Self-Organized Magnetic Recording System

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A new theoretical formulation is proposed on a spin system, which is related to the self-organization of spatially modulated phases and the magnetic multi-valued recording. In this work, macroscopic spin state of the system is determined as a result of a competition between exchange and dipole interactions during the time evolution. Macroscopic patterns are described in linear combinations of periodic functions in Fourier transformed equations with linearized spin interactions. Spins are placed on the lattice points to describe simply this system. Spin configurations can have various symmetrical and asymmetrical patterns even under a uniform external field, according to the balance between interactions. We show that a spatially modulated phase can spontaneously appear by applying just a uniform magnetic field for a finite time interval.

Key words: magnetic recording, thin film, spin system, multi-valued recording, spatially modulated phase

1. Introduction

Various magnetic domains (or patterns) appearing in optomagnetic recording are full of mystery, which are dependent on the temporal and spatial changes of environment such as an applied magnetic field and the temperature. Magnetic domains interact with each other via the wall-interaction and the magnetic dipole-dipole interaction. The former interaction stems from the short-range exchange interaction among spins, while the latter is essentially of long-range. The competition of these two interactions can be regarded as the main mechanism for the existence of the wide variations in optomagnetic recording patterns. As well as spatially homogeneous domains, there are also many kinds of spatially modulated domains which occur especially under a weak external field. Although the interactions among micro-domains can be calculated explicitly from microscopic models, detailed investigations of magnetic recording patterns have not yet been reported so far from the viewpoint of the empirical theory of micro-domains. Only exception is a study based on Ginzburg-Landau theory on a microscopic Ising model, whose Hamiltonian is time- and temperature-independent¹⁾. Similar theories have been developed first in the investigation of a micro-emulsion phase in water-surfactant-oil systems²⁾. In case of Ising model, each spin is fixed to $\pm S$ values. Mapping of a microscopic Ising model onto a macroscopic model of interacting magnetic domains is not so straightforward

and generally not easy, because every domains have different amplitudes and angles of magnetic moments according to the magnetic field, the temperature, and, moreover, to the history of optomagnetic recording. In order to understand hysteresis behaviors in real samples, a magnetic pinning effect due to impurities cannot be ignored. It causes a tendency to prevent large and abrupt changes in magnetic structures, and should be treated as a magnetic friction. Here we present a new formulation of interacting magnetic domains to understand these interesting behaviors.

2. Formulation

In general, if one considers only a system near thermal equilibrium, its temporal evolution occurs in a direction of the minimum free energy, according to the theory of non-equilibrium thermodynamics³⁾. The local free energy of the system is defined to be

$$F = \frac{1}{2} \left(E_1 \sum_{\langle i,j \rangle}^{n,n} \mathbf{S}_i(t) \cdot \mathbf{S}_j(t) + E_2 \sum_{\langle i,j \rangle}^{n,n,n} \mathbf{S}_i(t) \cdot \mathbf{S}_j(t) - E_3 \sum_{\langle i,j \rangle} \left(\frac{\mathbf{S}_i(t) \cdot \mathbf{S}_j(t)}{|\mathbf{r}_i - \mathbf{r}_j|^3} - 3 \frac{[\mathbf{S}_i(t) \cdot (\mathbf{r}_i - \mathbf{r}_j)][\mathbf{S}_j(t) \cdot (\mathbf{r}_i - \mathbf{r}_j)]}{|\mathbf{r}_i - \mathbf{r}_j|^5} \right) + E_4 \sum_i \mathbf{S}_i(t) \cdot \mathbf{H}(t) \right) \quad (1)$$

where n,n indicates nearest neighbor sites, n,n,n indicates next nearest neighbor sites, $\mathbf{S}_i$ is the D -dimensional spin at each site, $\mathbf{H}$ is the external field and E 's are coefficients of interactions. The temporal evolution of $\mathbf{S}$ is governed by

$$\frac{d\mathbf{S}_j(t)}{dt} = -\gamma \frac{\delta F}{\delta \mathbf{S}_j(t)} + \lambda(t) \mathbf{S}_j(t), \quad (2)$$

where $\lambda(t)$ is Lagrange's multiplier to ensure that the amplitude of spin is held fixed to the value determined by the temperature and γ is the inverse of a magnetic friction coefficient. Inserting Eq.(1) into Eq.(2), we have

$$\begin{aligned} \frac{dS_j^{(\alpha)}(t)}{dt} = & -\gamma E_1 \sum_i^{n,n} S_i^{(\alpha)}(t) - \gamma E_2 \sum_i^{n,n,n} S_i^{(\alpha)}(t) \\ & + 2\gamma E_3 \sum_i \left(\frac{S_i^{(\alpha)}(t)}{|\mathbf{r}_i - \mathbf{r}_j|^3} - 3 \frac{[S_i(t) \cdot (\mathbf{r}_i - \mathbf{r}_j)] (r_i^{(\alpha)} - r_j^{(\alpha)})}{|\mathbf{r}_i - \mathbf{r}_j|^5} \right) \end{aligned}$$

$$+\lambda(t)S_j^{(\alpha)}(t) - \gamma E_4 \sum_i H_i^{(\alpha)}(t) \quad (3)$$

where (α) or (β) indicates each component of vectors. Here, the equation in the bracket (the third term) is rewritten as

$$\frac{1}{2} \sum_{\beta=1}^D S_i^{(\alpha)}(t) \frac{\partial}{\partial r_i^{(\alpha)}} \frac{\partial}{\partial r_i^{(\beta)}} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}. \quad (4)$$

Now we transform Eq.(3) into Fourier space by integrating both sides of this equation with respect to $\mathbf{r}_j$:

$$\int d\mathbf{r}_j e^{-i\mathbf{G}_j \cdot \mathbf{r}_j} \dots, \quad (5)$$

where $\mathbf{G}$'s are the reciprocal lattice vectors; we decompose $\mathbf{S}$ and $\mathbf{H}$ into Fourier components, respectively, as

$$\mathbf{S} = \sum_{\mathbf{G}} e^{i\mathbf{G} \cdot \mathbf{r}} \tilde{\mathbf{S}}(\mathbf{G}, t) \quad (6)$$

and

$$\mathbf{H} = \sum_{\mathbf{G}} e^{i\mathbf{G} \cdot \mathbf{r}} \tilde{\mathbf{H}}(\mathbf{G}, t). \quad (7)$$

Then we obtain

$$\begin{aligned} \frac{\partial}{\partial t} \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) &= -\gamma E_1 \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) \sum_i^{n.n.} e^{i\mathbf{G}_j \cdot \boldsymbol{\delta}_i} \\ &\quad - \gamma E_2 \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) \sum_i^{n.n.n.} e^{i\mathbf{G}_j \cdot \boldsymbol{\delta}_i} \\ &\quad + \gamma E_3 \sum_{\beta=1}^D \tilde{S}^{(\beta)}(\mathbf{G}_j, t) G_j^{(\alpha)} G_j^{(\beta)} \\ &\quad \times \sum_i \frac{1}{|\mathbf{r}_i|} e^{\mathbf{G}_j \cdot \mathbf{r}_i} \\ &\quad + \lambda(t) \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) \\ &\quad - \gamma E_4 \tilde{\mathbf{H}}(\mathbf{G}_j, t), \end{aligned} \quad (8)$$

where $\boldsymbol{\delta}_i$ denotes a vector pointing to the nearest neighbor or next nearest neighbor sites. This summation with respect to $\boldsymbol{\delta}_i$ can be performed easily and becomes, e.g. for the case of the nearest neighbors,

$$\sum_i^{n.n.} e^{i\mathbf{G}_j \cdot \boldsymbol{\delta}_i} = 2 \sum_{\alpha=1}^d \cos a G_j^{(\alpha)} \quad (9)$$

for any spatial dimensionality d . Here we put the lattice constant as a . In particular, for small value of a , we may expand Eq.(9) to get

$$2 \sum_{\alpha=1}^d \cos a G_j^{(\alpha)} \sim 2d - a^2 \mathbf{G}_j^2 + \frac{1}{12} a^4 \sum_{\alpha} \left(G_j^{(\alpha)} \right)^4. \quad (10)$$

Similarly, in the case of the next nearest neighbors, we have

$$\begin{aligned} \sum_i^{n.n.n.} e^{i\mathbf{G}_j \cdot \boldsymbol{\delta}_i} &= 2 \sum_{\alpha=1}^d \cos a \left(G_j^{(\alpha)} + G_j^{(\alpha+1)} \right) \\ &\sim 2d - 2a^2 \mathbf{G}_j^2 + \frac{4}{5} a^4 (\mathbf{G}_j^2)^2, \end{aligned} \quad (11)$$

where we took a spherical average. In the third term of the right-hand-side in Eq.(8), the summation with respect to j can be performed independently as follows:

$$\begin{aligned} f(\mathbf{G}) &= \sum_i \frac{1}{|\mathbf{r}_i|} e^{i\mathbf{G} \cdot \mathbf{r}_i} \\ &\sim \frac{1}{a^d} \int_a^\infty r^{d-1} dr \int d\Omega \frac{e^{-i\mathbf{G} \cdot \mathbf{r}}}{r} \\ &= \frac{1}{a^d} \int_a^\infty r^{d-2} dr \int d\Omega e^{-i\mathbf{G} \cdot \mathbf{r}} \\ &= \frac{1}{a^d} \int_a^\infty r^{d-2} e^{-\eta r} dr \int d\Omega e^{-i\mathbf{G} \cdot \mathbf{r}}, \end{aligned} \quad (12)$$

which becomes, in two dimensions ($d=2$),

$$\begin{aligned} f(\mathbf{G}) &\sim \frac{\pi}{a^d} \left[\frac{2}{G} - a J_0(Ga) \right. \\ &\quad \left. + a\pi \{ J_0(Ga) \mathcal{H}_1(Ga) - J_1(Ga) \mathcal{H}_0(Ga) \} \right]. \end{aligned} \quad (13)$$

Here, we used

$$\begin{aligned} \int d\Omega e^{-i\mathbf{G} \cdot \mathbf{r}} &= \int_0^{2\pi} e^{-iGr \cos \varphi} d\varphi \\ &= 2\pi I_0(-iGr) \\ &= 2\pi J_0(Gr), \end{aligned} \quad (14)$$

where $J_n(x)$ and $\mathcal{H}_n(x)$ are the Bessel and Struve functions, respectively. All the above equations are general enough to discuss spin systems with arbitrary initial conditions and arbitrary external magnetic fields which may be time dependent. If one may regard $\mathbf{G}$ as spherically symmetric and take for granted the continuous limit where a goes to zero in Eqs.(10) and (11), the equation of motion (8) becomes

$$\begin{aligned} \frac{\partial}{\partial t} \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) &= (A(t) + B G_j^2 + C G_j^4) \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) \\ &\quad + \gamma E_3 f(\mathbf{G}_j) \sum_{\beta=1}^D G_j^{(\alpha)} G_j^{(\beta)} \tilde{S}^{(\beta)}(\mathbf{G}_j, t) \\ &\quad - \gamma E_4 \tilde{\mathbf{H}}^{(\alpha)}(\mathbf{G}_j, t), \end{aligned} \quad (15)$$

where we put coefficients $A(t)$, B , C as

$$\begin{aligned} A(t) &= \lambda(t) - 2\gamma d(E_1 + E_2) \\ B &= a^2 \gamma (E_1 + 2E_2) \\ C &= -\frac{1}{5} a^4 \gamma (E_1 + 4E_2). \end{aligned} \quad (16)$$

In real systems, the amplitude of each spin, $\mathbf{S}_j$, strongly depend on the temperature $T = T(t)$ through

$\lambda(t)$ according to an empirical condition such as the magnetic phase diagram. In our formulation based on Eq.(2), we may set the condition

$$\begin{aligned} \sum_j |\mathbf{S}_j(t)|^2 &= \sum_{\mathbf{G}} \sum_{\alpha=1}^D \tilde{S}^{(\alpha)}(\mathbf{G}, t) \tilde{S}^{(\alpha)}(-\mathbf{G}, t) \\ &= M^2, \end{aligned} \quad (17)$$

where $M = M(t, T(t))$ is the averaged amplitude of the magnetization. This equation can be used to fix the amplitude of $\mathbf{S}_j(t)$ which is determined by Eq.(15) through $A(t)$. Multiplying $\tilde{S}^{(\alpha)}(-\mathbf{G}_j, t)$ to the both sides of Eq.(15) and summing it up with respect to $\mathbf{G}_j$ and α , we have an equation

$$\begin{aligned} \frac{1}{2} \frac{\partial}{\partial t} M^2 &= A(t) M^2 \\ &+ \sum_{\mathbf{G}_j} \sum_{\alpha=1}^D \left[(BG_j^2 + CG_j^4) \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) \right. \\ &\left. + \gamma E_3 f(\mathbf{G}_j) \sum_{\beta=1}^D G_j^{(\alpha)} G_j^{(\beta)} \tilde{S}^{(\beta)}(\mathbf{G}_j, t) \right] \tilde{S}^{(\alpha)}(-\mathbf{G}_j, t) \\ &- \gamma E_4 H^{(1)}(t) \tilde{S}^{(1)}(\mathbf{0}, t), \end{aligned} \quad (18)$$

which determines $A(t)$. Here we assumed that the external magnetic field is homogeneous and is applied in the first direction:

$$\mathbf{H}(\mathbf{r}, t) = \begin{pmatrix} H^{(1)}(t) \\ 0 \\ 0 \end{pmatrix}. \quad (19)$$

In Eq.(18) the $\mathbf{G} = \mathbf{0}$ component of the spin means the spatial average

$$\begin{aligned} \tilde{S}^{(\alpha)}(\mathbf{0}, t) &= \frac{1}{N} \sum_{\mathbf{r}_i} S_i^{(\alpha)}(t) \\ &= \overline{S_i^{(\alpha)}}(t). \end{aligned} \quad (20)$$

3. Results and discussions

Time evolutions of spin configurations can be determined according to the formulation presented in the previous section. For example, let us set the temperature to $T < T_c$. Starting from an equilibrium ordered phase at $t < 0$, which is spatially homogeneous ($\mathbf{G} = \mathbf{0}$), we apply an external field

$$\begin{cases} H(t) \neq 0 & (0 \leq t_1 \leq a) \\ H(t) = 0 & (t < 0, a < t_1). \end{cases} \quad (21)$$

Since the system relaxes to a steady state at $t > t_1$, the time-derivative, i.e., the left-hand-side of Eqs.(15) and (18) approaches to 0 in the limit $t \rightarrow \infty$. The last term in the right-hand-side of Eqs.(15) and (18) is constantly zero for $t > t_1$. Then Eq.(15) becomes

$$\begin{aligned} 0 &= (A(t) + BG_j^2 + CG_j^4) \tilde{S}^{(\alpha)}(\mathbf{G}_j, t) \\ &+ \gamma E_3 f(\mathbf{G}_j) \sum_{\beta=1}^D G_j^{(\alpha)} G_j^{(\beta)} \tilde{S}^{(\beta)}(\mathbf{G}_j, t) \end{aligned} \quad (22)$$

in the limit $t \rightarrow \infty$. Since $f(\mathbf{G}_j) G_j^{(\alpha)} G_j^{(\beta)}$ is zero at $\mathbf{G} = \mathbf{0}$, Eqs.(18) and (22) have obviously a solution such that $\tilde{\mathbf{S}}(\mathbf{G}, t)$ is nonzero only at $\mathbf{G} = \mathbf{0}$. There are many other possible solutions with $\mathbf{G} \neq \mathbf{0}$, which correspond to spatially modulated phases. Such spatial modulations are possible as a result of the competition among four terms in Eq.(22). Even in the case C is small, the competition occurs between the rest terms. The values of $\mathbf{G}$ which satisfy Eq.(22) depend on the explicit form of the function $f(\mathbf{G})$ which is evaluated in Eq.(13). If the spatially modulated phase with $\mathbf{G} \neq \mathbf{0}$ corresponds to the minimum of the free-energy of the system, the system would have a strong tendency to approach to this phase. It may indeed happen in the present situation, once the spatial average of the first component of the spin, $\tilde{S}^{(1)}(\mathbf{0}, t)$, becomes small, because of the on-set of the external field (21).

Consequently, we have shown theoretically that magnetic ordering with spatial modulations may appear in the system starting even from a homogeneously ordered phase. We expect that the presently proposed formulation can be applicable to an analysis a self organized multi-valued spin system. This detailed analysis using numerical integrations will be reported elsewhere⁴⁾.

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