Fault Detection of Railway Vehicle Suspension Systems Using Multiple-Model Approach*

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Abstract
This paper demonstrates the possibility to detect suspension failures of railway vehicles using a multiple-model approach from on-board measurement data. The railway vehicle model used includes the lateral and yaw motions of the wheelsets and bogie, and the lateral motion of the vehicle body, with sensors measuring the lateral acceleration and yaw rate of the bogie, and lateral acceleration of the body. The detection algorithm is formulated based on the Interacting Multiple-Model (IMM) algorithm. The IMM method has been applied for detecting faults in vehicle suspension systems in a simulation study. The mode probabilities and states of vehicle suspension systems are estimated based on a Kalman Filter (KF). This algorithm is evaluated in simulation examples. Simulation results indicate that the algorithm effectively detects on-board faults of railway vehicle suspension systems.

Key words: Condition Monitoring, Fault Detection, Estimation, Kalman Filter, Extended Kalman Filter, Multiple-Model, Railway

1. Introduction

Railway maintenance is important for ensuring safety and avoiding accidents. An inspection to detect faults in railway vehicles is especially important. Some faults could cause serious accidents while the vehicle is traveling.

Condition monitoring is necessary in order to immediately detect vehicle faults. For condition monitoring, it is necessary to detect the fault from the signals of sensors attached to the vehicles.

Condition monitoring can be considered to be a part of the well-established and well-developed area of Fault Detection and Isolation (or Identification) (FDI). Many research works on FDI are summarized in numerous publications (1)-(6). Condition monitoring is mainly applicable to systems that deteriorate with time and seeks to detect and identify deterioration before it causes a failure. This is a key element of condition-based maintenance (7)(8).

Often, only output signals can be measured, in which case signal-based methods can be applied. A measured signal, essentially the response to a disturbance input(s), is analyzed in the time domain, frequency domain, or time-frequency domain. Band-pass filters, spectral analysis, maximum-entropy estimation, and wavelet analysis can all be used as signal-processing methods.

If the relation between the input signal and the output signal is known, abrupt faults can be detected using the model-based fault detection method, which can be defined as
detection and decision-making based on the evaluation of residuals.

A recent project has recognized the value of model-based processing, examining both residual-based (i.e. using the estimation errors) and direct parameter estimation for identifying degradation in lateral dampers, yaw dampers and the wheel profile, the three aspects industry has identified as the most common causes of maintenance activity. The parameter estimation technique in particular has been investigated using data collected from a set of inertial sensors fitted to a service vehicle, and it has been possible to obtain good, representative values for both lateral and secondary anti-yaw dampers (9)(10).

This study demonstrates the possibility to detect railway vehicle suspension failures using the multiple-model approach. In this study, we consider suspension malfunction such as damper failure or spring failure. The railway vehicle model used includes the lateral and yaw motions of the wheelsets and bogie, and the lateral motion of the vehicle body, with sensors measuring lateral acceleration and yaw rate of the bogie, and lateral acceleration of the body. The IMM method (11)(12) has been applied for detecting vehicle suspension faults in a simulation study.

2. Modeling

Figure 1 depicts the railway vehicle model (9). Lateral and yaw motion of wheelsets and bogie are considered. Lateral motion is considered for a vehicle body. The equations of motion for a vehicle traveling on a straight track can be written as follows:

\[ m_{aw1} \ddot{y}_{w1} + \frac{2f_2}{v} \dot{y}_{w1} + K_y y_{w1} + 2f_2 y_{w1} + K_y y_b + a_y K_y y_b = 0 \]  
(1)

\[ I_{aw1} \ddot{\psi}_{w1} + \frac{2f_1 l_0}{v} \dot{\psi}_{w1} + K_y \psi_{w1} + 2f_1 l_0 \psi_{w1} = 0 \]  
(2)

\[ m_{aw2} \ddot{y}_{w2} + \frac{2f_2}{v} \dot{y}_{w2} + K_y y_{w2} + 2f_2 y_{w2} + K_y y_b + a_y K_y y_b = 0 \]  
(3)

\[ I_{aw2} \ddot{\psi}_{w2} + \frac{2f_1 l_0}{v} \dot{\psi}_{w2} + K_y \psi_{w2} + 2f_1 l_0 \psi_{w2} = 0 \]  
(4)

\[ m_{b} \ddot{y}_{b} + (C_{y_{bd}} + C_{y_{br}}) \dot{y}_{b} + (2a_y^2 K_y + 2K_y) y_{b} = 0 \]  
(5)

\[ I_{b} \ddot{\psi}_{b} + (C_{y_{bd}} + C_{y_{br}}) \dot{\psi}_{b} + (2a_y^2 K_y + 2K_y) \psi_{b} = 0 \]  
(6)

\[ m_{bd} \ddot{y}_{bd} + (C_{y_{bd}} + C_{y_{br}}) \dot{y}_{bd} + (K_y + K_{y_{br}}) \dot{y}_{bd} = 0 \]  
(7)

Here, \( y_{w1} \) and \( y_{w2} \) are the lateral displacements of the leading and trailing wheelsets, \( y_b \) is the lateral displacement of the bogie, \( y_{bd} \) is the lateral displacement of the vehicle body, \( \psi_{w1} \) is the yaw angle of leading wheelsets, \( \psi_{w2} \) is the yaw angle of the trailing wheelsets, and \( \psi_b \) is the yaw angle of the bogie. \( y_{1f} \) represents the lateral track displacement at the leading wheels, and \( y_{2f} \) represents the lateral track displacement at the trailing wheels.
3. Multiple-Model Approach

The multiple-model approach is an adaptive-estimation technique proposed in the field of target tracking. This approach enables a wide variety of adaptive estimation to be used while changing both parameters and the model structure.

In the multiple-model approach, it is assumed that the system obeys one of a finite number of models \( M \in \{ m_j \}_{j=1}^M \) that include possible modes. Using Bayes’ formula, the mode probability, a posteriori probability that the model \( j \) is correct, can be calculated by the following equation under the measurement \( Y^t = \{ y_1, y_2, \cdots, y_T \} \):

\[
p(m_j | Y^t) = \frac{p(y_t | m_j, Y^{t-1})p(m_j | Y^{t-1})}{\sum_j p(y_t | m_j, Y^{t-1})p(m_j | Y^{t-1})}
\]  

(8)

Here, \( p(y_t | m_j, Y^{t-1}) \) is the likelihood function of model \( j \) at time \( t \) calculated by following equation under the assumptions of Gaussian distribution.

\[
p(y_t | m_j, Y^{t-1}) = \frac{1}{\sqrt{2\pi S_{(t)}}} \exp \left\{ -\frac{1}{2} \left( y_t - \hat{y}_{t|t} \right)^T \left( S_{(t)}^{-1} \right) \left( y_t - \hat{y}_{t|t} \right) \right\}
\]  

(9)

Here, \( S_{(t)} \) is innovation covariance (see Eq.(22)). \( \hat{y}_{t|t} \) is the measurement prediction of model \( j \) at time \( t \).

Assuming a Gaussian distribution, the likelihood function can be evaluated by residual and covariance from mode-matched filter \( j \). An overall estimate \( p(x_t | Y^t) \) can be obtained by using the mode-conditioned state estimate obtained from each filter \( p(x_t | m_j, Y^t) \) and each mode probability \( p(m_j | Y^t) \) as

\[
p(x_t | Y^t) = \sum_{j=1}^M p(x_t | m_j, Y^t)p(m_j | Y^t)
\]  

(10)

Figure 2 illustrates the basic concept of the multiple-model approach.

When the system mode (model) switches in time, it is necessary to formulate the multiple-model approach dynamically. In the dynamic formulation, the mode-jump process, which is considered the mode-transition probability, should be taken into account.

The possible model history through time \( t \) is denoted by the mode history, \( M^t = \{ M_1, M_2, \cdots, M_w \} \).
The mode probability based on the mode history is
\[
p(M^t | Y^t) = \frac{p(y_t | M^t, Y^{t-1}) p(m_t | M^{t-1}, Y^{t-1}) p(M^{t-1} | Y^{t-1})}{\sum p(y_t | M^t, Y^{t-1}) p(M^t | Y^{t-1})}
\] (11)

The overall estimate is obtained by the mode-conditioned estimate and the mode probability as
\[
p(x_t | Y^t) = \sum_{j=1}^{\infty} p(x_t | M^t, Y^t) p(M^t | Y^t)
\] (12)

The number of the mode history \(M^t\) increases exponentially with time, which is a fatal problem for implementation. In order to avoid the exponentially increasing number of mode history, generalized pseudo-Bayesian of first order (GPB1) and second order (GPB2) and Interacting Multiple-Model (IMM) algorithm (see Fig. 4) (12)-(14) are proposed.

4. Fault Detection of Vehicle Suspension Using IMM

Figure 3 illustrates the concept of fault detection of vehicle components using the multiple-model approach.

Figure 4 depicts the algorithm of the IMM estimator. Estimations were conducted using the Kalman filter (KF) described below.

\(m\) sets of models were considered as the system modes. The \((i,j)\) element of the mode
transition matrix $p_{ij}$ representing mode transition probabilities among the modes expresses the transition probability from Mode $i$ to Mode $j$.

The following sections describe the details of the IMM estimator for railway vehicle suspension systems.

4.1 Mixing

Expressing the estimated value obtained by the KF for Mode $i$ ($i = 1, \ldots, m$) as $\hat{x}_{i(t)}$ and that obtained at time $t$ ($t = 0, 1, 2, \ldots$) as $P_{ij(t)}$, the mixed estimated value $\hat{X}_{ij(t)}$ and the mixed estimated covariance matrix as $P_{ij(t)}$ can be expressed by the following equations.

\[
\hat{X}_{ij(t)} = \sum_{k=1}^{m} \hat{x}_{k(t-1)} p_{kj(t-1)} \quad j = 1, \ldots, m
\]

\[
P_{ij(t)} = \sum_{k=1}^{m} p_{kj(t-1)} \left[ p_{ij(t-1)} + \left[ \hat{x}_{i(t-1)} - \hat{X}_{ij(t-1)} \right] \left[ \hat{x}_{i(t-1)} - \hat{X}_{ij(t-1)} \right]^T \right]
\]

Here, $\rho_{ij(t-1)}$ is the mixed mode probability at time $t$, and is expressed by

\[
\rho_{ij(t-1)} = \frac{1}{\bar{c}_j} p_{ij(t-1)} \quad i, j = 1, \ldots, m
\]

where

\[
\bar{c}_j = \sum_{i=1}^{m} p_{ij(t-1)} \quad j = 1, \ldots, m
\]

4.2 Mode-Matched Filtering

In this study, the KF was designed based on the reduced-order linear model illustrated in Fig. 5, where the motions of wheelsets are excluded.

The discrete system is expressed by

\[
x_{(t+1)} = Fx_{(t)} + Gu_{(t)} + w_{(t)}
\]

\[
y_{(t)} = Hx_{(t)} + Lu_{(t)} + v_{(t)}
\]

where

\[
x_{(t)} = \left[ \dot{y}_b \quad \psi_b \quad \dot{\psi}_b \quad \dot{y}_{bd} \quad \psi_{bd} \right]^T
\]
and matrices $F$, $G$, $H$ and $L$ are described in Appendix.

The following KF algorithms are then obtained.

Filter equations

$$\hat{x}_{(t-1)} = F^j(\hat{x}_{(t-1)}^j) + D^j u_{(t-1)}$$

(19)

$$\hat{x}_{(t-1)} = \hat{x}_{(t-1)} + K_{(t)} \left[ y_{(t)} - \left( H^j(\hat{x}_{(t-1)}^j) + L^j u_{(t)} \right) \right]$$

(20)

Kalman gain

$$K_{(t)} = P_{(t-1)} H_{(t-1)}^j R_{(t-1)}^{-1}$$

(21)

$$S_{(t)} = H_{(t-1)}^j P_{(t-1)} H_{(t-1)}^j + R_{(t-1)}$$

(22)

Covariance equations

$$P_{(t-1)} = F_{(t-1)} P_{(t-1)} F_{(t-1)}^j + G_{(t-1)}^j Q_{(t-1)}^j G_{(t-1)}^j$$

(23)

$$P_{(t-1)} = P_{(t-1)} - K_{(t)} S_{(t)} K_{(t)}^j$$

(24)

Here, $\hat{x}_{(t)}^j$ is the estimated state obtained by using the KF.

The system noise $w_{(t)}$ and measurement noise $v_{(t)}$ are assumed to be white Gaussian with a zero mean and covariance of $Q(w_{(t)})$ and $R(v_{(t)})$.

![Fig. 5 Estimation model](image)

4.3 Calculation of Mode Probability

The likelihood function for each mode is expressed as

$$L_{(t)} = \frac{1}{2\pi S_{(t)} \left| \hat{y}_{(t)} \right|^2} \exp \left[ -\frac{1}{2} \left( y_{(t)} - \left( H^j(\hat{x}_{(t-1)}^j) + L^j u_{(t)} \right) \right)^T \left( S_{(t)} \right)^{-1} \left( y_{(t)} - \left( H^j(\hat{x}_{(t-1)}^j) + L^j u_{(t)} \right) \right) \right]$$

(25)
Therefore, the mode probability for Mode $j$ at time $t$ is given by

$$
\rho_{j(t)} = \frac{\Lambda_{j(t)} \bar{c}_j}{\sum_{m} \Lambda_{m(t)} \bar{c}_m}
$$

(26)

The mode probabilities obtained vary with time and are smoothed by using a moving-average window.

4.4 Estimate and Covariance Combination

The estimated state $\hat{x}_{(t)}$ and combination of covariance $P_{(t)}$ are finally obtained by weighting the estimated state $\hat{x}_{j(t)}$ and the mixed covariance $P_{j(t)}$ for each mode with the mode probabilities.

$$
\hat{x}_{(t)} = \sum_{j=1}^{n} \hat{x}_{j(t)} \rho_{j(t)}
$$

(27)

$$
P_{(t)} = \sum_{j=1}^{n} \rho_{j(t)} \left[ P_{j(t)} + \left( \hat{x}_{j(t)} - \hat{x}_{(t)} \right) \left( \hat{x}_{j(t)} - \hat{x}_{j(t)} \right)^T \right]
$$

(28)

5. Simulation Examples

5.1 Simulation Condition

We verified the validity of the proposed method for two cases: (1) a secondary lateral damper failure (the damping coefficient reduced from its nominal value) occurs while the train is running; (2) a lateral accelerometer failure occurs when the train is running. In this simulation, the vehicle response is generated when passing along a track with an irregularity. We created a track irregularity by using white Gaussian noise passed through a shaping filter.

In case 1, the damping coefficient changes as shown in Fig. 6.

![Fig. 6 Normalized damping coefficient](image)

In case 2, we modeled a sensor failure for increasing covariance of the measurement noise. Figure 7 depicts the measurement noise used in the simulation.

![Fig. 7 Measurement noise](image)
IMM estimator is designed based on following models.

Mode 1  No malfunction in vehicle
Mode 2  A secondary lateral spring failure (100% reduction in spring rate)
Mode 3  A secondary lateral damper failure (20% reduction)
Mode 4  A secondary lateral damper failure (40% reduction)
Mode 5  A secondary lateral damper failure (60% reduction)
Mode 6  Failure of lateral accelerometer of bogie.
Mode 7  Failure of yaw rate sensor of bogie.
Mode 8  Failure of lateral accelerometer of body.

In Modes 6, 7 and 8, we provide a different covariance model of the measurement noise. Covariance R of each mode is described in Appendix.

First, we assumed that the bogie lateral damper and lateral accelerometer were normal, which means \( \rho_{j0} = 1.0 \) for \( j = 1 \) (Mode1) and \( \rho_{j0} = 0 \) for the other cases.

In case 1, the estimated value of the damping coefficient \( \hat{C}_{jl}(t) \), at time \( t \), was obtained by Eq. (29) by weighting the damping coefficient \( C_{jl} \) for each mode with the mode probabilities.

\[
\hat{C}_{jl}(t) = \sum_{j=1}^{m} C_{jl} / \rho_{j}
\]  (29)

The vehicle parameters and transition matrix used in the simulations are described in Appendix.

5.2 Simulation Result

5.2.1 Detection of a Secondary Lateral Damper Failure

Figure 8 presents the measurement data of this case used for the suspension fault detection. Figure 9 depicts the calculation results of mode probabilities; the modes where the probability was almost zero are excluded.

It is difficult to directly detect a lateral damper failure from these measurement data. However, mode probabilities indicate that a lateral damper failure can be detected in 3s with an estimation delay of 1s. This can be supported by the fact that Modes 3, 4 and 5 (lateral damper failure mode) exhibited higher mode probabilities after 3s, and Mode 1 (no malfunction mode) indicated a lower mode probability after 3s.

It should be noted that the type of failure (lateral spring failure or lateral damper failure) can be detected because Mode 2 (lateral spring failure) exhibited lower mode probabilities.

The reason for the delay in mode probabilities is that the weight of the \( P_{11} \)
element (transition probabilities from Mode 1 to Mode 1) of the transition matrix is much higher than that of other modes.

The estimation delay can be improved by changing the transition matrix, but there is a trade-off between delay and estimation accuracy.

Figure 10 presents the estimated value of the lateral damper coefficient. It can be seen that the proposed method exhibited good estimation performance. In addition, we estimated the parameter using EKF for comparison.

EKF is the most widely applied state-estimation algorithm for nonlinear systems. EKF can be used to estimate unknown parameters in dynamic systems. When using EKF for parameter estimation, it should be noted that selecting the state vector and initial values of parameters is important as partial linearization is employed and that the convergence of estimated parameters is not always guaranteed. It should be noted that the estimation using EKF failed in this simulation example.

5.2.2 Detecting a Lateral Accelerometer Failure

Figure 11 illustrates the lateral acceleration of bogie data of this case used for the sensor fault detection as an example. Figure 12 presents the calculation results of mode probabilities; modes where the probability was almost zero are excluded.

We can see that mode probabilities indicate a lateral acceleration sensor failure in 2 to 6s.

In this approach, we have a trade off between estimation accuracy and isolation of fault mode. The large number of model causes a good estimation accuracy of parameters but it causes the less faults isolation performance. One of the solutions for this problem is to include the limited number of fault modes according to the order of priority.
6. Conclusions

We studied methods of detecting malfunctions in vehicle suspension systems using on-board measurement data. We adapted the IMM method, which is a multiple-model based estimation method, for detecting faults in railway vehicles. We described the interacting multiple-model (IMM) method for detecting faults in railway vehicles from the measured lateral acceleration of the bogie and body and from the yaw rate of the bogie. A parameter-estimation technique using EKF was also compared with the IMM-based method.

We examined the validity of the proposed approach by performing two simulations, a secondary lateral damper failure (the damping coefficient reduced from its nominal value) and a lateral accelerometer failure (covariance of the measurement noise is increase) in railway vehicle suspension systems.

Simulation results indicate that the IMM-based method is an effective on-board fault detection technique for railway vehicle suspension systems.

We are preparing to evaluate the proposed method using multibody system simulation software SIMPACK in realistic situation. In this study, multiple failure modes are not considered. The applicability of the proposed method for multiple failures should be studied as the future work.

Acknowledgements

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References

(5) Blanke, M., Kinnairt, M., Lunze, J. and Staroswiecki, M., Diagnosis and Fault-Tolerant Control, (2003), Springer.
Appendix

A1. Transition Matrix

Transition matrix $P_\theta$ is described as

\[
\begin{bmatrix}
0.999 & s & s & s & s & s & s & s \\
0.999 & s & s & s & s & s & s & s \\
0.433 & 0.333 & 0.233 & k & k & k & k & k \\
0.233 & 0.433 & 0.333 & k & k & k & k & k \\
0.233 & 0.333 & 0.433 & k & k & k & k & k \\
s & s & s & s & s & 0.999 & s & s \\
s & s & s & s & s & s & s & s \\
\end{bmatrix}
\]

where $s = (0.001/7)$, $k = (0.001/5)$


Matrices $F$ and $H$ are expressed as in the following equation:

\[
F = \begin{bmatrix}
F_{11} & F_{12} & 0 & 0 & F_{15} & F_{16} \\
T & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & F_{33} & F_{34} & 0 & 0 \\
0 & 0 & T & 1 & 0 & 0 \\
F_{41} & F_{42} & 0 & 0 & F_{35} & F_{36} \\
0 & 0 & 0 & 0 & T & 1 \\
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
H_{11} & H_{12} & 0 & 0 & H_{15} & H_{16} \\
0 & 0 & 1 & 0 & 0 & 0 \\
H_{31} & H_{32} & 0 & 0 & H_{35} & H_{36} \\
\end{bmatrix}
\]

where

\[
F_{11} = (m_b - (C_{yib} + C_{yib})T) / m_b \\
F_{12} = -(2K_y + K_{yib} + K_{yib})T / m_b \\
F_{15} = (C_{yib} + C_{yib})T / m_b \\
F_{16} = (K_{yib} + K_{yib})T / m_b \\
F_{33} = (I_b - (C_{yib} + C_{yib})T) / I_b \\
F_{34} = -2a_o \hat{K}_y + 2(K_{yib} + K_{yib})T / I_b \\
F_{31} = (C_{yib} + C_{yib})T / m_{bd} \\
F_{32} = (K_{yib} + K_{yib})T / m_{bd} \\
F_{35} = (m_{bd} - (C_{yib} + C_{yib})T) / m_{bd} \\
F_{36} = -(K_{yib} + K_{yib})Tm_{bd}
\]

\[
H_{11} = -(C_{yib} + C_{yib}) / m_b \\
H_{12} = -(2K_y + K_{yib} + K_{yib}) / m_b \\
H_{15} = (C_{yib} + C_{yib}) / m_b \\
H_{16} = (K_{yib} + K_{yib}) / m_b \\
H_{31} = (C_{yib} + C_{yib}) / m_{bd} \\
H_{32} = (K_{yib} + K_{yib}) / m_{bd} \\
H_{35} = -(C_{yib} + C_{yib}) / m_{bd} \\
H_{36} = -(K_{yib} + K_{yib})m_{bd}
\]

and $T$ is the sampling period for measurements.
Matrices $G$ and $L$ are written as follows.

\[
G = \begin{bmatrix}
K_y / m_b & K_y / m_b \\
0 & 0 \\
a_0 K_y / I_b & -a_0 K_y / I_b \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
K_y / m_b & K_y / m_b \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

A3. Covariance R of each mode

Covariance R of each mode used in the simulations is below

<table>
<thead>
<tr>
<th>Mode</th>
<th>Covariance R of Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lateral accelerometer</td>
</tr>
<tr>
<td>Mode 1 to 5</td>
<td>9.0×10^{-4}</td>
</tr>
<tr>
<td>Mode 6</td>
<td>81×10^{-4}</td>
</tr>
<tr>
<td>Mode 7</td>
<td>9.0×10^{-4}</td>
</tr>
<tr>
<td>Mode 8</td>
<td>9.0×10^{-4}</td>
</tr>
</tbody>
</table>

A4. Vehicle Parameters

Vehicle parameters used in the simulations are listed below.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_w$</td>
<td>Wheelset mass</td>
<td>kg</td>
<td>1000</td>
</tr>
<tr>
<td>$I_w$</td>
<td>Wheelset yaw inertia</td>
<td>kgm^2</td>
<td>600</td>
</tr>
<tr>
<td>$m_b$</td>
<td>Bogie mass</td>
<td>kg</td>
<td>2000</td>
</tr>
<tr>
<td>$I_b$</td>
<td>Bogie yaw inertia</td>
<td>kgm^2</td>
<td>2300</td>
</tr>
<tr>
<td>$m_{bd}$</td>
<td>Body mass</td>
<td>kg</td>
<td>8720</td>
</tr>
<tr>
<td>$K_y$</td>
<td>Primary lateral stiffness per wheelset</td>
<td>kN/m</td>
<td>4000</td>
</tr>
<tr>
<td>$K_{y,b}$</td>
<td>Secondary lateral stiffness per bogie of left</td>
<td>kN/m</td>
<td>80</td>
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<tr>
<td>$K_{y,b}$</td>
<td>Secondary lateral stiffness per bogie of right</td>
<td>kN/m</td>
<td>80</td>
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<tr>
<td>$C_{y,b}$</td>
<td>Secondary lateral damping per bogie of left</td>
<td>kN/m</td>
<td>8</td>
</tr>
<tr>
<td>$C_{y,b}$</td>
<td>Secondary lateral damping per bogie of right</td>
<td>kN/m</td>
<td>8</td>
</tr>
<tr>
<td>$C_{y,rb}$</td>
<td>Secondary anti-yaw damping per bogie of left</td>
<td>kN/m</td>
<td>250</td>
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<tr>
<td>$C_{y,rb}$</td>
<td>Secondary anti-yaw damping per bogie of right</td>
<td>kN/m</td>
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</tr>
<tr>
<td>$f_{11}$</td>
<td>Longitudinal creep coefficient</td>
<td>MN</td>
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<tr>
<td>$f_{22}$</td>
<td>Lateral creep coefficient</td>
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<td>Semi wheel-wheel spacing</td>
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<tr>
<td>$l_0$</td>
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<td>m</td>
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</tr>
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<td>$\lambda$</td>
<td>Conicity</td>
<td></td>
<td>0.15</td>
</tr>
<tr>
<td>$v$</td>
<td>Vehicle forward velocity</td>
<td>m/s</td>
<td>20</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Wheelset radius</td>
<td>m</td>
<td>0.37</td>
</tr>
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</table>