ocean each represented by a one-layer linear shallow-water model), where three initial oceanic fields (SLH or thermocline depth perturbation h_0 , and the current components u_0 and v_0) were estimated. Our purpose was to examine the potential of the adjoint method in assimilating the tropical atmospheric and oceanic data. A series of identical twin experiments of seasonal scale (90 days) was designed to study the sensitivity of

retrieving the initial conditions to data type (wind and SLH), data sparsity, first guess of the initial conditions, and data noise.

Usefulness of data type was tested by retrieving the initial conditions with first guess taken to be zero. Assimilated wind data were found to be useful in retrieving the initial oceanic conditions, with the overall retrieval quality sensitive to the temporal sparsity of the wind data. The SLH seemed to be more effective than the wind in retrieving the three initial fields. Among the three, the retrieval of the initial SLH was least sensitive to data type and to the temporal sparsity of data assimilated, while the retrieval of current fields was sensitive to the temporal sparsity of the data assimilated. A temporal resolution of 10 days for the wind data was found necessary for achieving an acceptable retrieval quality. Since the current fields were determined by both wind and SLH (see A.2), the better results with the SLH data in retrieving current fields might be attributed to the fact that (1) the wind was not a state variable of the ocean model, and (2) the oceanic motion had a slower time scale and therefore a longer memory to retain the information obtained from the SLH data, while the faster atmospheric motion had a shorter memory to carry the information obtained from the wind data. The current fields needed more information for retrieval as the structure of the currents was more complicated, with smaller spatial scales than the SLH, especially for the regions outside the equatorial wave guide area and the regions along the boundary.

Wind and SLH in the equatorial wave guide area were used to test the sensitivity of retrieving the initial oceanic fields to the combined (temporal and spatial) sparsity of data, and the sufficiency of the TAO array observing system in retrieving the initial conditions of simple models. Assuming no prior information was available for the ocean (*i.e.* zero initial guess for the oceanic fields), wind and SLH data were found to be sufficient for retrieving the initial SLH, but not enough to retrieve the current fields outside the equatorial wave guide area. The current fields in the equatorial wave guide area could not be well retrieved if the temporal resolution of the observations was longer than one day.

First guess in adjoint data assimilation plays an important role of providing initial background information on the ocean — a good first guess can provide useful information for retrieving the initial conditions, while a poor guess can be detrimental. The data sufficiency for retrieving the initial fields is therefore associated with the quality of the first guess. (1) Systematic bias in the magnitude of the initial conditions used for the first guess was easily overcome, without the need of a large amount of data, as temporally very sparse wind or SLH data in the equatorial wave guide area were enough to well

retrieve the initial conditions for the whole model domain. (2) Noisy perturbations in the magnitude of the initial conditions used for the first guess, however, were more difficult to handle. The retrieval of the initial conditions was found to be sensitive to the data sparsity for this case, which implied data insufficiency. For these two cases, the overall quality in retrieving the initial conditions was still better than those achieved from first zero guesses, suggesting that initial ocean fields with error in the magnitude were still useful for initializing simple models. A possible explanation is that for a linear system, perturbing the magnitude of the initial guesses does not change the spatial structure of the true initial conditions, so that the model and control run output have the same spatial structure as well. When the first guesses had large spatial phase difference from the true initial conditions (*i.e.* wrong zonal location of the initial anomaly in cases 3 and 4), the true initial conditions could not be retrieved easily by assimilating data with the TAO array resolution. With current observations available from the TAO system, one can expect improvements in the poor current field retrieval.

While SLH data was superior to wind data in retrieving the oceanic initial conditions, it was ironic that noise in the SLH data was far more detrimental than noise in the wind data. The reason is that when there is noise in the wind data, the ocean does not adjust quickly enough to the erroneous wind data, and with the atmosphere mainly a slave to the ocean at low frequencies, the wind soon adjusts to the ocean state — hence little damage is done by the noise in the original wind data. In contrast, when there is noise in the ocean data, the slave atmosphere adjusts rapidly to the erroneous ocean data, hence oceanic noise can cause far more damage to the coupled system.

Problems for further study: (1) The problem of retrieving model parameters and initial conditions together in coupled models by adjoint data assimilation has recently been studied by Lu and Hsieh (1998). (2) The procedure we have developed here assumed that the whole coupled system was differentiable, and we have used a differentiable simple model to test the procedure. However, for some complicated parameterizations involving step functions and cumulus heating processes, the model may not be differentiable nor invertible, so careful treatments and approximations are required (Xu, 1996). (3) Since the model has greatly simplified the coupling processes, it cannot be integrated for a long time. Therefore, the knowledge gained in this study is based on relatively short assimilation periods and might not be entirely valid for real operational models. (4) For most simple coupled models, the oceanic models are nonlinear, as such some of the results from our linear model may not hold for the nonlin-

ear models.

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Appendix

The atmosphere (denoted by subscript a) and the ocean (subscript o) of the local equilibrium model are each governed by a set of shallow water equations in spherical coordinates:

$$\begin{aligned} \frac{\partial u_a}{\partial t} - f v_a + \frac{g_a}{r \cos \phi} \frac{\partial h_a}{\partial \lambda} &= -A u_a, \\ \frac{\partial v_a}{\partial t} + f u_a + \frac{g_a}{r} \frac{\partial h_a}{\partial \phi} &= -A v_a, \\ \frac{\partial h_a}{\partial t} + \frac{1}{r \cos \phi} \left(\frac{\partial u_a}{\partial \lambda} + \frac{\partial (v_a \cos \phi)}{\partial \phi} \right) &= -B h_a - \alpha h_o, \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} \frac{\partial u_o}{\partial t} - f v_o + \frac{g_o}{r \cos \phi} \frac{\partial h_o}{\partial \lambda} &= -a u_o + \gamma u_a, \\ \frac{\partial v_o}{\partial t} + f u_o + \frac{g_o}{r} \frac{\partial h_o}{\partial \phi} &= -a v_o + \gamma v_a, \\ \frac{\partial h_o}{\partial t} + \frac{1}{r \cos \phi} \left(\frac{\partial u_o}{\partial \lambda} + \frac{\partial (v_o \cos \phi)}{\partial \phi} \right) &= -b h_o, \end{aligned} \quad (\text{A.2})$$

where u_a and v_a are the zonal and meridional wind velocity components, h_a the depth perturbation against the equivalent depth d_a , u_o and v_o are the zonal and meridional current components, and h_o the depth perturbation against the equivalent depth d_o . Motion is damped by Rayleigh friction and Newtonian cooling with respective parameters A and B for the atmosphere and a and b for the ocean. The dynamic coupling coefficient is γ and the thermodynamic coupling parameter is α . The Coriolis parameter is f , gravity is g , the reduced gravities for the atmosphere and the ocean are g_a and g_o respectively, and the coordinates (r, λ, ϕ) are the radial distance, and the longitude and the latitude, respectively. The parameter values are given in Table 1. The atmospheric long gravity wave speed $c_a = 66 \text{ m s}^{-1}$ is chosen so that the equatorial radius of deformation is about 2000 km and the corresponding equivalent depth is around 400 m. For the ocean, a long gravity wave speed of $c_o = 1.4 \text{ m s}^{-1}$ gives a deformation equatorial radius of 250 km and an equivalent depth of about 20 cm. This value of the equivalent depth is appropriate if the sharp, shallow

tropical thermocline is considered to be an interface between the warm surface water and the cold deep water. Before the cost function J is evaluated in (2.9), we have also scaled the 6 model variables by using the typical values in our model, which are 400 m and 20 cm for the atmospheric and oceanic equivalent depths, and 1 m s^{-1} for the wind and the ocean current.

Equations (A.1–A.2) were solved numerically following PYP. The ocean model was run in a 200° longitudinal extent between 30°S and 30°S . Zonally, the atmospheric domain was larger by 20° and was cyclic. The solid-wall boundary condition was set for the ocean and for the atmosphere in the northern and southern boundaries. The grid spacing was 1° latitudinally and longitudinally in both the ocean and the atmosphere. The coupling between the atmosphere and the ocean took place once a day. For the rest of the day, the driving forces for the atmosphere and ocean were held fixed.

The continuous adjoint model is derived from (A.1–A.2) with the cost function specified in (2.9):

$$\begin{aligned} \frac{\partial u_a^*}{\partial t} - f v_a^* + \frac{d_a}{r \cos \phi} \frac{\partial h_a^*}{\partial \lambda} &= A u_a^* - \gamma u_o^* + (u_a - u_a^{\text{obs}}), \\ \frac{\partial v_a^*}{\partial t} + f u_a^* + \frac{d_a}{r} \frac{\partial h_a^*}{\partial \phi} &= A v_a^* - \gamma v_o^* + (v_a - v_a^{\text{obs}}), \\ \frac{\partial h_a^*}{\partial t} + \frac{g_a}{r \cos \phi} \left(\frac{\partial u_a^*}{\partial \lambda} + \frac{\partial (v_a^* \cos \phi)}{\partial \phi} \right) &= B h_a^*, \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \frac{\partial u_o^*}{\partial t} - f v_o^* + \frac{d_o}{r \cos \phi} \frac{\partial h_o^*}{\partial \lambda} &= a u_o^*, \\ \frac{\partial v_o^*}{\partial t} + f u_o^* + \frac{d_o}{r} \frac{\partial h_o^*}{\partial \phi} &= a v_o^*, \\ \frac{\partial h_o^*}{\partial t} + \frac{g_o}{r \cos \phi} \left(\frac{\partial u_o^*}{\partial \lambda} + \frac{\partial (v_o^* \cos \phi)}{\partial \phi} \right) &= b h_o^* + \alpha h_a^* + (h_o - h_o^{\text{obs}}). \end{aligned} \quad (\text{A.4})$$

Variables with an asterisk denote the adjoint variables (*i.e.* Lagrange multipliers) of their counterparts in (A.1–A.2). In the adjoint model, the forcing terms on the RHS of the atmospheric momentum equations and the oceanic mass equation measure the misfit between the model solution and the data, and the damping processes take opposite signs because of backward integration. There exists an inverse coupling in the coupled adjoint model, in which the atmospheric adjoint model is driven “dynamically” by the “adjoint current” and the oceanic adjoint model is driven “thermodynamically” by the atmospheric “adjoint depth perturbation.” These terms allow information exchanges in the coupled adjoint model, so the model trajectories of the two single models can be tuned together.

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大気海洋結合モデルにおけるアジョイント法によるデータ同化： 単純な赤道域モデルの初期条件の決定

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大気海洋結合モデルの初期条件を、アジョイント・データ同化法によって求める一般的な問題が定式化された。特に、大気と海洋それぞれを線形浅水モデルで表現した単純な赤道域結合モデルに対して、海洋の3個の初期条件(海面高度と2個の水平流速成分)を決定することが、identical twin 実験によってテストされた。(i) データの種類と密度、(ii) 第一推定値、(iii) ノイズのあるデータが海洋の初期値の決定に及ぼす影響を調べるために、warm event における不安定な局所的成長についての90日間のシミュレーションから得られた、風と海面高度のデータが同化された。

海洋の初期値を求めるうえで、海面高度データが風データより効果的であることがわかり、海面高度の初期値は流速の初期値より正確に決定された。流速の初期値の決定はデータ、特に風データの時間密度に敏感であり、1日1回の観測が最低限必要である。海洋の状態の第一推定値には、warm event のアノマリーの大きさや位相(位置)の誤差が含まれ得るが、データ同化は第一推定値の大きさの誤差を容易に修正するが、大きな位相の誤差についてはそうでないことがわかった。ノイズのあるデータの同化実験は、初期条件の決定が風データより海面高度データのノイズに敏感であることを示した。