Interpretation of Thompson's Mechanism for Venus' Atmospheric Super-Rotation and its Extension to the Spherical Geometry

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(Manuscript received 3 August 1998, in revised form 28 May 1999)

Abstract

Thompson's mechanism is reconsidered in a two-dimensional plane. It is shown that a mean shear flow can be generated by the positive feedback of a mean shear flow, tilting convective motion and tilting temperature distribution. In this instability, a small perturbation of mean shear flow tilts the basic temperature distribution. The tilting temperature distribution induces tilting of convective motion, which transports horizontal momentum vertically so as to reinforce the mean flow shear.

In order to examine the validity of Thompson's mechanism on the sphere, a three-dimensional nonlinear numerical model is constructed. Steady solutions which represent convection between the day and night sides on the sphere, are numerically obtained for various values of parameters, and their stability is examined. It is shown that, if the planetary rotation is neglected, three-dimensional convection on the sphere is stable and a mean shear flow cannot be generated. This result suggests that Thompson's mechanism proposed for two-dimensional convection cannot be applied to three-dimensional convection on the sphere.

1. Introduction

The rotation of Venus, whose period is about 243 days, is so slow that convection axisymmetric about a line between the subsolar and antisolar points was expected to be predominant. However, it is known from many observations that the fast zonal flow (super-rotation) is predominant in the Venus atmosphere. There were some attempts to explain the generation of the fast zonal flow from convection between the day and night sides: the moving flame mechanism (Schubert and Whitehead, 1969), and Thompson's mechanism (Thompson, 1970).

In the former mechanism, the motion of solar heating is taken into account. It is assumed that the lower atmosphere is warmed by solar heating, and heat is transported upward by eddy motions and/or thermal conduction. Since the heating in the upper atmosphere retards, the temperature distribution and associated ascending motion will tilt in the direction opposite to the solar motion. This tilted convection cell transports horizontal momentum verti-

cally, due to the correlation between horizontal and vertical velocities. As a result, a mean shear flow will be generated. Schubert and Whitehead (1969), conducted a laboratory experiment in which a rotating flame heats a cylindrical annulus filled with liquid mercury from below, and found that the rotation rate of the fluid is several times greater than that of the flame. However, this seems to be much smaller than that of the super-rotation in the Venus atmosphere (about 30 times greater than that of solar heating).

In Thompson's (1970) study, unlike the moving flame mechanism, the motion of solar heating is not required for the generation of fast zonal flow. Thompson's two-dimensional numerical experiment showed that the convection cell induced by solar heating is unstable for the tilting convective motion, and a mean shear flow is produced from this instability. The mechanism of this instability proposed by Thompson is referred to as the Thompson mechanism in the present paper. According to his interpretation of this result, the mean shear flow produced by the tilted convective motion tilts the original convection further. However, this is not necessarily correct, because the effect of the vertical advection of mean zonal flow by the convective motion may pre-

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vent the tilt of the convection cell (see below). The first purpose of this paper is to correctly interpret the results of Thompson's experiment. Thompson's mechanism will be carefully reexamined in the following sections in detail.

The second purpose of this paper is to examine the stability of a three-dimensional convection cell on the sphere. Though convection induced by solar heating on a real planet is three-dimensional, Thompson's and other studies were conducted in a two-dimensional vertical plane. Convection treated by these studies corresponds only to that on the equator in a three-dimensional model. On the other hand, for three-dimensional Bénard convection in a basic flow with vertical shear, it is shown by Kuo (1963), and Asai (1970), that roll-type convection whose axis is perpendicular to the basic flow is suppressed by shear of the basic flow, while convection whose axis is parallel to the basic flow is not influenced by it. Though the convective motion is produced by unstable stratification in their numerical experiments, their result of the study on the interaction between the convective motion and the basic flow may be applicable to the case in which the convective motion is produced by the horizontally differential heating. In convection between the day and night sides on the sphere, there is a meridional flow perpendicular to the mean zonal flow. It is not clear whether the interaction between a mean zonal flow with vertical shear, and this meridional flow, excites or suppresses the instability. It is not obvious that Thompson's mechanism can work, and a mean flow can be generated in three-dimensional convection on the sphere. In the present study, we examine the stability of a *three-dimensional* convection cell on the sphere in detail.

In the present stage, Thompson's mechanism does not appear to be a promising theory for explaining the super-rotation of Venus atmosphere. It is confirmed that the meridional circulation is predominant in the cloud layer (45–65 km), and convection between the day and night sides is not observed there. However, apart from the application to Venus atmosphere, Thompson's mechanism is very interesting itself as a theory in geophysical fluid dynamics. We consider that it is well worthy to make clear Thompson's mechanism sufficiently. In the Venus atmosphere near the surface, there are scarce observations which suggest a generation mechanism of the super-rotation in the lower atmosphere. Also in this context, it is important to determine theoretically whether or not Thompson's mechanism can work in a three-dimensional fluid layer.

This paper is organized as follows. In Section 2, the two-dimensional Thompson's mechanism is reexamined. It is shown that the instability mechanism is classified into two categories. In the first category only the velocity field is considered, while in

the second category both the velocity and temperature fields are involved. The mechanism working in Thompson's numerical experiments is shown to belong to the second category. In Section 3, a three-dimensional model is constructed in order to examine the stability of three-dimensional convection between the day and night sides on the sphere. This stability problem is formulated as a linear eigenvalue problem of the perturbation equation about three-dimensional convection. In Section 4, steady solutions representing convection between the day and night sides are numerically obtained, and results of the linear analysis are discussed.

2. Thompson's mechanism in a two-dimensional model

2.1 Thompson's mechanism

Thompson (1970) examined the stability of two-dimensional convection forced by horizontally differential heating in a vertical plane. It was shown from his numerical experiments that this convection cell was unstable, and a mean flow could be produced in association with this instability. His interpretation of this instability was so-called tilting instability, which was caused by an interaction between the tilting convective motion and mean shear flow. The mechanism considered by Thompson is summarized as follows: if a horizontal mean flow with vertical shear ($\bar{u}(z)$) arises, its horizontal advection tilts the convection cells; the tilting cells transport horizontal momentum vertically by the correlation between the horizontal and vertical velocities ($\overline{u'v'} \neq 0$). This vertical momentum transport reinforces the original shear of the mean flow.

Recently, Soloviev *et al.* (1996) examined the highly truncated system of the vorticity equation which does not include the temperature field, and showed that the two-dimensional tilting instability develops only in the case of sufficiently tall convective structures. The vertical scale of convection in Thompson is sufficiently small in comparison with the horizontal one. Hence, according to Soloviev *et al.*, the convective motion cannot be unstable, namely, his above interpretation is not adequate for his own numerical experiments. The combination of wavenumbers treated in Soloviev's *et al.* analysis is very restricted. However, the purely dynamical process cannot explain the instability shown in Thompson's numerical experiments, even if truncation of wavenumbers are relaxed. This can be shown in the following manner.¹ We consider a two-dimensional fluid layer whose horizontal and vertical scales are L and D , respectively. Assuming that horizontal and vertical boundary conditions are periodic and slippery as in Soloviev *et al.*, the stream-

¹ This analysis, which is a direct extension of the analysis by Soloviev *et al.*, would make it easy to understand the following analysis including the temperature field.

functions of tilting convection can be represented by a superposition of the following:

$$\begin{aligned}\psi_B(t, x, z) &= \psi_B(t) \cos x \sin lz, \\ \psi'(t, x, z) &= \psi'(t) \sin x \sin(l+1)z,\end{aligned}\quad (2.1)$$

where ψ_B and ψ' are streamfunctions of the basic state (convection), and a disturbance. The streamfunctions of these modes are depicted in Fig. 1 for $l = 2$. The streamfunction of mean shear flow can be represented by

$$\bar{\psi}(t, z) = \bar{\psi}_1 \sin z + \bar{\psi}_2 \sin(2l+1)z. \quad (2.2)$$

It should be noted that Soloviev's *et al.* analysis is restricted to $l = 1$, while we examine stability below for arbitrary l . By scaling the streamfunction by LD/T , where T is a time scale, we obtain the nondimensional vorticity equation,

$$\frac{\partial \Delta \psi}{\partial t} + J(\psi, \Delta \psi) = 0, \quad (2.3)$$

where $\Delta = \delta^2 \partial^2 / \partial x^2 + \partial^2 / \partial z^2$ and $\delta = D/L$. From this equation, we can obtain the mode equations (ordinary differential equations for expansion coefficients ψ_B , ψ' , $\bar{\psi}_1$ and $\bar{\psi}_2$) which represent the process of tilting instability:

$$\begin{aligned}\frac{\partial}{\partial t} \psi' &= -\frac{\delta^2 + (l^2 - 1)}{2[\delta^2 + (l+1)^2]} \psi_B \bar{\psi}_1 \\ &\quad + \frac{(2l+1)[\delta^2 - (l+1)(3l+1)]}{2[\delta^2 + (l+1)^2]} \psi_B \bar{\psi}_2, \quad (2.4)\end{aligned}$$

$$\frac{\partial}{\partial t} \bar{\psi}_1 = -\frac{2l+1}{4} \psi_B \psi', \quad (2.5)$$

$$\frac{\partial}{\partial t} \bar{\psi}_2 = -\frac{1}{4} \psi_B \psi'. \quad (2.6)$$

The first equation represents the interaction between the convection cells (ψ_B) and the shear flows ($\bar{\psi}_1$, $\bar{\psi}_2$) that produces the tilt of convection cells (ψ'). By putting the solution of this equation as proportional to $e^{\gamma t}$, Eqs. (2.4)–(2.6) constitute an eigenvalue problem with respect to γ . γ is given by the following equation,

$$\gamma^3 - \frac{(2l+1)[\delta^2 - (l+1)^2] \psi_B^2}{4[\delta^2 + (l+1)^2]} \gamma = 0. \quad (2.7)$$

For instability, the condition $\delta > l+1$ must be fulfilled. According to this result, the two-dimensional tilting instability can take place only for a sufficiently tall convective structure, *i.e.*, $\delta > l+1$, in the case of a purely dynamical system neglecting effects of the temperature field.

2.2 Interaction between the velocity and temperature fields

In this subsection, the temperature field is introduced into our system. It is shown that there exists

Fig. 1. Two modes representing tilting convection cells. (a) and (b) are streamfunctions ψ_B and ψ' . The tilting cells (c) are expressed as a superposition of (a) and (b).

an unstable mode such that the tilting convective motion, and mean shear flow, reinforce each other through the tilting temperature field.

We adopt equations treated by Thompson, which describe the velocity and temperature fields in a shallow fluid layer,

$$\left. \begin{aligned} \psi_{zzt} + J(\psi, \psi_{zz}) \\ = GT_x + E_H \psi_{zzxx} + E_V \psi_{zzzz} \\ T_t + J(\psi, T) \\ = Q + K_H T_{xx} + K_V T_{zz} - \Gamma \psi_x \end{aligned} \right\}, \quad (2.8)$$

where G is a scaled gravity, E_H and E_V scaled horizontal and vertical viscosity, K_H and K_V scaled horizontal and vertical thermal conductivity, Q a scaled solar differential heating and Γ a scaled temperature stratification. The fundamental modes of streamfunction, and temperature deviation associated with convection produced by solar heating, can be assumed to be:

$$\psi_B = \psi_1^1 \cos x \sin z, \quad T_B = T_{-1}^1 \sin x \sin z, \quad (2.9)$$

respectively. The modes representing tilts of the convective motion and temperature distribution are

$$\psi' = \psi_{-1}^2 \sin x \sin 2z, \quad T' = T_1^2 \cos x \sin 2z, \quad (2.10)$$

respectively, and that of mean shear flow is

$$\bar{\psi} = \psi_0^1 \sin z. \quad (2.11)$$

By substituting (2.9), (2.10) and (2.11) into (2.8), the mode equations governing the expansion coefficients of each mode are obtained:

$$\left. \begin{aligned} \frac{\partial}{\partial t} \psi_1^1 &= -GT_{-1}^1 + 3/2 \psi_0^1 \psi_{-1}^2 \\ \frac{\partial}{\partial t} \psi_{-1}^2 &= 1/4 GT_1^2 \\ \frac{\partial}{\partial t} \psi_0^1 &= -3/4 \psi_1^1 \psi_{-1}^2 \\ \frac{\partial}{\partial t} T_{-1}^1 &= \Gamma \psi_1^1 - 1/2 \psi_0^1 T_1^2 + Q \\ \frac{\partial}{\partial t} T_1^2 &= -\Gamma \psi_{-1}^2 + 1/2 \psi_0^1 T_{-1}^1 \end{aligned} \right\}, \quad (2.12)$$

where $E = 0$ and $K = 0$ are put for simplicity. Though terms of viscosity and thermal conductivity are very influential in a final state of nonlinear solution produced by instability, in the linear stability analysis, these terms merely reduce the growth rates of eigenmodes. Therefore E and K are put to 0 in Eq. (2.12). By dividing dependent variables into the time-independent basic parts (denoted by subscript B) and the perturbation parts (denoted by superscript '), the perturbation equations are obtained:

$$\frac{\partial}{\partial t} \begin{pmatrix} \psi_1^1 \\ \psi_{-1}^2 \\ \psi_0^1 \\ T_{-1}^1 \\ T_1^2 \end{pmatrix} =$$

$$\begin{pmatrix} 0 & 0 & 0 & -G & 0 \\ 0 & 0 & 0 & 0 & G/4 \\ 0 & -3\psi_{1,B}^1/4 & 0 & 0 & 0 \\ \Gamma & 0 & 0 & 0 & 0 \\ 0 & -\Gamma & T_{-1,B}^1/2 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_1^1 \\ \psi_{-1}^2 \\ \psi_0^1 \\ T_{-1}^1 \\ T_1^2 \end{pmatrix}. \quad (2.13)$$

By putting the solution of this equation as proportional to $e^{\gamma t}$ and $\Gamma = G = 1$ (see Thompson), it is found that there is one unstable (growing) mode, whose corresponding eigenvector is

$$(\psi_1^1, \psi_{-1}^2, \psi_0^1, T_{-1}^1, T_1^2) \\ = (0, -0.33, -0.87, 0, -0.37). \quad (2.14)$$

As shown in the preceding subsection, the velocity field represented only by the three modes ψ_B , ψ' and $\bar{\psi}$ does not give any unstable mode. Hence the unstable mode is obtained in this case by including the effect of the temperature field. The mechanism working in this instability is illustrated in Fig. 2. Basic convection is represented by streamfunction ψ_1^1 associated with temperature deviation T_{-1}^1 ; if the horizontal flow with vertical shear, ψ_0^1 , is superposed on the basic field. This advection can tilt the temperature distribution, producing T_1^2 , but cannot tilt the velocity field associated with convection, as discussed above. Instead, the tilting temperature distribution can produce tilting convection cells, which transport horizontal momentum vertically. Then the original shear of the mean zonal flow will be reinforced.

Thus, it is shown that the instability obtained in Thompson's numerical experiment can be explained not by purely dynamical effects, but by an interaction between the dynamical and thermal effects. Though this result is obtained by the highly truncated model, it is confirmed that the same instability took place in a high resolution grid model and a spectrum model involving many harmonics. In Fig. 3, the basic streamfunction and temperature fields obtained by a spectrum model with sufficient resolution are depicted. The solution of the perturbation equation about this basic field shows that there exists only one unstable mode. The streamfunction and temperature fields of this unstable mode are depicted in Fig. 4. The streamfunction in Fig. 4(a) shows that this unstable mode includes a mean shear flow. The streamfunction in Fig. 4(c), which is obtained by excluding the zonal component from (a), shows that this mean shear flow is maintained by tilting convection cells. Further, the temperature field depicted in Fig. 4(d) suggests that this tilting convective motion is maintained by the tilting temperature distribution. These features are not only similar to Thompson's result, but also consistent with the result obtained by the above highly truncated model.

Fig. 2. Schematic diagram showing how the interaction between velocity and temperature fields induces an instability, which generates a mean shear flow.

Fig. 3. The streamfunction (a) and temperature (b) fields associated with convection obtained in a two-dimensional spectrum model. The parameters E , K and G represent dimensionless viscosity, thermal conductivity and gravity acceleration, the values of which are assumed to be the same as used in Thompson (1970). The maximum horizontal and vertical wavenumbers are set to 2 and 9, respectively.

These facts suggest that the instability working in Thompson's experiments is caused by the interaction between velocity and temperature fields. This suggestion can be confirmed also in the following way. The velocity and temperature fields are connected by dimensionless acceleration coefficient of gravity G in our system, Eq. (2.8). Hence, by changing the value of G , the intensity of interaction between the two fields can be controlled. In Fig. 5, the growth rates of unstable mode are shown as a function of G . As G is decreased, the growth rate is also decreased, and unstable mode vanishes for $G \leq 0.1$. If the instability in Thompson's numerical experiment takes place independently of the temperature field, an unstable mode must be obtained even for $G = 0$. So it can be concluded that the instability in Thompson's numerical experiments is caused by the interaction between the velocity and

temperature fields.

3. Three-dimensional model

As described in the introduction, many studies on an interaction between the tilting convective motion and mean shear flow have been conducted (*i.e.*, Schubert and Whitehead, 1969; Thompson, 1970; Soloviev *et al.*, 1996). Most of them have adopted a two-dimensional model, in which the velocity component of convection perpendicular to mean flow cannot be taken into account. It is, however, obvious that a three-dimensional model must be adopted to examine the interaction between the convective motion, and mean (shear) flow in a real three-dimensional atmosphere. For examples, Asai (1970) showed that, in a fluid layer with unstable stratification, the mean shear flow has a stabilizing effect. This effect is most striking for transverse roll-type

Fig. 4. Unstable mode obtained for basic convection depicted in Fig. 3. Streamfunction (a), temperature (b), streamfunction deviated from the zonal mean (c) and temperature deviated from it (d) are shown.

Fig. 5. Relation between G and growth rate. Growth rates are obtained for various G . It is noted that the basic state is obtained for $G = 1$. No unstable mode is found for $G \leq 0.1$.

convection whose axis is perpendicular to the mean flow. On the other hand, longitudinal convection, whose axis is parallel to the mean flow, has no interaction with the mean flow, and will be the most preferred mode. Though, in the present problem,

convection is induced by horizontal temperature difference in a stably stratified fluid layer, it is expected that the interaction between the convective motion and mean shear flow in a three-dimensional model differs from that in the two-dimensional model.

Further, the three-dimensional studies such as Asai (1970) have been conducted by using a model *on the plane*. In order to examine the stability of planetary-scale convection cells correctly, the problem should be treated *on the sphere*. The zonal mean flow and convection treated in a two-dimensional model correspond to those in a vertical plane on the equator. However, convection between the day and night sides on the sphere involves not only a flow along the equator, but also that along meridians. The zonal mean flow is parallel to the flow of convection on the equator, but perpendicular to the one along meridians. The question then is whether or not the instability mechanism discussed in a two-dimensional model can work for convection on the sphere.

For the above reasons, a three-dimensional model on the sphere is constructed. In Thompson, the instability of steady convection was investigated by the time-integration of the numerical model. However, it is preferable to examine this instability in terms of the linear eigenvalue problem. The basic equations are equations of vorticity and temperature:

$$\left. \begin{aligned} \frac{\partial \text{rot } \mathbf{v}}{\partial t} &= \text{rot}(\mathbf{v} \times \text{rot } \mathbf{v}) - \nu \text{rot rot rot } \mathbf{v} \\ &\quad + \alpha g \text{rot} \left(\frac{\mathbf{r}}{r} \Theta \right) \\ \frac{\partial \Theta}{\partial t} &= Q - \text{div}(\Theta \mathbf{v}) + \kappa \Delta \Theta \end{aligned} \right\}, \quad (3.1)$$

where $\mathbf{r}$ is the vector from the center of sphere, ν the kinematic viscosity, α the thermal expansion coefficient, g the gravity, Q the solar heating and κ the thermal conductivity.

Under the Boussinesq approximation, the velocity field is solenoidal and can be expressed as a linear combination of certain basic toroidal (T) and poloidal (S) fields given by

$$\mathbf{T} = \text{rot} \left(\frac{\mathbf{r}}{r} T \right) \quad \text{and} \quad \mathbf{S} = \text{rot rot} \left(\frac{\mathbf{r}}{r} S \right), \quad (3.2)$$

where T and S are scalar functions of position $\mathbf{r}$ and time t (Chandrasekhar, 1961). T , S and temperature field, Θ are expanded by spherical harmonic functions. Instead of the usual definition of spherical harmonics, $Y_l^m(\theta, \phi) = P_l^{|m|}(\cos \theta) e^{im\phi}$, $Y_l^m(\theta, \phi)$ is defined as

$$Y_l^m(\theta, \phi) = \begin{cases} P_l^m(\cos \theta) \cos m\phi & (m \geq 0) \\ P_l^{|m|}(\cos \theta) \sin |m|\phi & (m < 0) \end{cases}, \quad (3.3)$$

in the present study, where θ and ϕ are co-latitude and longitude, respectively. Then, T , S and Θ are expanded as

$$\left. \begin{aligned} T(t, r, \theta, \phi) &= \sum_{l,m} T_l^m(t, r) Y_l^m(\theta, \phi) \\ S(t, r, \theta, \phi) &= \sum_{l,m} S_l^m(t, r) Y_l^m(\theta, \phi) \\ \Theta(t, r, \theta, \phi) &= \sum_{l,m} \Theta_l^m(t, r) Y_l^m(\theta, \phi) \end{aligned} \right\}. \quad (3.4)$$

Here, the symmetry about the equator is assumed for simplicity (see the discussion below). By substituting (3.2) with (3.4) into (3.1), equations for T_l^m , S_l^m and Θ_l^m are obtained. By replacing $\partial/\partial r$ with the finite difference in (3.1), the mode equations with respect to time t are obtained for T_α^k , S_α^k and Θ_α^k , where subscripts α and k denote a pair of wavenumbers (l, m) and a level index in the vertical discretization. The derivation of the mode equations and its numerical treatment are discussed in detail in Appendix a.

The following boundary conditions are adopted:

$$\frac{\partial u}{\partial r} = \frac{\partial v}{\partial r} = w = \frac{\partial \Theta}{\partial r} = 0 \quad \text{at } z = 0, D, \quad (3.5)$$

where z and D are the vertical coordinate and depth of the fluid layer. These boundary conditions are the same as used in Thompson. With T_l^m , S_l^m and Θ_l^m , these conditions can be written as

$$\frac{\partial T_l^m}{\partial r} = S_l^m = \frac{\partial \Theta_l^m}{\partial r} = 0 \quad \text{at } z = 0, D. \quad (3.6)$$

Since the planetary rotation is neglected in the present study, the distribution of solar heating and radiative cooling is axisymmetric about a line between the subsolar and antisolar points. In this study, these two points are taken on the equator. Hence, it can be expressed as

$$Q(z, \theta, \phi) = Q_0 \frac{z}{D} P_1^1(\cos \theta) \cos \phi$$

(Q_0 is a constant), (3.7)

where $P_1^1(\cos \theta)$ equals to $\sin \theta$. The vertical dependency, z/D , is the same as used in Thompson.

Using the model described above, we examine the interaction between the mean shear flow parallel to the equator, and convective motion axisymmetric about a line between the subsolar and antisolar points on the equator. It is evident that all mean flows parallel to an arbitrary great circle crossing the subsolar and antisolar points are equivalent to each other in the condition of the present problem. Hence, it is sufficient to examine the stability of convection between the day and night sides perturbed by the mean flow parallel to the equator. Thus, the symmetry about the equator can be assumed in our instability problem without loss of generality. The numerical calculations are performed as follows. First, a steady solution representing convection between the day and night sides is sought for. In order to obtain steady solutions as the continuous function of parameter values (*e.g.*, values of viscosity or heating, *etc.*), the continuation method based on Newton-Raphson method is adopted (Itoh, 1993). Once a steady solution is found, the perturbation equation about the steady solution is readily obtained. It should be noted that the perturbation

equation can be formulated directly from the mode equations for T_α^k , S_α^k and Θ_α^k , since the nonlinear terms are explicitly written down as products of expansion coefficients, T_α^k , S_α^k and Θ_α^k in the mode equations. By solving this perturbation equation, stability of the steady solution can be determined.

4. Results

In our computations, the triangular truncation $l \leq 6$ and $|m| \leq 6$ is adopted for the expansion by spherical harmonics. The five level model is used for the vertical discretization.

The parameter values involved in the equations are assumed as follows: the radius of Venus (a) is 6050km, the depth of fluid layer (D) is 10km, the gravity (g) is 8.9m/s^2 , the thermal expansion coefficient (α) is 0.002. The intensity of solar heating (Q_0) is put to $1\text{K}/10\text{days}$. For the range of vertical viscosity, $3 \leq \nu_V \leq 10\text{m}^2/\text{s}$, steady convection is calculated. For simplicity, the Prandtl number is fixed to 1. The horizontal viscosity and thermal conductivity, ν_H and κ_H , are put to $\nu_H = (a/D)^2 \nu_V$ and $\kappa_H = (a/D)^2 \kappa_V$.

The parameter range examined in this study corresponds to that in Thompson, because our main interest is to examine whether or not Thompson's mechanism proposed from a two-dimensional model can work also in a three-dimensional fluid layer on the sphere.

4.1 Steady solutions

Steady solutions are obtained in the range of $3 \leq E_V \leq 10$ (m^2/s), $3 \leq K_V \leq 10$ (m^2/s) for a fixed value of $\Gamma = 1.3$ (K/km). The relaxation time due to diffusion is about 1 year for $E_V = K_V = 3\text{m}^2/\text{s}$. This relaxation time is sufficiently larger than that of convection. The velocity and temperature fields obtained for the value of $E_V = K_V = 9, 7, 5, 3$ (K/km) are depicted in Fig. 6. The velocity and temperature fields of these solutions are axisymmetry about a line between the subsolar and antisolar points. In the upper layer, the flow is divergent due to the ascending flow near the subsolar point, and convergent due to the descending flow near the antisolar point. Since the ascending (descending) flow is located in the warm (cold) region, heat is transported upward and the temperature stratification is strengthened by the convective motion. The convective motion and temperature deviation is stronger in the upper layer, owing to a stronger contrast of heating and cooling in the upper layer. The maximum horizontal velocity is about 1 m/s at $z = D$. It is found that the area of convergence is narrower than that of divergence. This is similar to the result obtained in the two-dimensional model. The predominance of wavenumber 2 in the middle layer and Θ_0^0 distribution are also seen in Fig. 2 of Thompson. In particular, Fig. 6(d) seems to be very similar to

it.

In Fig. 7, the steady solutions for $\Gamma = 0.7$ (K/km) are depicted. It can be seen that the temperature stratification produced by convection is more intensified in comparison with the case of $\Gamma = 1.3$. Though the initial stratification is weak in the case of Fig. 7, owing to this intensification of stratification, convection for $E_V = 3$ becomes very similar to Fig. 6(d). It should be noted, however, that the temperature stratification can be influenced also by processes except convection, *e.g.*, the radiative process. So this problem should be examined also in such a situation that the temperature stratification is determined by a cooperation between convection and other processes.

4.2 Stability analysis

Since the nonlinear terms in the mode equations are written down as products of expansion coefficients as discussed in the previous section, the perturbation equation can be constructed easily if the steady solution is obtained. This perturbation equation is solved as an eigenvalue problem with respect to the growth rate. We seek for an eigenvalue with a positive real part. However, it can be shown that there is no growing mode for the steady solutions, (a), (b), (c) and (d) depicted in Figs. 6 and 7. The steady solutions are calculated also for the parameter values $3 \leq E_V \leq 100$, $3 \leq K_V \leq 100$ and $0.1 \leq \Gamma \leq 1.3$. It is shown that there is no unstable mode for these steady solutions in these parameter ranges.

This result suggests that the instability due to the Thompson mechanism found in two-dimensional models cannot work at all in the three-dimensional conditions, especially on the sphere, though the convective pattern in the vertical section on the equator is very similar to that obtained in the two-dimensional models.

4.3 Discussion

The difference in the results between the two-dimensional and three-dimensional models is supposed to be connected with the fact that the region considered in the two-dimensional model is restricted to the equatorial one on the sphere. In Thompson's study, only convection on the equator is taken into account. Kuo (1963) and Asai (1964, 1970) studied the interaction between the convective motion and mean shear flow with a three-dimensional model on the plane, and showed that a constant vertical shear suppresses the transverse mode of convection whose axis is perpendicular to the basic shear flow. But, the longitudinal mode whose axis is parallel to the basic shear flow is not suppressed at all. The longitudinal convection grows fastest in the flow with constant shear. If the same dynamical mechanism works for convection on the sphere, it is expected that Thompson's instability

Fig. 6. Steady solutions for $E_V = K_V = 9, 7, 5, 3$ and $\Gamma = 1.3$. Horizontal velocity field at $z = D$ and vertical flow at $z = D/2$ are depicted in the upper panels for each case, (a), (b), (c) and (d). The horizontal and vertical velocities are indicated by arrows and contour lines, respectively. In the lower panels of each cases shown are wind vectors and temperature distribution in the vertical section on the equator. The temperature averaged over the sphere, $\Theta_0^0(r)$, is also depicted in the lower right hand corner of each case.

on the sphere is suppressed by the meridional motion of convection between the day and night sides on meridional planes.

5. Summary and concluding remarks

In the present study, we have reconsidered Thompson's mechanism in a two-dimensional plane, and examined whether this mechanism does work in three-dimensional convection on the sphere. The results can be summarized as follows:

First, for Thompson's mechanism of two-dimensional convection, it is shown that a mean shear flow cannot tilt convection cells dynamically, and the instability is caused by the interaction between the velocity and temperature fields; this interaction can be described as follows. If a small

perturbation of mean shear flow tilts the temperature distribution in the convection cells, this tilting temperature distribution induces tilting of convective motions, which transports horizontal momentum vertically so as to reinforce the original shear.

Second, the numerical result by a three-dimensional model shows that convection between the day and night sides on the sphere are always stable, and there is no unstable mode which produces a mean shear flow. Though convection in a two-dimensional plane corresponds to convective motion in the vertical section on the equator in a three-dimensional model, this result shows that Thompson's mechanism cannot work in convection between the day and night sides on the sphere.

Certainly, this investigation is made for restricted

Fig. 7. Same as Fig. 6 but for $\Gamma = 0.7$.

parameter ranges. However, our main interest is to examine whether or not Thompson's mechanism proposed from a two-dimensional model can also work in a three-dimensional fluid layer on the sphere. The stability is investigated in the parameter range which corresponds to that adopted in Thompson. Our result shows that no unstable mode can be found in this parameter range. From this result, it is confirmed that we cannot directly apply Thompson's mechanism to three-dimensional situation.

In the present study, the effect of planetary rotation is neglected. It is shown by Gill (1980), and Matsuda and Kato (1987) that, by the Coriolis force the circulation induced by tropical heating tends to be concentrated to the equatorial region. This concentration is controlled by the ratio of planetary angular velocity to phase speed of gravity waves. Though the rotation of Venus is very slow, it is possible that this ratio becomes large because of weak stratification. The effect of the Coriolis force may become important. The circulation may be re-

stricted to the equatorial region, and suppressed in the other area. In such a situation, it is expected that Thompson's mechanism may work in the circulation concentrated near the equator. The examination of this problem is a subject for future study.

Appendix

Mathematical formulation for numerical computation

a. Expansions by spherical harmonics

Under the Boussinesq approximation, the equations of vorticity and temperature are written as:

$$\left. \begin{aligned} \frac{\partial \text{rot } \mathbf{v}}{\partial t} &= \text{rot} (\mathbf{v} \times \text{rot } \mathbf{v}) - \nu \text{rot rot rot } \mathbf{v} \\ &\quad + \alpha g \text{rot} \left(\frac{\mathbf{r}}{r} \Theta \right) \\ \frac{\partial \Theta}{\partial t} &= Q - \text{div} (\Theta \mathbf{v}) + \kappa \Delta \Theta \end{aligned} \right\}. \quad (\text{A.1})$$

By the Boussinesq approximation, the velocity field, $\mathbf{v}$ is solenoidal and can be expressed as a linear

combination of toroidal ($\mathbf{T}$) and poloidal ($\mathbf{S}$) fields: $\mathbf{v} = \mathbf{T} + \mathbf{S}$. $\mathbf{T}$ and $\mathbf{S}$ are defined as

$$\mathbf{T} = \text{rot} \left(\frac{\mathbf{r}}{r} T \right), \quad \mathbf{S} = \text{rot rot} \left(\frac{\mathbf{r}}{r} S \right), \quad (\text{A.2})$$

where T and S are scalar functions of position and time (Chandrasekhar, 1961). $\mathbf{T}(T)$ and $\mathbf{S}(S)$ denote toroidal and poloidal fields defined by scalar functions, T and S . Components of $\mathbf{T}$ and $\mathbf{S}$ are:

$$T_r = 0, \quad T_\theta = \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi}, \quad T_\phi = -\frac{1}{r} \frac{\partial T}{\partial \theta} \quad (\text{A.3})$$

$$S_r = \frac{1}{r^2} L^2 S, \quad S_\theta = \frac{1}{r} \frac{\partial^2 S}{\partial r \partial \theta}, \quad S_\phi = \frac{1}{r \sin \theta} \frac{\partial^2 S}{\partial r \partial \phi}, \quad (\text{A.4})$$

where L^2 stands for the operator,

$$L^2 \equiv -\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial}{\partial \theta} - \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2}. \quad (\text{A.5})$$

From the definition, we can verify the following relation,

$$\text{rot } \mathbf{T}(T) = \mathbf{S}(T), \quad \text{rot } \mathbf{S}(S) = \mathbf{T}(S^*), \quad (\text{A.6})$$

where $S^* = -\partial^2 S / \partial r^2 + L^2 S / r^2$. Since the fluid layer considered in the present study is very shallow, the second term in the right hand side can be ignored.

Here, T , S and temperature field, Θ are expanded by spherical harmonic functions. In the present study, spherical harmonics are defined as

$$Y_l^m(\theta, \phi) = \begin{cases} P_l^m(\cos \theta) \cos m\phi & (m \geq 0) \\ P_l^{|m|}(\cos \theta) \sin |m|\phi & (m < 0) \end{cases}, \quad (\text{A.7})$$

where θ and ϕ are co-latitude and longitude, respectively. Then, T , S and Θ are written as

$$\left. \begin{aligned} T(t, r, \theta, \phi) &= \sum_{l,m} T_l^m(t, r) Y_l^m(\theta, \phi) \\ S(t, r, \theta, \phi) &= \sum_{l,m} S_l^m(t, r) Y_l^m(\theta, \phi) \\ \Theta(t, r, \theta, \phi) &= \sum_{l,m} \Theta_l^m(t, r) Y_l^m(\theta, \phi) \end{aligned} \right\}. \quad (\text{A.8})$$

Since the symmetry about the equator is assumed in this study, T_l^m is adopted when $l+m$ is odd, and S_l^m and Θ_l^m are adopted when $l+m$ is even. The wavenumber l is truncated by 6. It should be noted that the mode with $(l, m) = (0, 0)$ is included only in the expansion of $\Theta(t, r, \theta, \phi)$. With the aid of the orthogonality of toroidal and poloidal fields, substitution of (A.8) into (A.1) gives equations for expansion coefficients, T_l^m , S_l^m and Θ_l^m (Bullard and Gellman, 1954),

$$\begin{aligned} \frac{\partial T_\gamma}{\partial t} &= \nu_V \frac{\partial^2 T_\gamma}{\partial r^2} - \nu_H \frac{(l-1)(l+2)}{a^2} T_\gamma \\ &\quad - \frac{1}{a^2 N_\gamma} \sum_{\alpha, \beta} \left[\text{TSS} + \text{SSS} + \text{STS} \right], \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} \frac{\partial S_\gamma^*}{\partial t} &= -\nu_V \frac{\partial^4 S_\gamma}{\partial r^4} + \nu_H \frac{2(l^2 + l - 1)}{a^2} \frac{\partial^2 S_\gamma}{\partial r^2} + \alpha g \Theta_\gamma \\ &\quad - \frac{1}{a^2 N_\gamma} \sum_{\alpha, \beta} \left[\text{TST} + \text{TTT} + \text{SST} + \text{STT} \right], \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} \frac{\partial \Theta_\gamma}{\partial t} &= \kappa_V \frac{\partial^2 \Theta_\gamma}{\partial r^2} - \kappa_H \frac{l(l+1)}{a^2} \Theta_\gamma + Q_\gamma \\ &\quad - \frac{1}{a^2 M_\gamma} \sum_{\alpha, \beta} \left[\text{T}\Theta\text{Y} + \text{S}\Theta\text{Y} \right] \\ &\quad - \frac{l(l+1)}{a^2} \left(\Gamma + \frac{\partial \Theta_0^0}{\partial r} \right) S_\gamma, \end{aligned} \quad (\text{A.11})$$

where a is a radius of planet, subscripts α , β and γ denote pairs of (l, m) symbolically. The terms of TSS, SSS, STS, TST, TTT, SST, STT, T Θ Y and S Θ Y represent nonlinear terms in the equations for vorticity and temperature. These terms are given by

$$\text{TSS} = -L_{\alpha, \beta, \gamma} D_\beta T_\alpha T_\beta \quad (\text{A.12})$$

$$\begin{aligned} \text{SSS} &= -\frac{K_{\alpha, \beta, \gamma}}{2} \left[D_\alpha (D_\alpha - D_\beta - D_\gamma) S_\alpha \frac{\partial T_\beta}{\partial r} \right. \\ &\quad \left. + D_\beta (D_\alpha - D_\beta + D_\gamma) \frac{\partial S_\alpha}{\partial r} T_\beta \right] \end{aligned} \quad (\text{A.13})$$

$$\text{STS} = L_{\alpha, \beta, \gamma} D_\alpha S_\alpha \frac{\partial^2 S_\beta}{\partial r^2} \quad (\text{A.14})$$

$$\begin{aligned} \text{TST} &= -\frac{K_{\alpha, \beta, \gamma}}{2} \left[\{ D_\beta (D_\alpha - D_\beta + D_\gamma) \right. \\ &\quad \left. + D_\gamma (D_\alpha + D_\beta - D_\gamma) \} T_\alpha \frac{\partial T_\beta}{\partial r} \right. \\ &\quad \left. + D_\beta (D_\alpha - D_\beta + D_\gamma) \frac{\partial T_\alpha}{\partial r} T_\beta \right] \end{aligned} \quad (\text{A.15})$$

$$\text{TTT} = L_{\alpha, \beta, \gamma} D_\gamma T_\alpha \frac{\partial^2 S_\beta}{\partial r^2} \quad (\text{A.16})$$

$$\begin{aligned} \text{SST} &= L_{\alpha, \beta, \gamma} \left[D_\alpha S_\alpha \frac{\partial^2 T_\beta}{\partial r^2} + (D_\alpha + D_\beta - D_\gamma) \right. \\ &\quad \left. \times \frac{\partial S_\alpha}{\partial r} \frac{\partial T_\beta}{\partial r} + D_\beta \frac{\partial^2 S_\alpha}{\partial r^2} T_\beta \right] \end{aligned} \quad (\text{A.17})$$

$$\begin{aligned} \text{STT} &= -\frac{K_{\alpha, \beta, \gamma}}{2} \left[D_\alpha (-D_\alpha + D_\beta + D_\gamma) \right. \\ &\quad \left. \times S_\alpha \frac{\partial^3 S_\beta}{\partial r^3} + \{ D_\alpha (-D_\alpha + D_\beta + D_\gamma) \right. \\ &\quad \left. + D_\gamma (D_\alpha + D_\beta - D_\gamma) \} \frac{\partial S_\alpha}{\partial r} \frac{\partial^2 S_\beta}{\partial r^2} \right] \end{aligned} \quad (\text{A.18})$$

$$\text{T}\Theta\text{Y} = -L_{\alpha, \beta, \gamma} T_\alpha \Theta_\beta \quad (\text{A.19})$$

$$\text{S}\Theta\text{Y} = K_{\alpha, \beta, \gamma} D_\alpha S_\alpha \frac{\partial \Theta_\beta}{\partial r} + X_{\alpha, \beta, \gamma} \frac{\partial S_\alpha}{\partial r} \Theta_\beta \quad (\text{A.20})$$

where $D_\alpha = l_\alpha(l_\alpha + 1)$. M_γ and N_γ are defined as

$$M_\gamma = \iint Y_\gamma^2 \sin \theta d\theta d\phi$$

$$= \begin{cases} \frac{2\pi}{2l_\gamma + 1} \frac{(l_\gamma + |m_\gamma|)!}{(l_\gamma - |m_\gamma|)!} & (m \neq 0) \\ \frac{4\pi}{2l_\gamma + 1} & (m = 0) \end{cases} \quad (\text{A.21})$$

$$N_\gamma = l_\gamma(l_\gamma + 1)M_\gamma \quad (\text{A.22})$$

$K_{\alpha,\beta,\gamma}$, $L_{\alpha,\beta,\gamma}$ and $X_{\alpha,\beta,\gamma}$ are called interaction coefficients defined as

$$K_{\alpha,\beta,\gamma} = \iint Y_\alpha Y_\beta Y_\gamma \sin \theta d\theta d\phi \quad (\text{A.23})$$

$$L_{\alpha,\beta,\gamma} = \iint Y_\alpha \left(\frac{\partial Y_\beta}{\partial \theta} \frac{\partial Y_\gamma}{\partial \phi} - \frac{\partial Y_\beta}{\partial \phi} \frac{\partial Y_\gamma}{\partial \theta} \right) d\theta d\phi \quad (\text{A.24})$$

$$X_{\alpha,\beta,\gamma} = \iint \left(\frac{\partial Y_\alpha}{\partial \theta} \frac{\partial Y_\beta}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial Y_\alpha}{\partial \phi} \frac{\partial Y_\beta}{\partial \phi} \right) \times Y_\gamma \sin \theta d\theta d\phi \quad (\text{A.25})$$

(Bullard and Gellman, 1954). The method for obtaining these coefficients is described in Silberman (1954).

b. Finite difference scheme in the vertical direction

Five levels are taken in the vertical direction for the numerical computations. Taking the upper and lower boundary conditions into account, T_l^m and Θ_l^m are defined at $z = D/10, 3D/10, 5D/10, 7D/10$ and $9D/10$, and S_l^m is at $z = D/5, 2D/5, 3D/5$ and $4D/5$.

c. Treatment of T_1^0 and Θ_0^0

Toroidal mode, T_1^0 represents a total angular momentum associated with a zonal flow. The vertical integral of advection terms of the equation for T_1^0 must vanish, and the dissipation term by horizontal viscosity vanishes for T_1^0 mode. Therefore, the vertical integration of the equation for T_1^0 gives

$$\frac{\partial}{\partial t} \int T_1^0 dr = \nu_V \left[\frac{\partial T_1^0}{\partial r} \right]_0^D. \quad (\text{A.26})$$

The right hand side of this equation vanishes by the boundary conditions. Though the total angular momentum remains zero in time-integration if it is zero initially, it is indefinite in stationary solutions ob-

tained by the Newton-Raphson method. The average temperature, Θ_0^0 has the same problem. Because of the absence of net heating, $\int \Theta_0^0 dr$ must remain zero. However, it is also indefinite in stationary solutions obtained by the Newton-Raphson method. So, in order to obtain stationary solutions in which $\int T_1^0 dr$ and $\int \Theta_0^0 dr$ are damped to zero, Rayleigh friction is introduced for T_1^0 and Newtonian cooling for Θ_0^0 .

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金星大気のスーパーローテーションに関する Thompson・メカニズムの解釈と、その球面上への拡張

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先ず、二次元面内での Thompson メカニズムを再検討した。少数のモードによる考察により「傾いた対流がシアにより更に傾く」という力学過程における正のフィードバックが存在しないことを示した。更に、平均流を生成する不安定が傾いた対流と傾いた温度場の間の相互作用によって生じることを示し、高解像度のモデルを用いてそれを検証した。この結果から Thompson(1970) の数値実験で示された不安定は、力学場と温度場の相互作用により生じると考えられる。

次に、球面上における Thompson メカニズムの有効性 — 三次元的な夜昼間対流の不安定により平均流が生成されるのか — を調べるための数値モデルを作成した。惑星の自転がない場合、定常解は夜側と昼側の間の軸対称な夜昼間対流となる。いろいろな条件の下で定常解を求めその安定性を調べた所、それらは安定であり、平均流を生成するような不安定モードが存在しないことが示された。この結果は Thompson メカニズムが二次元面内の対流でのみ働き、球面上の対流ではうまく働かないことを示唆している。