Analysis of Magneto-Impedance in Multilayers with Crossed Anisotropy

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The magneto-impedance (MI) behavior has been analyzed in multilayers having an inner conductor lead and two identical magnetic layers with a uniaxial anisotropy which is crossed in the upper film with respect to that in the lower one. Emphasis is placed on the evolution of MI characteristics under the effect of a dc biasing current, which is of interest for developing auto-biased magnetic sensors. Various types of MI can be realized in this case. Among them, asymmetrical anhysteretic MI has special importance for linear magnetic sensing. For analysis, an approximation of a weak skin effect, as well as an exact solution of Maxwell's equations, is used, demonstrating that for a CoSiB/Cu/CoSiB film of 1- μm thickness the skin effect can be neglected for frequencies up to the GHz band. The insertion of a conductor lead, by decreasing the resistance of the system, makes the skin effect unimportant for obtaining large impedance changes: MI of 100–250% takes place at frequencies of 10–50 MHz.

Key words: multilayers, magneto-impedance, biasing field, rotational hysteresis, ac rotational permeability.

1. Introduction

The impedance of a multilayer film consisting essentially of two outer soft ferromagnetic layers and an inner metallic layer (Cu or Ag) carrying a high frequency current can be very sensitive to the application of a dc magnetic field.^{1)–5)} The magneto-impedance (MI) ratio of about 300% has been reported³⁾ for CoSiB/Cu/CoSiB multilayers of several micron thick, for a current frequency of 10 MHz and a magnetic field of 830 A/m applied along the current (and transverse to the magnetic anisotropy). In this structure, the large change in impedance is not related to the skin-effect, since the skin depth δ in both the magnetic and metallic layers is larger than 10 μm at 10 MHz. It implies that in a similar single CoSiB layer the impedance change would be very small (experimentally, about 5%). The resistivity of amorphous CoFeSiB alloy is about 50 times larger than that of Cu, therefore, the resistance of the multilayer structure is mainly determined by the inner lead resistance. The inductive term of the impedance which comes largely from the flux change in the magnetic layers can become dominant at relatively low frequencies when the skin effect is not essential.²⁾ The possibility to neglect the skin effect and still obtain a very

large change in impedance leads to a great advantage: for operational frequencies in the MHz band the size of an MI element can be decreased down to several microns without loss in sensitivity. This meets the requirements of micro-magnetic sensor development and such MI elements can well compete with magnetoresistive sensor heads since in the latter case the sensitivity to the field is relatively low and does not exceed 1.5%. Besides, there are hysteresis and temperature instability problems.^{6)–8)}

The theoretical and experimental studies on MI in multilayers are mainly restricted to the case of a transverse in-plane anisotropy induced in ferromagnetic layers. In this case the impedance exhibits two sharp symmetrical peaks at the external field equal to the anisotropy field. The present analysis is directed to a specific type of anisotropy which is crossed in one film with respect to that in the other. Such anisotropy can be induced by current annealing in the presence of a longitudinal field, and can be assigned to a helical type (existing in a torsioned wire or a torsion annealed wire). The MI behavior in materials with the helical anisotropy can be modified by the effect of a dc current inducing a circulatory magnetic field,^{9), 10)} which is important for developing auto-biased magnetic sensors. With increasing the bias current, the hysteresis portion of impedance plots shrinks and shifts and finally disappears, resulting in highly sensitive asymmetric MI characteristics needed to realize linear sensing characteris-

Fig. 1 Geometry of a multilayer MI element.

tics.

2. Mathematical Background

The geometry of the multilayer MI element is illustrated in Fig. 1. A conductive lead of thickness $2d_1$ is covered by two identical magnetic layers of thickness d_2 . The combination of an ac current $i = i_0 \exp(-j\omega t)$ and a dc one I_{dc} is flowing along the conductive lead in the length direction ($I = I_{dc} + i$). A dc magnetic field H_{ex} is applied parallel to the current. The coordinates are defined such that the layers are parallel to yz -plane and z -axis is along the current. The anisotropy axes in the ferromagnetic layers are directed at an angle $\pm\alpha$ to the z -direction, respectively for the upper ($x > 0$) and lower ($x < 0$) magnetic layers. The magnetic layers are single domains. The application of H_{ex} and I_{dc} which induces a circulatory field $H_b(x > 0) = -H_b(x < 0)$ results in the antisymmetric arrangement of the dc magnetization $M_0(M_{0y}(x > 0) = -M_{0y}(x < 0))$. Edge effects will be ignored, that is we treat the structure as if it extends from $y = -\infty$ to $y = +\infty$. Our goal is to find the impedance of this multilayer structure as a function of H_{ex} and to investigate the modification to this dependence due to the effect of H_b .

2.1 Weak skin effect approximation

First we consider that the film dimensions d_1, d_2 are such that the skin effect can be neglected. If the inner lead conductivity σ_1 is considerably higher than the conductivity σ_2 of the magnetic layers (to be exact, $\sigma_1 d_1 \gg \sigma_2 d_2$), the current mainly flows along the conductive lead. With these assumptions, the expression for the impedance can be written in the form:

$$Z = R_m - j\omega\Phi/i, \quad (1)$$

where $R_m = l/2d_1 b \sigma_1$ is the resistance of the inner conductor, b and l are the film size in the y - and z -directions, respectively, Φ is the total transverse magnetic flux generated by the drive current i in the magnetic layers. Since the edge effect is neglected the ac magnetic field h in the magnetic layers is uniform. Then:

$$\Phi = \mu_0(\hat{\mu}h)_y d_2 l, \quad (2)$$

where $\hat{\mu}$ is the relative ac permeability tensor and $\mu_0 = 4\pi \times 10^{-7}$ H/m. The field h induced by the ac current has y - and x -components due to the tensor form of the magnetic permeability. The value of the y -component is related to the ac current as $|h_y| = i/2b$, and the relation between h_y and h_x is determined from the condition that the normal flux is zero ($(\hat{\mu}h)_x = 0$). This yields

$$\Phi = \mu_0 \mu_t d_2 l h_y = \mu_0 \mu_t d_2 l \frac{i}{2b}, \quad (3)$$

$$\mu_t = \mu_{yy} - \mu_{xy}^2 / \mu_{xx}.$$

Using (3) the impedance can be written in the form:

$$Z = R_m \left(1 - 2j\mu_t \frac{d_1 d_2}{\delta_1^2} \right), \quad (4)$$

$$\delta_1 = \sqrt{2/\mu_0 \sigma_1 \omega},$$

where δ_1 is the skin depth in the metallic inner lead. Expression (4) shows that the MI ratio in the sandwich

film can be very large even at relatively low frequencies when the skin effect is not essential, having a linear dependence on μ_t . This conclusion can be illustrated as following. For a frequency of 10 MHz, taking $d_1 = d_2 = 0.5 \mu\text{m}$ and $\sigma_1 = 5.7 \times 10^7$ S/m (conductivity of Cu), the parameter $d_1/\delta_1 = d_2/\delta_1 = 0.045$. A typical low frequency change in μ_t (having a rotational mechanism) under the application of $H_{ex} = H_K \cong 800$ A/m (H_K is the anisotropy field) is from 1 to 500, then, according to (4) the impedance varies over 200%.

The impedance for the present geometry can be calculated exactly for any frequency, without neglecting the skin effect, as demonstrated in Appendix. For a film of $1 \mu\text{m}$ thick with $\sigma_1 d_1 / \sigma_2 d_2 = 50$ (corresponds to CoSiB/Cu/CoSiB system with amorphous magnetic films) the approximate solution almost coincides with the exact one up to a frequency of 1 GHz.

2.2 Transverse permeability

The impedance field behavior is determined by the field dependence of the transverse permeability, which is related to certain magnetization processes. Assuming a single domain state for the magnetic layers we deal with the rotational mechanism of magnetization alone. Even in multidomain films, the restriction to the rotational processes is reasonable because in films the magnetization vector during its rotation is held parallel to the surface, without going through high energy demagnetization states. Besides, the critical field of the irreversible magnetization flip quickly drops as the transverse bias is increased. Regarding to an ac magnetization, the wall motion becomes strongly damped with increasing frequency and gives little contribution to μ_t . For example, in the case of an amorphous film of $1 \mu\text{m}$ thick having a stripe domain structure a characteristic relaxation frequency of domain wall oscillations is estimated to be about 10–30 MHz¹¹⁾ if the dc relative permeability is about 10^3 .

Customarily, the magnetization is decomposed into a static part M_0 and a variable part $m = \hat{\chi}h$ where $\hat{\chi} = \hat{\mu} - 1$ is the ac susceptibility tensor. Such representation corresponds to a linear approximation over the ac variables. The static magnetization M_0 is directed at an angle $\pm\phi$ to the z -axis for $x > 0$ and $x < 0$, respectively. Neglecting the interaction between the layers, the stable direction of M_0 is found by minimizing the energy U of each layer:

$$\partial U / \partial \phi = 0, \quad (5)$$

$$U = -K \cos^2(\alpha - \phi) - \mu_0 M_0 H_{ex} \cos \phi - \mu_0 M_0 H_b \sin \phi,$$

where K is the anisotropy constant.

The susceptibility tensor $\hat{\chi}$ found from the linearized Landau-Lifshitz equation has the simplest form in the co-ordinate system (x, y', z') with the z' -axis parallel to M_0 :

$$\hat{\chi} = \begin{bmatrix} \chi_1 & -j\chi_a & 0 \\ j\chi_a & \chi_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (6)$$

Fig. 2 Magnetization curves $M_{0z}(H_{ex})$ for (a) different values of H_b and (b) critical fields of the irreversible magnetization flip vs. H_b .

where^{(11), (12)}

$$\begin{aligned}\chi_1 &= \Omega_M(\Omega_1 - j\beta\omega)/D, \\ \chi_2 &= \Omega_M(\Omega_2 - j\beta\omega)/D, \\ \chi_a &= \omega\Omega_M/D, \\ D &= (\Omega_1 - j\beta\omega)(\Omega_2 - j\beta\omega) - \omega^2, \\ \Omega_1 &= \gamma\mu_0[H_{ex} \cos \phi + H_b \sin \phi + H_K \cos 2(\alpha - \phi)], \\ \Omega_2 &= \gamma\mu_0[H_{ex} \cos \phi + H_b \sin \phi + H_K \cos^2(\alpha - \phi)], \\ \Omega_M &= \gamma\mu_0 M_0.\end{aligned}$$

Here β is the spin relaxation parameter, γ is the gyromagnetic constant, $H_K = 2K/\mu_0 M_0$ is the anisotropy field. The susceptibility tensor is the same in both layers as $\hat{\chi}(\alpha, \phi, H_b) = \hat{\chi}(-\alpha, -\phi, -H_b)$. The form of $\hat{\chi}$ in the original co-ordinate system is easily found by rotating the prime system (x, y, z') by the angle $\pm\phi$. After a straight forward calculation the transverse relative permeability is determined as:

$$\mu_t = 1 + \cos^2 \phi \left[\chi_2 - \frac{\chi_a^2}{1 + \chi_1} \right]. \quad (7)$$

3. Numerical Results and Discussion

We now proceed to discuss the MI characteristics of the present multilayer structures, which are determined by the field dependence of the transverse permeability. According to (7), the permeability is a function of the static magnetization direction. The particular features of the dependence μ_t vs. H_{ex} in the presence of H_b are directly related to the modification to the dc magnetization process. Figure 2a shows the hysteresis loops $M_{0z}(H_{ex})$ with H_b as a parameter, calculated from (5). The combination of the crossed anisotropy and H_b in-

Fig. 3 (a) Real part of the transverse permeability as a function of H_{ex} with H_b as a parameter for $\alpha = 50^\circ$. (b) Case of a transverse anisotropy ($\alpha = 90^\circ$), for comparison. The permeability is normalized to the characteristic rotational parameter $\mu_K = M_0/H_K$. Other parameters: $\beta = 0.2$, $M_0 = 5 \times 10^5$ A/m, $H_K = 500$ A/m, and $f = 50$ MHz.

roduces asymmetry: the magnetization does not respond the same to positive and negative fields. Because of asymmetry there are two different critical fields $H_1^* > H_2^*$ for irreversible magnetization jump related to the ascendant and descended branches, respectively as shown in Fig. 2b (zero field in the direction of the anisotropy corresponds to $H_{ex} = -H_b \tan \alpha$). With increasing H_b the value of H_2^* decreases whereas H_1^* shows an initial rise followed by an abrupt fall. The sum of these critical fields which bounds the hysteresis area initially almost does not change with increasing H_b and then it sharply drops down to zero at $H_b = H_K \cos \alpha$. Accordingly, the hysteresis loops suddenly shift being very narrow and finally turn to anhysteretic asymmetric curves.

Figure 3 shows the ac transverse permeability as a function of H_{ex} for $H_b/H_K = 0, 0.4, 1$. The frequency is taken 50 MHz which is close to a characteristic frequency $\gamma\mu_0 H_K/2\pi = 20$ MHz. For smaller H_b , the plots show a hysteresis which is related to the hysteresis in the magnetization reversal. The application of H_b introduces asymmetry: the permeability peak is higher when H_{ex} is reversed from the minus direction than

that when H_{ex} is reversed from the plus direction. An interesting characteristic is the appearance of two different "residual" values μ_1 and μ_2 at $H_{ex}=0$ related to the ascendant and descended branches of the magnetization, respectively, which can be regarded as ac memory property. With increasing H_b the difference between μ_1 and μ_2 grows until the whole curve does not suddenly shift. For $H_b > H_K \cos \alpha$ the hysteresis disappears. This permeability behavior in the presence of H_b can be compared with that for the case of the transverse anisotropy, as shown in Fig. 3b. For the transverse magnetic configuration ($\alpha=90^\circ$) H_{ex} is a hard axis field, then both $M_{0z}(H_{ex})$ and $\mu_t(H_{ex})$ do not exhibit a hysteresis. The permeability plot has two symmetrical sharp peaks. The application of the transverse bias H_b makes the peaks smaller and broader, but does not lead to a characteristically different behavior. We can conclude that the combination of the crossed anisotropy and the dc circulatory field allows the principle changes in the ac permeability as a function of H_{ex} to be obtained.

Finally, we can discuss the impedance characteristics. Figure 4 plots the normalized impedance $\zeta = |Z(H_{ex})/Z(H_{sat})|$ vs. H_{ex} with H_b as a parameter, where H_{sat} is the saturation field. The calculation is made for a film of $1 \mu\text{m}$ thick having a Cu inner lead ($2d_1=d_2$), and a frequency of 50 MHz. This frequency is high enough to obtain very large impedance changes up to 250%. The modification to the MI characteristics due to the application of H_b is similar to that seen for the plots $M_{0z}(H_{ex})$ and $\mu_t(H_{ex})$. Figure 4a is related to values of H_b for which the hysteresis portion of the magnetization reversal almost does not change. In this case, the effect of the biasing field is a more profound asymmetry: with increasing H_b the impedance peak at negative fields becomes larger and sharper whereas it tends to be flatter at positive fields. The impedance peak value increases from 1.4 at $H_b=0$ to 2.9 at $H_b/H_K=0.45$ (just before a sharp drop in the critical field H_1^* , see Fig. 2b) while it becomes almost flat at the level $\zeta \cong 1$ at positive fields, indicating that the factor of field sensitivity increases considerably. Along with this, the difference between the two "residual" values of impedance ζ_1 and ζ_2 determined at $H_{ex}=0$ also grows: their ratio equals to 2.1 at $H_b/H_K=0.45$.

A similar field behavior of the voltage response measured across a twisted CoFeSiB amorphous wire having an induced helical anisotropy has been found when a pulse current of a short duration is driven through the wire.⁹⁾ As the current amplitude is increased (and hence, a low frequency bias field) the hysteresis in the voltage peak vs. H_{ex} alters in a way shown in Fig. 4a. Such field dependence of the voltage pulse has been proposed to utilize for sensitive non-destructive read-out heads of neural network memory, and switch-type sensors.

Further increase in $H_b/H_K > 0.45$ results in a sudden

Fig. 4 Normalized impedance $\zeta = |Z/Z(H_{sat})|$ as a function of H_{ex} for different values of H_b . The calculation is related to CoSiB/Cu/CoSiB multilayers with $2d = 1 \mu\text{m}$, $2d_1 = d_2$, and $f = 50 \text{ MHz}$.

shift of the hysteresis to negative fields with a simultaneous shrinkage of the hysteresis area, as shown in Fig. 4b. For $H_b > H_K \cos \alpha$ when the magnetization reversal has no hysteresis the impedance curves are non-hysteretic as well. If H_b is just slightly larger than $H_K \cos \alpha$, the field sensitivity of the impedance change is especially high. For $H_b = 0.62H_K$ the relative change in impedance can be more than 200% in a small vicinity of $H_{ex} \cong -0.75H_K$. However, with further increase in biasing field, the sensitivity gradually drops. These asymmetric sensitive magneto-impedance characteristics are needed to construct a linear field sensor.¹⁰⁾ Considering that the bias field H_b can be induced by a dc current flowing along the film the case can be related to auto-biased type sensors.

Figure 5 presents the frequency characteristics for the MI ratio defined as $\Delta Z/Z_{sat} = \zeta(H_{max}) - 1$, where $\zeta(H_{max})$ is the value of the reduced impedance in maximum. Typically, the MI spectra of multilayer films exhibit an initial rapid rise followed by a wide flat region. For parameters used and $H_b = 0.62H_K$, $\Delta Z/Z_{sat}$ increases from 30% to 275% for frequencies between 1 and 100 MHz with further little change up to a frequency of 1 GHz.

Fig. 5 Frequency spectrum of the MI ratio for $H_b/H_k=0.62$.

4. Conclusion

A modification to the MI behavior resulting from the effect of the dc current inducing a circulatory biasing field has been studied in multilayers having an inner conductor lead and two identical magnetic layers with crossed anisotropy. The combination of the crossed anisotropy and the transverse bias causes asymmetry in MI behavior, and the insertion of the conductor lead decreasing the resistance of the system results in a considerable enlargement of the MI ratio. Various types of sensitive MI characteristics can be realized in a small size film of micron thickness for operation frequencies higher than about 10 MHz. Increasing the biasing field leads to asymmetrical anhysteretic MI which presents a special interest for linear magnetic sensing.

Appendix

The considered geometry allows the problem to be solved exactly, without neglecting the skin effect. The calculation of Z is based on the solution of the Maxwell's equations for the ac electric $\mathbf{e}$ and magnetic $\mathbf{h}$ fields with a given ac permeability tensor $\hat{\mu}=1+\hat{\chi}$ defined by (6):

$$\text{curl } \mathbf{e} = j\omega(\mu_0 \hat{\mu} \mathbf{h}), \quad \text{curl } \mathbf{h} = \sigma \mathbf{e}, \quad (\text{A.1})$$

satisfying the boundary conditions on the external surface $x = \pm d$, ($d=d_1+d_2$):

$$h_y(\pm d) = \pm i/2b, \quad h_z(\pm d) = 0, \quad (\text{A.2})$$

as well as the symmetry conditions and the conditions of continuity across the boundaries. In a general case, the impedance is defined by considering the total energy flux entering through the surface of the film, which results in the relationship between Z and the surface value of the electric field¹³⁾:

$$Z = e_z(d)/i. \quad (\text{A.3})$$

Equations (A.1) applied to the magnetic layers are easily solved in the prime co-ordinate system (z' -axis is parallel to $\mathbf{M}_0$), where the susceptibility tensor $\hat{\chi}$ is of quasi-diagonal form (6). The solution is represented by two normal waves $\{e_z', h_y'\}$, $\{e_y', h_z'\}$ corresponding to a

scalar permeability $\mu = 1 + (\chi_z - \chi_a^2/(1 + \chi_1))$ and a unit permeability, respectively. Inside the magnetic layers ($d_1 < |x| < d$), the magnetic field $\mathbf{h}$ is written in the form:

$$\begin{aligned} h_y' &= A_m^+ \sinh(jk_m x) + B_m^+ \cosh(jk_m x), \\ h_z' &= A_0^+ \sinh(jk_0 x) + B_0^+ \cosh(jk_0 x), \end{aligned} \quad (\text{A.4})$$

where indices “ $\pm$ ” are related to the upper and lower magnetic layers, respectively, and

$$\begin{aligned} k_0 &= (1+j)/\delta_0, \quad k_m = (1+j)/\delta_m, \\ \delta_0 &= \sqrt{2/\sigma_2 \omega \mu_0}, \quad \delta_m = \sqrt{2/\sigma_2 \omega \mu \mu_0}. \end{aligned}$$

Here δ_m and δ_0 are the magnetic and non-magnetic skin depth in the outer layers. The solutions of equations (A.1) for the inner conductive layer ($|x| < d_1$) satisfying the symmetry conditions are written in the original co-ordinate system (x, y, z):

$$\begin{aligned} h_y &= C_y \sinh(jk_1 x), \quad h_z = C_z \cosh(jk_1 x), \\ k_1 &= (1+j)/\delta_1. \end{aligned} \quad (\text{A.5})$$

The electric field is found from the first Maxwell's equation. General solutions (A.4) and (A.5) contain 10 constants, which can be determined from boundary conditions (A.2), the circularity condition for h_y , e_y and matching the tangential field components across the boundary $x = \pm d_1$. Calculating the z -component of $\mathbf{e}$ on the external surface, the impedance is determined by (A.2) as:

$$\begin{aligned} Z &= \frac{jk_0}{\sigma_2 D} \left[\sqrt{\mu_0 \mu} Q_1 Q_8 \cos^4 \phi + Q_4 Q_5 \sin^4 \phi \right. \\ &\quad \left. + \left(\frac{2k_m k_1}{\sigma_1 \sigma} + Q_2 Q_7 + \sqrt{\mu_0 \mu} Q_3 Q_6 \right) \frac{\sin^2 2\phi}{4} \right], \end{aligned} \quad (\text{A.6})$$

where,

$$\begin{aligned} D &= Q_1 Q_2 \cos^2 \phi + Q_3 Q_4 \sin^2 \phi, \\ Q_1 &= k_1 \sinh(jk_1 d_1) \sinh(jk_0 d_2)/\sigma_1 \\ &\quad + k_0 \cosh(jk_1 d_1) \cosh(jk_0 d_2)/\sigma_2, \\ Q_2 &= k_1 \cosh(jk_1 d_1) \sinh(jk_m d_2)/\sigma_1 \\ &\quad + k_m \sinh(jk_1 d_1) \cosh(jk_m d_2)/\sigma_2, \\ Q_3 &= k_1 \cosh(jk_1 d_1) \sinh(jk_0 d_2)/\sigma_1 \\ &\quad + k_0 \sinh(jk_1 d_1) \cosh(jk_0 d_2)/\sigma_2, \\ Q_4 &= k_1 \sinh(jk_1 d_1) \sinh(jk_m d_2)/\sigma_1 \\ &\quad + k_m \cosh(jk_1 d_1) \cosh(jk_m d_2)/\sigma_2, \\ Q_5 &= k_1 \cosh(jk_1 d_1) \cosh(jk_m d_2)/\sigma_1 \\ &\quad + k_m \sinh(jk_1 d_1) \sinh(jk_m d_2)/\sigma_2, \\ Q_6 &= k_1 \cosh(jk_1 d_1) \cosh(jk_m d_2)/\sigma_1 \\ &\quad + k_0 \sinh(jk_1 d_1) \sinh(jk_0 d_2)/\sigma_2, \\ Q_7 &= k_1 \sinh(jk_1 d_1) \cosh(jk_0 d_2)/\sigma_1 \\ &\quad + k_0 \cosh(jk_1 d_1) \sinh(jk_0 d_2)/\sigma_2, \\ Q_8 &= k_1 \sinh(jk_1 d_1) \cosh(jk_m d_2)/\sigma_1 \\ &\quad + k_m \cosh(jk_1 d_1) \sinh(jk_m d_2)/\sigma_2. \end{aligned}$$

If d_1 tends to be zero, formula (A.6) reduces to that obtained in Ref. 14 for a bilayer magnetic film having a similar magnetic configuration. Approximation of a weak skin effect corresponds to $d_1/\delta_1 \ll 1$, $d_2/\delta_m \ll 1$. Making expansion of (A.6) as far as the second power of these parameters, the impedance is represented as:

$$\begin{aligned} Z &= R_{dc} \left[1 - \frac{k_0^2 d_1 d_2 (\sigma_1^2 d_1^2)}{3(\sigma_1 d_1 + \sigma_2 d_2) \sigma_2 d_2} \right. \\ &\quad \left. - \frac{k_0^2 d_2^2 [\sin^2 \phi + \mu \cos^2 \phi]}{3} q \right], \end{aligned} \quad (\text{A.7})$$

Fig. 6 Impedance as a function of H_{ex} obtained from the exact solution (solid line) and from the weak skin-effect approximation (dashed line) $f=1$ GHz, $H_b=0$.

where

$$q = \frac{3d_1^2\sigma_1^2 + 3d_1d_2\sigma_1\sigma_2 + d_2^2\sigma_2^2}{(\sigma_1d_1 + \sigma_2d_2)\sigma_2d_2}.$$

We are interested in a multilayer structure with $\sigma_1d_1 \gg \sigma_2d_2$. Then, (A.7) simplifies to:

$$Z = R_{dc} \left[1 - k_1^2 d_1^2 \left(\frac{1}{3} + \frac{d_2}{d_1} \mu_t \right) \right], \quad (\text{A.8})$$

where μ_t is the transverse permeability defined by (7). If the condition $\mu_t \gg 1$ is used, formula (A.8) coincides with (4) obtained under the same approximations. The comparison of the impedance plots *vs.* H_{ex} calculated from exact solution (A.6) and approximate (A.7) is presented in Fig. 6. The calculation is related to a film of $1 \mu\text{m}$ thick having a Cu inner lead and CoFeSiB

amorphous magnetic layers. Even for a frequency of 1 GHz the discrepancy between the two curves is hardly noticeable.

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Received Oct. 19, 1998; Accepted Jan. 21, 1999