ANALYSIS OF INTEGRATED OPTICAL ISOLATOR USING A SINGLE SECTION OF MAGNETIC GARNET WAVEGUIDE LAYER —ON NON- AND RECIPROCITY IN IRON-GARNET WAVEGUIDES—

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ABSTRACT

This paper shows results of analyses about nonreciprocal and reciprocal properties in magnetic garnet films in order to deal with the single section unidirectional mode converter (SSUMC) required for integrated optical waveguide isolators. The analyses are made from a viewpoint of relative permittivity tensors, $[K_{ij}]$, of anisotropic materials according to the method shown by Yamamoto et al.. The direction of magnetization which gives the maximum effect on reciprocity is shown for each of the K_{xy} and the K_{yz} perturbation, respectively. In these two perturbations, it is expected that the reciprocal effect obtained from the K_{yz} perturbation is almost the same amount with the nonreciprocal effect due to the same K_{xy} perturbation. From the obtained results, a possibility of realization of the SSUMC using Bi-substituted iron garnet films as a gyrotropic waveguide is discussed.

INTRODUCTION

Recently, magnetic garnet films prepared by liquid phase epitaxy (LPE) for applications to magneto-optical elements have highly been developed. Especially, their properties on a Faraday rotation in a visible and a near infrared light region have remarkably been improved by substituting bismuth for yttrium or gadolinium [1]-[4]. Bulk isolators using thick LPE films substituted by bismuth are intensively investigated for laser diodes operating in a wavelength region of 780 - 930 nm [5]. The Bi-substituted LPE thin films are expected also for optical waveguide isolators in the infrared light region as well as the bulk ones noted above. The waveguide isolator is an isolator in the strict sense of the word, because it does not change propagating modes between the input light and the output one. The waveguide isolator is obtained from a combination of a unidirectional mode converter (UMC) and a mode selector (MS) [6]. The magnetic UMC developed today is composed of a serial connection of the Faraday and the Cotton-Mouton section [7],[8]. These two sections will be combined into one region, if the region serves both of the nonreciprocal and the reciprocal effect [9],[10]. The single section UMC has a favorable simplicity in the structure. However, basic analysis for the operation of UMC of this type has insufficiently been made until now.

We discuss the nonreciprocity and the reciprocity about the single section UMC from a viewpoint of permittivity tensors in iron garnet films, and the direction of magnetization to be controlled in the film in order to design the UMC.

TREATMENT OF RECIPROCITY AND NONRECIPROCITY DUE TO THE TRANSMISSION MATRIX

General treatments as to gyrotropic waveguides have previously been shown by Yamamoto et al. [6],[11], Miyazaki et al. [12], and Torfeh et al. [13]. We here show one approach to the problem of the magnetic garnet waveguides. Signs and their definitions required for later discussions are given in the Table 1. The signs which are almost the same as those having been used by Yamamoto are employed for convenience when referring to their papers. The waveguide treated here is a two dimensional structure consisting of three layers as shown in Fig. 1. Two portions, I and III, operating as a clad in the waveguide are assumed isotropic. A portion, II, is a magnetic garnet film, and is assumed optically anisotropic due to its magnetic and dielectric properties. The direction of optical wave propagation is parallel to the z-axis. It is well known that internal stresses induced in films as the magnetic garnet film is epitaxially grown on a substrate cause birefringence. The induced birefringence will effectively be utilized for phase matching between TE and TM modes. This paper deals with the operation in the phase matching state from the above point of view.

TABLE I. Expressions of fields and definitions of parameters.

Fields:	$\begin{bmatrix} E \\ H \end{bmatrix} = \begin{bmatrix} e(x) \\ h(x) \end{bmatrix} \exp\{j(\omega t - \beta z)\}$
TE-mode:	$\{(e^{(h)}(x), h^{(h)}(x); \beta^{(h)})\}$
TM-mode:	$\{(e^{(e)}(x), h^{(e)}(x); \beta^{(e)})\}$
Perturbation coefficient:	$\alpha = N/ N $
Mean phase constant:	$\beta_m = \frac{1}{2}(\beta^{(h)} + \beta^{(e)})$
Degeneracy factor:	$\delta = (\beta^{(h)} - \beta^{(e)})/(\beta^{(h)} + \beta^{(e)})$
Coupling coefficient:	$\kappa = N /\beta_m$

Superscripts (h) and (e) denote the TE-mode and the TM-mode, respectively. The parameter N is the overlap integral.

Fig. 1. Geometry of a waveguide. I and III are isotropic. II is gyrotropic and anisotropic.

A. Modes of perturbation

Dielectric tensors in anisotropic waveguide films are expressed as

$$[\epsilon] = \epsilon_0 [K] = \epsilon_0 \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{xy}^* & K_{yy} & K_{yz} \\ K_{xz}^* & K_{yz}^* & K_{zz} \end{bmatrix}, \quad (1)$$

where $[K]$ is the relative permittivity tensor. In the perturbation method due to the two mode approximation, in which only the basis for each of TE and TM waves in basic system modes is employed, off-diagonal elements, K_{xy} , K_{yz} and K_{xz} , are assumed to operate as perturbation for the basic system having only diagonal terms of K_{xx} , K_{yy} and K_{zz} . The following three modes of perturbation exist with respect to three off-diagonal elements noted above.

$$(a) K_{xy} \text{ perturbation: } N_{xy} = \omega \epsilon_0 \int (K_{xy}^* e_y^{(h)} * e_x^{(e)}) dx. \quad (2)$$

$$(b) K_{yz} \text{ perturbation: } N_{yz} = \omega \epsilon_0 \int (K_{yz}^* e_y^{(h)} * e_z^{(e)}) dx. \quad (3)$$

$$(c) K_{xz} \text{ perturbation: } N_{xz} = 2 \omega \epsilon_0 \int R [K_{xz}^* e_x^{(e)} * e_z^{(e)}] dx. \quad (4)$$

In both cases of (a) and (b), mode coupling occurs between a TE and a TM wave. In the case of (c), TE-TM coupling does not occur, and only the propagation constant varies from $\beta^{(e)}$ to $\beta^{(e)} + N_{xz}$. Values of off-diagonal elements appearing in dielectric tensors of magnetic garnets are usually a factor of 10^{-3} to 10^{-4} less than those of on-diagonal ones. It is considered that the perturbation method employed here as an approximation is adequate to analyses of magnetic garnet waveguide layers.

B. Reciprocity and nonreciprocity in relation to the transmission matrix

The transmission matrix is defined as follows [11]:

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}. \quad (5)$$

In a TE-TM degenerate system ($\delta=0$), $[T]$ for forward-traveling waves (F-waves) due to K_{xy} and K_{yz} perturbations is described as

$$[T_{d \ xy, yz}^F] = \begin{bmatrix} \cos(\beta_m K z) & j(-\alpha_d) \sin(\beta_m K z) \\ j(-\alpha_d^*) \sin(\beta_m K z) & \cos(\beta_m K z) \end{bmatrix}, \quad (6)$$

where a subscript d in the perturbation coefficient α denotes the mode degeneracy. Similarly, that of backward-traveling waves (B-waves) due to K_{xy} perturbation is expressed as

$$[T_{d \ xy}^B] = \begin{bmatrix} \cos(\beta_m K z) & j(-\alpha_d) \sin(\beta_m K z) \\ j(-\alpha_d^*) \sin(\beta_m K z) & \cos(\beta_m K z) \end{bmatrix} = [T_{d \ xy}^F], \quad (7)$$

and that on the K_{yz} perturbation as

$$[T_{d \ yz}^B] = \begin{bmatrix} \cos(\beta_m K z) & j\alpha_d \sin(\beta_m K z) \\ j\alpha_d^* \sin(\beta_m K z) & \cos(\beta_m K z) \end{bmatrix}. \quad (8)$$

If a relation of $[T^F]^t = [T^B]$ holds between F-waves and B-waves, the TE-TM mode conversion is reciprocal.

CHARACTERISTICS OF MODE CONVERSION IN MAGNETIC GARNET WAVEGUIDE FILMS

The permittivity tensor in a (111) face epitaxial garnet film are explicitly given in Eq. (4) in Ref. 12 as a function of the magnetization direction of polar and azimuthal angles γ and ξ . If we neglect the anisotropic term Δf , K_{xy} and K_{yz} are simply expressed as

$$K_{xy} = -jf_1^e (\sin \gamma \cos \xi) M + f_{44} (\sin^2 \gamma \sin^2 \xi) M^2, \quad (9)$$

and

$$K_{yz} = +jf_1^e (\cos \gamma) M + f_{44} (\sin^2 \gamma \sin^2 \xi) M^2, \quad (10)$$

respectively. A relation between a coordinate system and the angle γ and ξ is shown in Fig. 2. Assuming off-diagonal terms of relative permittivity tensors in the top layer III and the substrate I to be zero, the overlap integrals N_{xy} and N_{yz} with respect to the K_{xy} and K_{yz} perturbations are described as

$$\begin{aligned} N_{xy} &= A^{(h)} A^{(e)} \beta^{(e)} \frac{K_{xy}^*}{2K_{xx}} \left[\frac{\sin(\psi^{(h)} + \psi^{(e)}) - \sin(\phi^{(h)} + \phi^{(e)})}{h^{(h)} + h^{(e)}} \right. \\ &\quad \left. + \frac{\sin(\psi^{(h)} - \psi^{(e)}) - \sin(\phi^{(h)} - \phi^{(e)})}{h^{(h)} - h^{(e)}} \right] \\ &= [f_{44} (\sin^2 \gamma \sin^2 \xi) M^2 + jf_1^e (\sin \gamma \cos \xi) M] [F(A^{(h)}, (e), \beta^{(e)}, \psi^{(h)}, (e), \phi^{(h)}, (e), h^{(h)}, (e))] / 2K_{xx} \\ &= a + jb, \end{aligned} \quad (11)$$

and

$$\begin{aligned} N_{yz} &= jA^{(h)} A^{(e)} \frac{K_{yz}^*}{2K_{zz}} \left[\frac{-\cos(\psi^{(h)} + \psi^{(e)}) + \cos(\phi^{(h)} + \phi^{(e)})}{h^{(h)} + h^{(e)}} + \frac{\cos(\psi^{(h)} - \psi^{(e)}) - \cos(\phi^{(h)} - \phi^{(e)})}{h^{(h)} - h^{(e)}} \right] \\ &= [-f_1^e (\cos \gamma) M + jf_{44} (\sin^2 \gamma \sin^2 \xi) M^2] [G(A^{(h)}, (e), \beta^{(e)}, \psi^{(h)}, (e), \phi^{(h)}, (e), h^{(h)}, (e))] / 2K_{zz} \\ &= -c + jd, \end{aligned} \quad (12)$$

Fig. 2. Coordinate system. A relation between the direction of magnetization and polar and azimuthal angles, γ and ξ .

where A is the normalization constant, $\beta^{(e)}$ the propagation one, ψ and ϕ are the phase one, and h is the constant as to K and $\beta^{(h)}$. These quantities appear when solving the wave equation. From (11) and (12), α_d is expressed with respect to the K_{xy} and the K_{yz} perturbation as

$$\alpha_{d,xy} = N_{xy}/|N_{xy}| = a' + jb', \quad (13)$$

and

$$\alpha_{d,yz} = N_{yz}/|N_{yz}| = -c' + jd', \quad (14)$$

respectively. The transmission matrices in this case are described using (13) and (14) as follows.

(a) K_{xy} perturbation. For F-waves,

$$[T_{d,xy}^F] = \begin{bmatrix} \cos(|N_{xy}| z) & (b' - ja') \sin(|N_{xy}| z) \\ (-b' - ja') \sin(|N_{xy}| z) & \cos(|N_{xy}| z) \end{bmatrix}; \quad (15)$$

and for B-waves,

$$[T_{d,xy}^B] = [T_{d,xy}^F]. \quad (16)$$

(b) K_{yz} perturbation. For F-waves,

$$[T_{d,yz}^F] = \begin{bmatrix} \cos(|N_{yz}| z) & (d' + jc') \sin(|N_{yz}| z) \\ (-d' + jc') \sin(|N_{yz}| z) & \cos(|N_{yz}| z) \end{bmatrix}; \quad (17)$$

and for B-waves,

$$[T_{d,yz}^B] = \begin{bmatrix} \cos(|N_{yz}| z) & (-d' - jc') \sin(|N_{yz}| z) \\ (d' - jc') \sin(|N_{yz}| z) & \cos(|N_{yz}| z) \end{bmatrix}. \quad (18)$$

From expressions (15)-(18), the reciprocity related to the magnetization direction in the film, or parameters a, b, c and d, is summarized in Table 2.

Table 2. The relation between the reciprocity and parameters a, b, c and d.

K_{xy} perturbation	K_{yz} perturbation
<p>(1) $a \neq 0, b = 0$ (a term of $M = 0$, that of $M^2 \neq 0$) $[T_{d,xy}^F]^t = [T_{d,xy}^B]$: purely reciprocal</p> <p>(2) $a = 0, b \neq 0$ (a term of $M \neq 0$, that of $M^2 = 0$) $[T_{d,xy}^F]^t \neq [T_{d,xy}^B]$: purely nonreciprocal</p> <p>(3) $a \neq 0, b \neq 0$ (a term of $M \neq 0$, that of $M^2 \neq 0$) $[T_{d,xy}^F]^t \neq [T_{d,xy}^B]$: nonreciprocal containing the reciprocity</p>	<p>(1) $c \neq 0, d = 0$ (a term of $M \neq 0$, that of $M^2 = 0$) $[T_{d,yz}^F]^t \neq [T_{d,yz}^B]$: purely nonreciprocal</p> <p>(2) $c = 0, d \neq 0$ (a term of $M = 0$, that of $M^2 \neq 0$) $[T_{d,yz}^F]^t = [T_{d,yz}^B]$: purely reciprocal</p> <p>(3) $c \neq 0, d \neq 0$ (a term of $M \neq 0$, that of $M^2 \neq 0$) $[T_{d,yz}^F]^t \neq [T_{d,yz}^B]$: nonreciprocal containing the reciprocity</p>

DISCUSSION

A. Mode conversion efficiency

The mode conversion efficiency required for the UMC is 50% for each effect of reciprocity and nonreciprocity. The efficiency depends only on the ratio δ/K according to the method employed here [6], and the 50% conversion efficiency can be obtained when $\delta/K = 1$. Therefore, δ must be non-zero, that is, waveguides must be in the state of non-degenerate operation. Dotted lines in Fig. 3 show that, and a point, c, denotes the phase at which 50% conversion can be obtained. On the other hand, 100% conversion can always be achieved in the state of degeneracy (phase-match operation), and in this case we will be able to obtain the 50% conversion by reducing a guide length for the 100% conversion to half (a point, a, shown in Fig. 3). If the direction of magnetization is changed from the one which gives the minimum length, it only makes the guide length achieving 50% conversion longer as a relation of two points between a and b shown in Fig. 3. On the basis of these discussions, we will be able to employ the following expression for the SSUMC:

$$L = \pi/4 |N_{TE-TM}|_{\text{reciprocal}} = \pi/4 |N_{TE-TM}|_{\text{nonreciprocal}}, \quad (19)$$

where L is the waveguide length for a 45° rotation of polarized planes, and N the overlap integral.

B. Magnetization direction and guide lengths in the SSUMC

Two typical cases in order to constitute the SSUMC by using the K_{xy} and the K_{yz} perturbation are as follows.

Fig. 3. Variation of the TE-TM conversion efficiency with $c_0 z$ ($c_0 = \beta K$) taking δ/K as a parameter.

Fig. 4. The direction of magnetization when using the K_{yz} perturbation as a reciprocal effect.

1) The case in which the K_{xy} perturbation is used as a nonreciprocal effect and the K_{yz} one as a reciprocal effect. In this case, $a = 0$, $b \neq 0$, $c = 0$, and $d \neq 0$. The maximum effect for the reciprocity is obtained from $\gamma = 90^\circ$ and $\xi = 45^\circ$, as shown in Fig. 4. The magnitude of nonreciprocal effect is $1/\sqrt{2}$ comparing with that in the case where the magnetization lies in the z -axis. This case is simple, since roles of the K_{xy} and the K_{yz} perturbation are separated with respect to the nonreciprocity and the reciprocity. However, the SSUMC will not be obtained from this condition, because guide lengths for each effect are entirely different as shown in Fig. 5 in which the $f_{44}^e M_s^2$ dependence of the guide lengths for each of the reciprocal (LMB; linear magnetic birefringence) and nonreciprocal (CMB; circular magnetic birefringence) effects are calculated by assuming $K_{xy} = 6 \times 10^{-3}$.

2) The case in which the K_{xy} perturbation is used for both of the reciprocal and the nonreciprocal effect. In this case, $a \neq 0$, $b \neq 0$, $c = 0$, and $d = 0$. The parameter, a , corresponds to the reciprocal effect, and b to the nonreciprocal one. The effect of maximum magnitude for the reciprocity is obtained from $\gamma = 45^\circ$ and $\xi = 90^\circ$, as shown in Fig. 6, where the effect of the nonreciprocity is zero. In this case, by decreasing the angle ξ from 90° , the guide length required for 50% conversion will be equalized for both effects. Results of sample calculations in Fig. 7 show that two effects due to the K_{xy} perturbation exhibit the same order of magnitude, and at $\xi = 77^\circ$ the lengths become equal. These results are considered to qualitatively support data having been obtained from experiments on TE-TM mode conversion properties in Bi-substituted iron garnet films[8].

Fig. 5. $f_{44}^e M_s^2$ dependence of the guide length at an optical wave length of $1.15 \mu\text{m}$. M_s is the saturation magnetization.

Fig. 6. Magnetization direction when using the K_{xy} perturbation as both of a nonreciprocal and a reciprocal effect.

Fig. 7. ξ dependence of the guide length at $1.15 \mu\text{m}$.

CONCLUSION

The reciprocity and the nonreciprocity due to off-diagonal elements, K_{xy} and K_{yz} , of relative permittivity tensors in the magnetic garnet films used as gyrotropic waveguides have been investigated in relation to the direction of magnetization in the films. The perturbation due to K_{xz} has not been considered here. That effect will out of the question, since the TE-TM mode conversion is directly unaffected from that perturbation. The following have been clarified. 1) The arrangement of the magnetization in which the K_{xy} and the K_{yz} perturbation contribute respectively to the nonreciprocal and the reciprocal effect, or vice versa, is possible. In this case, however, it will be impossible to make the waveguide lengths equal for both of the reciprocal and the nonreciprocal effect required for the TE-TM conversion. The effect due to K_{yz} is about two orders of magnitude less comparing with that of K_{xy} . 2) A real part of K_{xy} and an imaginary part of the same off-diagonal element can be concerned with the reciprocal and the nonreciprocal effect, respectively. In this case, both of the effects are expected to be of the same order of magnitude, and it will give the possibility of making the waveguide lengths for both effects equal. However, this is not utilized immediately in the case of the degenerate operation. The nondegenerate one has to be discussed in order to make the best use of such a magnetization orientation.

Although, a few excellent investigations on linear magnetic birefringence (Cotton-Mouton effect) have been made as to bismuth substituted iron garnet films [8], the results obtained to date are not enough to develop the SSUMC. Experimental studies have to be continued in order to obtain further information on a value of f_{44}^e in a term of M_s^2 for the Bi-substituted films. It is noted that a technique for the arrangement of the magnetization in the films due to external fields is particularly important for the SSUMC.

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