Magnetic Circuit Approach to Magnetic Shielding

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It was found that the magnetic circuit approach, which is useful in situations where magnetic flux is interrupted by air gaps, is also helpful in illustrating magnetic shielding. Equivalent magnetic circuits of single and multiple cylindrical shields were developed and analyzed. Simple and explicit equations describing the shielding factors in terms of the reluctances of the shielded area, the shells themselves, and the spaces between them were obtained. To simplify evaluation of corresponding reluctances, flux density is assumed to be distributed uniformly throughout the shield parts, including the shells and the spaces between them. The resulting equations exactly reproduce well-known approximate analytical formulas. An important advantage of the proposed method over numerical ones is that it allows a straightforward physical insight into the shielding problem.

Key words: magnetic shielding, magnetic circuit concept, cylindrical shields, transverse shielding factor

1. Introduction

Magnetic shielding is of growing importance to modern physics and electronics because of increasing resolution and sensitivity of scientific instruments, micro miniaturization of electronic devices, and steadily intensifying electro-magnetic interference.

Although magnetic shielding is an old science it is still mathematically difficult. Exact description of even basic principles of shielding with multiple concentric cylinders is rather mathematically complicated.

On the other hand, modern computers and software based on numerical methods, such as ANSYS[®] and Vector Fields software packages, permit a relatively easy and rapid evaluation of shielding enclosures even for complex geometries.

Although numerical methods are technically quite accurate and powerful, they often hide physics of problems they solve. There are many options even when a relatively simple shield is designed, and it is difficult to find immediately the right strategy for the numerical simulation. It is also difficult to validate results obtained numerically. A simple physical model would serve a more conscious and effective employment of numerical methods.

Unfortunately, neither old^{1)–6)} nor recent^{7)–12)} literature suggest a clear and comprehensive illustration of the

magnetic shielding mechanism. It seems that a great potential of approximation and analogy is still not well recognized and employed for a deeper and more straightforward physical insight into the shielding problem. An example of such approximation in the magnetic circuit concept. This concept is based on the analogy existing between magnetic and electric circuits and is an effective tool for illustrating and solving many practical problems such as the design of electromagnets. Since the magnetic circuit concept is useful in very similar situations where a magnetic flux is interrupted by air gaps, it is intriguing to investigate whether this approach can be useful in illustrating magnetic shielding as well.

2. Magnetic Shielding Formulas

For a long time (perhaps since 1589 when the first publication by della Porta¹⁾ appeared) it had been thought that the only way to improve shielding efficiency was simply to increase the thickness of a single iron shell. This lasted three centuries until Rücker²⁾ in 1894 pioneered the case of transverse shielding with infinitely long multiple concentric cylinders and showed that dividing the iron into thin concentric shells with air gaps between them can provide higher shielding efficiency than that provided by a single thick solid shell. In 1897–1898, du Bois^{3)–5)} treated the case of two concentric spherical shells. Results obtained by Rücker and du Bois were elaborated and adapted for practical use by Wills in 1899.⁶⁾ Thus, during the last decade of the past century, the main fundamental principles of magnetic shielding had been formulated.

Unfortunately, exact mathematical solutions for the static magnetic field penetration into multiple concentric shielding structures are rather cumbersome. As a result, they are of a limited practical value. Approximate solutions obtained for the case of a high-permeability shielding material and relatively small thicknesses of the shells are more widely used in practice and will be reviewed in this section. For the sake of simplicity, it is generally assumed that a magnetic shield is placed in a uniform external field, and the permeability does not depend on the magnetic induction.

As was already mentioned, exact solutions are known only for the ideal shielding assemblies, an example of those is a set of infinitely long concentric cylinders.

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Fig. 1 An ideal shielding enclosure: an infinitely long cylindrical shell in a uniform transverse field.

ders. Such shielding assembly is considered ideal since it does not distort the internal field while it shields against a uniform dc transverse field.¹²⁾ For a single cylinder of thickness t and a constant permeability μ , the transverse shielding factor (see Fig. 1) is given as follows¹²⁾:

$$S_{t1} = \frac{\Phi_{ext}}{\Phi_{int}} = \frac{1}{4\mu} \left[(\mu+1)^2 - \left(\frac{a_1}{A_1} \right) (\mu-1)^2 \right], \quad (1)$$

where Φ_{ext} and Φ_{int} , respectively, are the magnetic fluxes outside the shield and within the shielded area, a_1 and A_1 , respectively, are the cross sectional areas defined by the inner and outer cylindrical surfaces.

For high-permeability, $\mu \gg 1$, and relatively thin, compared to outer diameter cylinders, $t \ll D$, (1) becomes:

$$S_{t1} \approx 1 + \frac{\mu t}{D}. \quad (2)$$

Exact and explicit analytical solutions for double and triple concentric cylinders are given in Ref. 6. Development of these formulas is rather complicated. Assuming $\mu \gg 1$ and $t \ll D$, the equations described in Ref. 6 can be approximated as follows:

for double-shell shields

$$S_{t2} \approx 1 + S_1 + S_2 + S_1 S_2 \left(1 - \frac{A_1}{A_2} \right), \quad (3)$$

and for triple-shell shields

$$\begin{aligned} S_{t3} \approx & 1 + S_1 + S_2 + S_3 \\ & + S_1 S_2 \left(1 - \frac{A_1}{A_2} \right) + S_1 S_3 \left(1 - \frac{A_2}{A_3} \right) \\ & + S_1 S_2 \left(1 - \frac{A_2}{A_3} \right) + S_1 S_2 S_3 \left(1 - \frac{A_1}{A_2} \right) \left(1 - \frac{A_2}{A_3} \right), \end{aligned} \quad (4)$$

where $S_i \approx (\mu_i t_i / R_i) / 2$ are approximate transverse shielding factors for individual shells, A_i and A_j , respectively, are the cross sectional areas defined by the outer and inner cylindrical surfaces of neighboring shells i and j ($j = i + 1$).

The general approximate solution for n -shell system

of concentric cylinders can be obtained by the obvious extension of (3) and (4), namely:

$$\begin{aligned} S_{tn} \approx & 1 + \sum_{i=1}^n S_i + \sum_{i=1}^{n-1} \sum_{j>i}^n S_i S_j \left(1 - \frac{A_{j-1}}{A_j} \right) \\ & + \sum_{i=1}^{n-2} \sum_{j>i}^{n-1} \sum_{k>j}^n S_i S_j S_k \left(1 - \frac{A_{j-1}}{A_j} \right) \left(1 - \frac{A_{k-1}}{A_k} \right) \\ & + \sum_{i=1}^{n-3} \sum_{j>i}^{n-2} \sum_{k>j}^{n-1} \sum_{l>k}^n S_i S_j S_k S_l \\ & \times \left(1 - \frac{A_{j-1}}{A_j} \right) \left(1 - \frac{A_{k-1}}{A_k} \right) \left(1 - \frac{A_{l-1}}{A_l} \right) + \dots \end{aligned} \quad (5)$$

Equations (3)–(5) represent the main principle of magnetic shielding with multiple shells: decoupling the shells, $A_j/A_i \ll 1$, due to an introduction of air gaps between them allows for a multiplicative rather than an additive increase in the shielding. As a result, the shielding factor for a set of thin concentric shells can be much larger than that of a single thick shell built with the same amount of the material.

3. Equivalent Magnetic Circuits of Magnetic Shields

Our present aim is to analyze magnetic shielding from the point of view of the magnetic circuit concept. This concept employs the close mathematical analogy between magnetic and electric circuits. It allows one to analyze magnetic circuits in terms of magnetic analogous of Ohm's and Kirchhoff's laws, where the electric current I is replaced by the magnetic flux Φ and the electric resistance R is replaced by the magnetic reluctance.

Fig. 2 A single-shell shield. (a) Magnetic flux distribution. (b) Equivalent magnetic circuit.

Fig. 3 A double-shell concentric shield. (a) Magnetic flux distribution. (b) Equivalent magnetic circuit.

$$\mathcal{R} = \frac{l}{\mu A}, \quad (6)$$

where l is the length of magnetic flux path into a material of the permeability μ and the cross sectional area A .

In our further analysis of magnetic shields, we will employ the magnetic circuit approach, where magnetic flux is conserved. 2-D case is considered. Equivalent magnetic circuits of the shield will be developed first. Then the equivalent circuits will be solved for the shielding factor. The following shielding structures will be analyzed: a single infinitely long cylinder (Fig. 2 (a)); double (Fig. 3(a)) and triple (Fig. 4(a)) sets of concentric cylinders.

The magnetic flux lines in Fig. 2(a)–Fig. 4(a) (obtained numerically by using a standard ANSYS® software package) suggest that the examined shielding structures can be described by the corresponding equivalent magnetic circuits shown in Fig. 2(b)–Fig. 4(b). The magnetic flux Φ_{ext} in these figures represents an external flux entering the outermost surface of the shields, and Φ_{int} represents the magnetic flux within the shielded area (the innermost area of the shields). The reluctances $\mathcal{R}_{int}$, $\mathcal{R}_{sh i}$, and $\mathcal{R}_{g i,j}$ in Fig. 2(b)–Fig. 4(b) are appropriate to the shielded area, the shells themselves, and spaces between adjacent shells, respectively.

Theory²⁾⁻¹²⁾ show that the magnetic flux within the

Fig. 4 A triple-shell concentric shield. (a) Magnetic flux distribution. (b) Equivalent magnetic circuit.

Fig. 5 A hypothetical model used to calculate reluctance within cylindrical shields. To simplify calculations, flux density is assumed to be distributed uniformly throughout the shield parts (neglecting boundary conditions), including the shells and the spaces between them. (a) Equivalent magnetic flux distribution. (b) Element of a cylindrical shells.

shielded area is distributed uniformly (see Fig. 2(a)–Fig. 4(a)). Hence, $\mathcal{R}_{int}$ can be easily and exactly calculated in accordance to (6). On the other hand, the distribution of

magnetic flux within the shells themselves and within the spaces between the shells is not uniform. In order to simplify calculation of $\mathcal{R}_{sh i}$ and $\mathcal{R}_{g i, j}$, we introduce a *hypothetical* model of the shields (see Fig. 5(a) and Fig. 6) where the magnetic flux is distributed uniformly throughout the shield parts, including the shells themselves and the spaces between them.

A. Magnetic shielding with a single cylinder

Let us first consider the case of a single infinitely long cylinder (Fig. 2(a)), whose equivalent magnetic circuit is shown in Fig. 2(b). This circuit can be easily solved to give the ratio of Φ_{ext}/Φ_{int} , which defines the shielding factor:

$$S_t = \frac{\Phi_{ext}}{\Phi_{int}} = 1 + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}}. \quad (7)$$

In order to calculate the reluctances $\mathcal{R}_{sh 1}$ and $\mathcal{R}_{int}$, we examine a narrow, ring-like section of an infinite cylindrical shield, as shown in Fig. 5(b). According to (6), $\mathcal{R}_{int}$ and $\mathcal{R}_{sh 1}$ are defined by the corresponding average path lengths, $\bar{l}_{int}$ and $\bar{l}_{sh 1}$, and the cross sectional areas of the shielded space and that of the shell itself: A_1 and $A_{sh 1}$ (see Fig. 5 and Fig. 6). Assuming a uniform distribution of magnetic flux, the average path length can be calcu-

Fig. 6 A simplified *hypothetical* model used to calculate the average lengths of the magnetic flux path into the shielded area, $\bar{l}_{int i}$, and within the spaces between the shield shells, $\bar{l}_{g i, j}$. (The thickness of the shells are assumed to be relatively small in comparison with their diameters and air gaps between them.)

lated as the ratio between the volume of the corresponding part of the shield and the corresponding cross sectional area,

$$\bar{l}_{int} = \frac{V_1}{A_1}, \quad \bar{l}_{sh} = \frac{V_{sh 1}}{A_{sh 1}}. \quad (8)$$

Equation (7) can be rewritten now, by considering (6) and (8), as follows:

$$S_t = 1 + \frac{V_1}{V_{sh 1}} \frac{A_{sh 1}^2}{A_1^2} \mu. \quad (9)$$

Finally, after a simple calculation of the corresponding volumes and cross sectional areas, the following shielding formula can be obtained:

$$S_{t1} \approx 1 + \frac{\mu t}{D}. \quad (10)$$

It is interesting that (10) is exactly the same as (2). Thus, the simplified hypothetical model predicts exactly the same shielding performance of a single cylinder as the analytical model of real shields does.

B. Magnetic shielding with multiple concentric shields

Flux distribution within double and triple concentric shields and their equivalent magnetic circuits are shown in Figs. 3 and 4. Solving these circuits for Φ_{ext}/Φ_{int} results in the following formulas (note that, according to (6), (8) and Fig. 5(b), $\mathcal{R}_{int i} = \mathcal{R}_{int j} = \mathcal{R}_{int} = \pi/4$):

for double-shell shields

$$S_{t2} = 1 + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 2}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}} \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 2}} \frac{\mathcal{R}_{g 1,2}}{\mathcal{R}_{int}} \left(1 + \frac{\mathcal{R}_{sh 1}}{\mathcal{R}_{int}} \right), \quad (11)$$

and for triple-shell shields

$$S_{t3} = 1 + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 2}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 3}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}} \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 2}} \frac{\mathcal{R}_{g 1,2}}{\mathcal{R}_{int}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}} \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 3}} \frac{\mathcal{R}_{g 2,3}}{\mathcal{R}_{int}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}} \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 2}} \frac{\mathcal{R}_{g 1,2}}{\mathcal{R}_{int}} + \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 1}} \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 2}} \frac{\mathcal{R}_{int}}{\mathcal{R}_{sh 3}} \frac{\mathcal{R}_{g 1,2}}{\mathcal{R}_{int}} \frac{\mathcal{R}_{g 2,3}}{\mathcal{R}_{int}} \times \left[1 + \frac{\mathcal{R}_{sh 1}}{\mathcal{R}_{int}} \left(\frac{\mathcal{R}_{sh 2}}{\mathcal{R}_{g 1,2}} + \frac{\mathcal{R}_{sh 3}}{\mathcal{R}_{g 2,3}} \right) \right], \quad (12)$$

where $\mathcal{R}_{g i, j}$ are the reluctances of the spaces between corresponding shells. The terms other than 1 in the brackets in (11) and (12) can be neglected, and, assuming $\mathcal{R}_{sh i} \propto 1/\mu \ll 1$ and $\mathcal{R}_{sh j} < \mathcal{R}_{g j-1, j}$, (11) and (12) can be rewritten then as follows:

for double-shell shields

$$S_{t2} \approx 1 + S_1 + S_2 + S_1 S_2 \frac{\mathcal{R}_{g 1,2}}{\mathcal{R}_{int 2}}, \quad (13)$$

for triple-shell shields

$$S_{t3} \approx 1 + S_1 + S_2 + S_3 + S_1 S_2 S_3 \frac{\mathcal{R}_{g 1,2}}{\mathcal{R}_{int 2}} \frac{\mathcal{R}_{g 2,3}}{\mathcal{R}_{int 3}}, \quad (14)$$

where $S_i = \mathcal{R}_{int i} / \mathcal{R}_{sh i}$ are the shielding factors for individual shells. $\mathcal{R}_{g i j} / \mathcal{R}_{int j}$ in (13) and (14) represent the reluctances of air spaces between corresponding shells that are normalized to the reluctances of the shielded areas.

Reluctances $\mathcal{R}_{g i j}$ in (14) can be easily calculated according to (6) and (8), taking into account the previously made assumption about the uniformity of the magnetic flux distribution within air gaps between the shells and assuming that the thicknesses of the shells (see Fig. 6) are small in comparison with their diameters and air gaps between them (practical case):

$$\frac{\mathcal{R}_{g i j}}{\mathcal{R}_{int j}} = \frac{\tilde{l}_{g i j} / A_j}{\tilde{l}_{int j} / A_j} \approx \frac{V_j - V_i}{V_j} \approx 1 - \frac{V_i}{V_j}. \quad (15)$$

For multiple concentric cylinders, (15) can be rewritten as follows, considering that $V_i = A_i \times \Delta$ and $V_j = A_j \times \Delta$ (see Fig. 5(b)):

$$\frac{\mathcal{R}_{g i j}}{\mathcal{R}_{int}} \approx 1 - \frac{A_i}{A_j}. \quad (16)$$

Finally, by considering (15), (16) and the recursion in (13) and (14), a general solution for n concentric cylinders can be written:

$$\begin{aligned} S_n \approx & 1 + \sum_{i=1}^n S_i + \sum_{i=1}^{n-1} \sum_{j>i}^n S_i S_j \left(1 - \frac{A_{j-1}}{A_j}\right) \\ & + \sum_{i=1}^{n-2} \sum_{j>i}^{n-1} \sum_{k>j}^n S_i S_j S_k \left(1 - \frac{A_{j-1}}{A_j}\right) \left(1 - \frac{A_{k-1}}{A_k}\right) \\ & + \sum_{i=1}^{n-3} \sum_{j>i}^{n-2} \sum_{k>j}^{n-1} \sum_{t>k}^n S_i S_j S_k S_t \\ & \times \left(1 - \frac{A_{j-1}}{A_j}\right) \left(1 - \frac{A_{k-1}}{A_k}\right) \left(1 - \frac{A_{t-1}}{A_t}\right) + \dots \end{aligned} \quad (17)$$

Equation (17) exactly reproduces (5). We can conclude therefore, that the magnetic circuit approach enables both qualitative and quantitative descriptions of magnetic shielding with single and multiple concentric cylinders.

The developed equivalent magnetic circuits clearly demonstrate an advantage of introducing air spaces between the shield shells. For a shield with no air gaps (Fig. 2(a)), elements representing in Fig. 2(b) reluctances of the shielded area, $\mathcal{R}_{int}$, and the shield shell, $\mathcal{R}_{sh 1}$, are connected in parallel to the 'flux source' Φ_{ext} . For a relatively large value of the $\mathcal{R}_{int} / \mathcal{R}_{sh 1}$ ratio, magnetic flux, Φ_{int} , flowing through $\mathcal{R}_{int}$ is in direct proportion to $\mathcal{R}_{sh 1}$ (see (7)). According to (6), $\mathcal{R}_{sh 1}$ is in inverse proportion to the permeability and the shield thickness. Therefore, for a solid shield, with no air gaps, the shielding factor, Φ_{ext} / Φ_{int} , is proportional to the μ and t (see (8), (9)). Dividing a solid shell into two shells and introducing an air gap between them (Fig. 3(a)) principally changes the shield's equivalent scheme (Fig. 3(b)). If the air gap in Fig. 3(a) is relatively narrow and its reluctance, $\mathcal{R}_{g 1,2}$, is comparable to that of the shells, $\mathcal{R}_{sh 1}$ and $\mathcal{R}_{sh 2}$, then these two latter reluctances can be considered as not decoupled by the $\mathcal{R}_{g 1,2}$. In this case, the behavior of the equivalent scheme in Fig. 3(b) resem-

bles the behavior of the scheme in Fig. 2(a), and the effect of the permeability and thickness of the two shells on the shielding factor is additive. Increasing the gap width decouples the $\mathcal{R}_{sh 1}$ from the $\mathcal{R}_{sh 2}$ and causes magnetic flux, $\Phi_{sh 1}$, flowing through $\mathcal{R}_{sh 1}$ to be in direct proportion to the $\mathcal{R}_{sh 2}$. Magnetic flux flowing through $\mathcal{R}_{int}$ is in direct proportion to the $\mathcal{R}_{sh 1}$, as was mentioned above. Hence, the total effect of the $\mathcal{R}_{sh 1}$ and the $\mathcal{R}_{sh 2}$ on the Φ_{int} is multiplicative. Therefore, an introduction of spaces between the shield's shells allows one to save an amount of the magnetic material, keeping a constant shielding factor or *vice versa*, it allows one to increase the shielding factor, keeping a constant amount of the shielding material.

4. Conclusions

The usefulness of the magnetic circuit concept for illustrating magnetic shielding mechanism has been demonstrated. Equivalent magnetic circuits for both qualitative and quantitative descriptions of single and multiple cylindrical shields are developed and analyzed. The analysis resulted in formulas describing the transverse shielding factors in terms of the reluctances of the shielded area, the shield shells, and the spaces between the shells. In order to simplify further analysis, it is assumed that flux density is distributed uniformly throughout the shield parts, including the shielding material and the spaces between the shells. This allows a simple and straightforward evaluation of the corresponding reluctances. It is interesting that the finally developed equations exactly reproduce well-known approximate analytical shielding formulas. The described approach, assists in a simple and comprehensive illustration of magnetic shielding. It also enables a deeper and more straightforward physical insight into the shielding problem. The proposed simple physical model can assist in a more conscious and effective employment of numerical methods.

References

- 1) Della Porta: *Magiae Naturalis, Sive de Miraculus Rerum Naturalium* (Libri VII, 1589).
- 2) A. W. Rucker: *Phil. Mag.*, **23**, 225 (1887).
- 3) Du Bois: *Wied. Ann.*, **63**, 348 (1897).
- 4) Du Bois: *Wied. Ann.*, **65**, 1 (1897).
- 5) Du Bois: *The Electrician*, **40**, 1 (1898).
- 6) A. P. Wills: *Phys. Rev.*, **9**, 193 (1899).
- 7) A. J. Mager: *Z. Angew. Phys.*, **23**, 381 (1967).
- 8) A. J. Mager: *IEEE Trans. Magn.*, **6**, 67 (1970).
- 9) D. U. Gubser, S. A. Wolf, and J. E. Cox: *Rev. Sci. Instrum.*, **50**, 751 (1979).
- 10) D. Dubbers: *Nucl. Instrum. Methods A*, **243**, 511 (1985).
- 11) T. J. Sumner, J. M. Pendlebury, and K. F. Smith: *J. Phys. D: Appl. Phys.*, **20**, 1095 (1987).
- 12) A. K. Thomas: *IEEE Trans. Electromagnetic Compatibility*, **10**, 142 (1968).

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