3-D Analysis of the Influence of a Pile on Underground Induced Vibrations Using the Boundary Element Method*

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Abstract
This paper presents the influence of a pile on ground vibration induced by a moving train in an underground tunnel. A coupling of 2.5-D and 3-D boundary element method (BEM) is used to obtain displacement amplitudes on a surface around the pile top. First, a 2.5-D BEM is used to obtain an incident wave field in a 3-D half space, without a pile, subjected to an underground moving load. Second, 3-D BEM is carried out for a soil-pile model using the obtained incident wave field. Numerical results show that the presence of a pile may increase the ground vibration level around the pile top at low frequencies.

Key words : Environmental Vibration, Pile, Moving Load, BEM Analysis

1. Introduction
In urban areas around the world, underground railways play an important role in transportation. In general, using of an underground rather than a surface railway can help reduce ground vibrations and noise. Furthermore, it can improve the earthquake resistance of the railway system and relieve ground level rush-hour traffic congestion. However, underground trains can create strong ground vibrations in surrounding areas, especially when they travel in the vicinity of pile foundations. In addition, although the effects of surface moving loads have been studied extensively over the past decade (1)–(3), the influence of underground moving loads remains unconsidered; the effect of a surface moving load is still not understood, and an underground moving load presents a much more complex problem. It is important for both safety and environmental reasons to understand how a pile influences vibrations from an underground railway.

In general, a moving point load can be considered as the simplest analytical model of a train passing along a track. Even for such a simple model, no explicit solutions can be found in a closed form (4). Therefore, numerical methods are used to solve such problems. Since finite element method (FEM) and finite difference method (FDM) can deal with finite regions only, they have been combined with other numerical techniques to consider the radiation conditions in a half or infinite space (5). On the other hand, the boundary element method (BEM) can deal with such spaces without any modification. Therefore, BEM is more efficient for such problems.

The 3-D dynamic response of a model where the material parameters vary only in a 2-D geometry is common in many engineering problems. Dealing with a 3-D wave field in a 2-D medium is called a 2.5-D problem. A 2.5-D BEM has been developed to obtain solutions of wave fields in some engineering fields (6),(7). The wave field in the vicinity of an infinite vibration barrier located along the train path is a typical 2.5-D problem. In addition, the wave field around an infinite tunnel subjected to an underground moving load is also a 2.5-D problem. For example, a 2.5-D BEM has been utilized for investigating the effectiveness of...
a wave barrier along a track\(^8\). In practice, however, it is more realistic to conduct numerical calculations of a finite model with 3-D BEM, instead of an infinite model with 2.5-D BEM. For such a finite model, a full 3-D BEM analysis is required; however, a conventional 3-D BEM needs significant computational time and memory\(^9\). Therefore, an important key for such 3-D problems is finding a way to efficiently and accurately obtain numerical solutions with BEM.

In this study, therefore, the effects of the interaction between a pile and vibration induced by a moving train are investigated. Using a 3-D BEM in conjunction with 2.5-D analysis, the computational efficiency for the proposed problem\(^{10}\) can be improved. In this paper, the concept of a 2.5-D soil-pile problem and a 2.5-D BEM formulation is introduced by showing an analysis model. Next, a method for combining a 2.5-D BEM with a 3-D BEM is discussed. Finally, based on the numerical results, it can be concluded that, at low frequencies, there is an increase in the ground vibration level in the vicinity of the pile top.

2. Problem

A dynamic problem in a homogeneous isotropic elastic solid with a pile subjected to a moving load \(g\) is considered, as shown in Fig.1-(a). The moving load \(g\), with circular frequency \(\Omega\), moves at a constant speed \(V\) inside a tunnel along the \(x_3\) axis. It is assumed that the geometry, except for the pile, has a 2-D configuration with a uniform cross-section along the \(x_3\) direction, as shown in Fig.1-(a).

2.1. BEM Analysis

In this study, a numerical approach is used to obtain the wave fields around the pile top in Fig.1-(a). However, it is difficult to implement this from the standpoint of numerical analysis. In Japan, generally, a deep underground tunnel must be built more than 10m away from a pile, and the diameter of the pile should not be more than several meters. In such situations, the interaction between the tunnel and the pile may be negligible. Adapting this situation, the 3-D problem in Fig.1-(a) can be split as a radiation problem subjected to an underground moving load \(g\) (Fig.1 (b)), and a scattering problem by a pile and surface (Fig.1 (c)). The tunnel has a uniform cross-section along the \(x_3\) direction and thus a 2.5-D problem is observed as shown in

![Fig. 1 3-D separation analysis model.](image)
2.2. Governing Equations and Boundary Conditions

In general, the displacement field \( u_i(x, t) \) at point \( x \) and time \( t \) satisfies the Navier-Cauchy equations for a homogeneous isotropic elastic solid \( D \) as follows:

\[
L_{ik}(\partial_x)u_k(x, t) = \frac{\partial^2}{\partial t^2} u_i(x, t), \quad x \in D
\]

(1)

where \( \rho \) is the mass density and \( L_{ik}(\partial_x) \) is the differential operator defined by

\[
L_{ik}(\partial_x) = C_{ijkl} \frac{\partial}{\partial x_j} C_{ijkl}.
\]

(2)

In addition, \( C_{ijkl} \) are the elastic constants given by \( C_{ijkl} = \lambda \delta_{ij}\delta_{kl} + \mu (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \), where \( \lambda \) and \( \mu \) are Lamé constants, and \( \delta_{ij} \) is the Kronecker delta. The boundary conditions are given by

\[
u_i(x, t) = h_i(x, t), \quad x \in S_u
\]

(3)

\[
t_i(x, t) \equiv \sum_{k=1}^{n} T_{ik} (\partial_x)u_k(x, t) = g_i(x, t), \quad x \in S_t = S \setminus S_u
\]

(4)

where \( t_i(x, t) \) shows the traction components at the point \( x \) and time \( t \) corresponding to the displacement component \( u_i(x, t) \). In addition, \( h_i \) and \( g_i \) are given boundary values, and \( T_{ik}(\partial_x) \) are the traction operators defined by

\[
T_{ik}(\partial_x) = C_{ijkl} n_j(x) \frac{\partial}{\partial x_l}
\]

(5)

where \( n_j(x) \) are the components of a unit normal vector to the surface \( S \).

The Fourier transforms with respect to \( x_3 \) and time \( t \) are defined as follows:

\[
\tilde{f}(x_1, x_2, \xi_3, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1, x_2, x_3, t) e^{-i\xi_3 x_3} dx_3
\]

(6)

\[
\tilde{f}(x_1, x_2, x_3, \omega) = \int_{-\infty}^{\infty} f(x_1, x_2, x_3, t) e^{i\omega t} dt.
\]

(7)

Applying the Fourier transforms defined by Eqs. (6) and (7) to the governing equation (1) and the boundary conditions (4) for three dimensional space yields the following in a two dimensional space \( X = (x_1, x_2) \):

\[
L_{ik}(\partial_x)u_k(X, \xi_3, \omega) = -\rho \omega^2 \hat{u}_i(X, \xi_3, \omega), \quad X \in \tilde{D}
\]

(8)

\[
\hat{u}_i(X, \xi_3, \omega) = \hat{h}_i(X, \xi_3, \omega), \quad X \in \partial S_u
\]

\[
\hat{t}_i(X, \xi_3, \omega) = \hat{g}_i(X, \xi_3, \omega), \quad X \in \partial S_t = \partial S \setminus \partial S_u
\]

(9)

where \( \tilde{D} \) is the two dimensional domain bounded by the boundaries \( \partial S_u \) and \( \partial S_t \) in the \( x_1-x_2 \) plane.

The two dimensional solutions \( \hat{u}_i \) are obtained by solving the boundary value problems of Eq. (8) with boundary conditions defined in Eq. (9). Considering the relation of Eqs. (6) and (7), the solutions \( u_i \) at point \( x \) and time \( t \) are then calculated using the inverse Fourier transform with respect to the wave-number space \( \xi_3 \) and circular frequency \( \omega \), respectively, as follows:

\[
u_i(x, t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{u}_i(X, \xi_3, \omega) e^{i(x_3 - \xi_3 x_3)} d\xi_3 d\omega.
\]

(10)
For this calculation, however, the two dimensional solutions \( \hat{u}_i \) for all values of \( \xi_3 \) and \( \omega \) should be obtained, which may be time consuming.

As mentioned before, the moving load \( g \) travels along the \( x_3 \) direction, which is assumed to have a constant velocity \( V \) and circular frequency \( \Omega \). In addition, since the geometry of the model has a uniform cross-section in the \( x_3 \) direction, as shown in Fig.1-(a), both the displacement \( u(x, t) \) and traction \( t(x, t) \) have the same form, i.e.,

\[
u(x, t) = u(x, x_3 - Vt, t) e^{-\omega t} \quad \text{(11)}
\]

\[
t(x, t) = t(x, x_3 - Vt, t) e^{-\omega t} \quad \text{(12)}
\]

The Fourier transforms defined by Eqs. (6) and (7) are applied to Eq. (11), to obtain

\[
\hat{u}_i(X, \xi_3, \omega) = 2\pi \hat{u}_i(X, \xi_3, \omega) \delta(\omega - \Omega - \xi_3 V) \quad \text{(13)}
\]

where \( \delta(\cdot) \) is the Dirac delta function and \( \hat{u}_i \) denotes the solution of Eq. (8) with the circular frequency \( \omega = \xi_3 V + \Omega \). Substituting Eq. (13) into Eq. (10), the displacement \( u(x, t) \) can be obtained by

\[
u(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{u}_i(X, \xi_3, V \xi_3 + \Omega)e^{i(x_3 - Vt)\xi_3 - \omega t} d\xi_3 \quad \text{(14)}
\]

Consequently, three dimensional displacements \( u(x, t) \) due to a moving load in the tunnel can be constructed by solving the 2-D boundary value problem in the \( X = (x_1, x_2) \) space with the condition \( \omega = \xi_3 V + \Omega \) and taking the single Fourier transform for \( \hat{u}_i(X, \xi_3, V \xi_3) \) with respect to \( \xi_3 \), as shown in Eq. (14). In the numerical calculation, the inverse Fourier transform is carried out in a discretized form by the fast Fourier transform.

Needless to say, these formulations are beneficial if a pile whose cross-section is not uniform along the \( x_3 \) direction does not exist. The details of dealing with the presence of the pile will be described in § 4.

3. 2.5-D BEM Formulation

BEM is used to solve the 2-D boundary value problem defined by Eqs. (8) and (9). In this section, the Greek subscripts for summation convention take the values 1 and 2 only. In the BEM formulation, the fundamental solutions satisfying the equations of motion

\[
L_i(\delta_{x_i}, \delta_{x_j}, -i\xi_3)U_{ik}(X, Y) + \delta_{ik}\delta(X - Y) = -\rho c^2 U_{ik}(X, Y) \quad \text{(15)}
\]

are utilized. Explicit expressions of the fundamental solutions \( U_{ik} \) were obtained by Li et al. (11):

\[
U_{ik}(X, Y) = \frac{i}{4\mu} \left[ H_{0}^{(1)}(\tilde{k}_R)\delta_{ik} + M_{ik} \right] \left[ H_{0}^{(1)}(\tilde{k}_T R) - H_{0}^{(1)}(\tilde{k}_L R) \right] \quad \text{(16)}
\]

where \( H_{0}^{(1)}(\cdot) \) is the Hankel function of the zeroth order of the first kind, and \( M_{ik} \) is the following differential operator:

\[
M_{ik} = \frac{1}{k_T^2} \left[ \frac{\delta_{ik}\delta_{j\alpha}}{\partial x_3 \partial x_{\beta}} - i\xi_3(\delta_{3\alpha}\delta_{ik} + \delta_{3\beta}\delta_{ik}) \right] \frac{\partial}{\partial x_{\alpha}} - \xi_3^2 \delta_{3\alpha}\delta_{3\beta} \quad \text{(17)}
\]

The symbols in Eqs. (16) and (17) are defined by \( R = [(x_1 - y_1)^2 + (x_2 - y_2)^2]^{1/2}, \quad \tilde{k}_T = (k_T^2 - \xi_3^2)^{1/2}, \quad \tilde{k}_L = (k_L^2 - \xi_3^2)^{1/2}, \) where \( k_T \) and \( k_L \) are the transverse and longitudinal wave numbers defined by \( k_T = \omega/c_T \) and \( k_L = \omega/c_L \), respectively; \( c_T \) and \( c_L \) are the velocities of waves P and S, respectively.

Multiplying Eqs. (8) and (15) by \( U_{ik}(X, Y) \) and \( \hat{u}_i(X, \xi_3, \omega) \), respectively, subtracting the latter from the former and integrating over the domain \( D \), and after some manipulations, we have the following 2.5-D boundary integral equations:

\[
\int_{\partial S} \left\{ U_{ik}(X, Y)\hat{u}_i(Y, X, \omega) - T_{ik}(X, Y)\hat{u}_k(Y, \xi_3, \omega) \right\} d\Sigma_Y = \frac{1}{2} \hat{u}_i(X, \xi_3, \omega), \quad X \in \partial S \quad \text{(18)}
\]
Table 1 Material constants used in the analysis.

<table>
<thead>
<tr>
<th>Material</th>
<th>( c_L ) (m/s)</th>
<th>( c_T ) (m/s)</th>
<th>( \rho ) (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-space soil</td>
<td>1600</td>
<td>400</td>
<td>2000</td>
</tr>
<tr>
<td>Tunnel lining</td>
<td>1387</td>
<td>789</td>
<td>2300 (( \mu = 1435 \text{MPa}, \nu = 0.26 ))</td>
</tr>
<tr>
<td>Pile</td>
<td>4000</td>
<td>2500</td>
<td>2400</td>
</tr>
</tbody>
</table>

where the boundary \( \partial S \) is assumed to be smooth and \( T_{ik}(X, Y) = T_{ij} (\partial \gamma_i, \partial \gamma_j, -i \xi_3) U_{ik}(X, Y) \).
Eq. (18) is discretized into the system of equations and solved using the boundary conditions given by Eq. (9). The 2-D solution \( \hat{h}_i(Y, \xi_3, \omega) \) with \( \omega = \xi_3 V + \Omega \) is substituted into Eq. (14) to obtain the 3-D solution.

4. 3-D BEM Formulation

4.1. Problem
A 2.5-D modeling is very effective in investigating the dynamic behavior of the ground subjected to a moving load inside a tunnel. However, the 2.5-D modeling is subject to a strong restriction, in that the geometry must be a 2-D configuration with a constant cross-section along the moving direction of the load. In this section, a 3-D BEM formulation is presented for the scattering problem in 3-D half space with a pile subjected to an underground moving load, as shown in Fig. 1-(c). The conditions of the moving load and material are the same as that in the 2.5-D analysis except for the finite pile.

If there is no pile, the problem is reduced to a 2.5-D problem of a half space with an infinite tunnel subjected to an underground moving load, as shown in Fig. 1-(b). The 3-D scattering problem (Fig. 1-(c)) is formulated by considering the solution of the 2.5-D problem (Fig. 1-(b)) as the incident wave of the 3-D problem (Fig. 1-(c)).

4.2. 3-D BEM Formulation
In this section, the superscripts I and II represent physical quantities of the homogeneous elastic solid and pile, respectively. The incident wave due to a moving load is reflected and scattered by the pile and ground surface. For the homogeneous elastic solid, \( u^{3D}_{\text{I}} \) is shown as the sum of incident wave field \( u^{\text{lin}}_{\text{I}} \) and scattered field \( u^{\text{sc}}_{\text{I}} \);

\[
u^{3D}_{\text{I}} = u^{\text{lin}}_{\text{I}} + u^{\text{sc}}_{\text{I}}.
\]
If the 3-D scattering problem is formulated for the scattered wave fields of \( u^{\text{sc}}_{\text{I}} \) and \( t^{\text{sc}}_{\text{I}} \) in the frequency domain, the boundary integral equations on the ground surface \( S_1 \) and interface \( S_2 \), between the elastic solid and pile are expressed as:

\[
\int_{S_A} \left[ U^{3D}_{ik}(x, y) u^{\text{sc}}_{k}(y, \omega) - T^{3D}_{ik}(x, y) u^{\text{lin}}_{k}(y, \omega) \right] dS_y = \frac{1}{2} u^{\text{sc}}_{i}(x, \omega), x \in S_A = S_1 + S_2
\]
where \( U^{3D}_{ik}(x, y) \) is the fundamental solution of 3-D elastodynamics in the frequency domain, given by

\[
U^{3D}_{ik}(x, y) = \frac{1}{4 \pi \mu} \left[ \frac{e^{ikr}}{r} \delta_{ik} + \frac{1}{k L} \frac{\partial}{\partial y_i} \left( \frac{e^{ikr}}{r} - \frac{\partial e^{ikr}}{\partial y_i} \right) \right]
\]
where \( r \) is the distance between \( x \) and \( y \), given by \( r = |x - y| \). The double layer kernel \( T^{3D}_{ik}(x, y) \) is defined by

\[
T^{3D}_{ik}(x, y) = n_j(y) C_{kjpq} \frac{\partial}{\partial y_q} U^{3D}_{ip}(x, y).
\]
Substituting Eq. (19) into Eq. (20), and taking into account the boundary conditions of \( t^{3D}(x, \omega) = t^{\text{lin}}(x, \omega) = 0 \) on the ground surface \( S_1 \) yields the boundary integral equation...
for the 3-D wave field for the elastic solid as follows:

\[
\begin{align*}
\frac{1}{2} u_{i,3D}^{1D}(x, \omega) + \oint_{S_2} T_{ik}^{3D}(x, \omega) u_{j,3D}^{1D}(y, \omega) dS_y &= \oint_{S_3} U_{ik}^{3D}(x, \omega) t_{j,3D}^{1D}(y, \omega) dS_y \\
\frac{1}{2} u_{i,3D}^{1D}(x, \omega) + \oint_{S_2} T_{ik}^{3D}(x, \omega) u_{j,3D}^{1D}(y, \omega) dS_y &= \oint_{S_3} U_{ik}^{3D}(x, \omega) t_{j,3D}^{1D}(y, \omega) dS_y 
\end{align*}
\]  

(23)

On the other hand, we have the boundary integral equation on the pile with \(S_2\) and the surface of the pile top \(S_3\) as follows:

\[
\begin{align*}
\frac{1}{2} u_{i,3D}^{1D}(x, \omega) &= \oint_{S_2} U_{ik}^{3D}(x, \omega) t_{j,3D}^{1D}(y, \omega) dS_y \\
- \oint_{S_3} T_{ik}^{3D}(x, \omega) u_{j,3D}^{1D}(y, \omega) dS_y, x \in S_B = S_2 + S_3.
\end{align*}
\]  

(24)

Assuming the following continuity boundary conditions on \(S_2\),

\[
\begin{align*}
u_{i,3D}^{1D} = u_{j,3D}^{1D}, & \quad t_{i,3D}^{1D} = -t_{j,3D}^{1D} \text{ on } S_2
\end{align*}
\]  

(25)

and combining Eqs. (23) and (24), we obtain the 3-D solutions as \(u_{i,3D}^{1D}\) and \(t_{i,3D}^{1D}\).

As mentioned above, the incident wave \(u_{in}^{1D}\) used in the 3-D boundary integral equations (23) is the solution of the 2.5-D radiation problem in the frequency domain; a moving load \(g\) with uniform velocity \(V\) inside the tunnel, with infinite length and a uniform cross-section for the \(x_3\) direction. The 2.5-D solution in the frequency domain is obtained from Eq. (14). Changing the integral variable \(\xi_3\) to \(\omega = V\xi_3 + \Omega\) in Eq. (14), we have

\[
u_{i}(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{V} \hat{h}_{i}^{1D}(X, \omega - \Omega, V)e^{i\omega x_3} e^{-i\omega t} d\omega.
\]  

(26)

Here, we note that Eq. (26) has a relation with the inverse Fourier transform of \(u_{in}^{1D}(x, \omega)\) and it can be found that

\[
u_{i}(x, \omega) = \frac{1}{V} \hat{h}_{i}^{1D}(X, \omega - \Omega, V)e^{i\omega x_3}.
\]  

(27)

Thus, the boundary integral equations (23) and (24) for the 3-D problem can be solved in conjunction with the 2.5-D analysis.
5. Numerical Results

Numerical calculations implemented for the problems are described in this section. Note that the velocity of the moving load \( g \) is \( V = 75 \text{km/h} \). The material constants used in these analyses are given in Table 1. In general, a frequency range up to several hundred Hz should be considered in train-induced vibration problems; however, the actual high frequency component is relatively prone to attenuation. In addition, high frequency problems need a significant amount of computational time and memory; thus, the maximum considered frequency is set to 64 Hz.

5.1. Separation Analysis of Tunnel and Pile in 2-D

In this formulation, the tunnel-pile interaction is not considered. Numerical results for the analysis of the two models, the original Fig.2-(a) and separated Figs.2-(b) and (c), are compared to check the validity of the separation concept. In particular, it is desirable to implement these checks when the tunnel-pile interaction is as large as possible. In general, elastic waves generated from a train in an underground tunnel have an obliquely upward impact on the surface, and not directly overhead from the tunnel because of elastic wave diffraction. Therefore, the horizontal distance \( h \) between the centers of the tunnel and pile is set relatively large in the check. In the 2-D analysis, a unit distributed load with a circular frequency \( \Omega \) is assumed for
the underground moving load.

Figures 3-(a) and (b) show the horizontal and vertical surface displacements at several frequencies \( \omega = \Omega \) when the horizontal distance between the centers of the tunnel and pile is \( h = 20.5 \text{m} \) for the 2-D problem shown in Fig.2. In Figs.3-(a) and (b), the lines indicate solutions obtained from separation analysis and circles represent no separation analysis. Good agreement can be seen between the results obtained from each analysis. Therefore, if the distance between a tunnel and pile is sufficient, the problem can be solved separately.

5.2. Separation Analysis of Tunnel and Pile in 3-D

In this subsection, the solution for the 3-D problem shown in Fig.4-(a) is described. In the 3-D analysis, the pile \( S_2 + S_3 \) and ground surface \( S_1 \) are discretized into 376 and 1368 boundary elements, respectively, with piecewise constant approximation, as shown in Fig.4-(b). The mesh size was decided according to the wave length of S-wave \( \lambda_S \approx 6.25 \text{m} \), which corresponds to a frequency \( f = \frac{\omega}{(2\pi)} = 64 \text{Hz} \). In addition, the fundamental solutions defined in equations (21) and (22) are for full space. Therefore, the ground surface is truncated for the numerical calculations. A unit distributed load with circular frequency \( \Omega \) and velocity \( V = 75 \text{km/h} \) is assumed for the underground moving load, as shown in Fig.4.

The BEM analysis can obtain displacements of the ground surface. In general, the vibration level is evaluated as a ratio of the measured vibration amplitude on the ground surface to that on the side of a rail track in actual measurements. Therefore, in the latter results, note that the numerical results are scaled with the vertical displacements \( u_{ref} \) at reference point A (-0.135m,-46.3m,0m) in Fig.4-(a).

5.2.1. Separation Analysis of Tunnel and Pile in 3-D when \( h = 0 \text{ m} \)

Figures 5-(a) and (b) show the vertical surface displacements at several frequencies \( \omega = \Omega \) with \( h = 0 \text{ m} \) along the (a) \( x_1 \) and (b) \( x_3 \) axes. It can be seen that the vertical displacements around the pile top are relatively large at low frequencies \( \omega = \Omega = 6.3, 8.0, \) and 10.0Hz, and relatively small at other frequencies. Figures 6-(a),(b),(c) and (d) indicate the vertical displacement amplitudes on the ground surface around the top of the pile with \( h = 0 \text{ m} \) at the frequencies of \( \omega = \Omega = 8, 10, 40, \) and 50Hz, respectively. Note that the legend scales for Figs.6 and 8 are different. As shown in Figs.6-(a) and (b), the vertical displacement amplitudes are larger at the pile top at the low frequencies of 8 and 10Hz, and smaller at the higher frequencies of 40 and 50Hz.
5.2.2. Separation Analysis of Tunnel and Pile in 3-D when $h = 20.5$ m

Taking the same problem, but this time considering only the horizontal distance $h$ between the tunnel and the pile, which is horizontally separated from the tunnel by $h = 20.5$ m, Figures 7 and 8 illustrate the same results as in Figs.5 and 6, respectively, but for the horizontal position of the pile, $h = 20.5$m. As shown in Fig.7, the vertical displacement amplitudes show slightly large values around the pile top, compared with the other areas at frequencies $\omega = \Omega = 6.3$ and 16.0Hz. However, the vertical displacement amplitudes around the pile top are smaller than those in other areas at other frequencies. Also, the presence of the pile constrains the amplitude amplification in the vicinity of the pile top. Moreover, large values can be observed at the pile top area at low frequencies (Figs.8-(a) and (b), Figs.6-(a) and (b)).

Note that elastic waves in a pile can propagate as guided waves. This fact can cause large amplitude vibration at the ground surface around the pile top for some specific frequencies. Therefore, it is necessary to investigate ground vibrations in such situations from the standpoint of elastic wave theory.

6. Conclusions

In this study, the effect of interactions between a pile and ground vibration, induced by a moving load inside a tunnel, was investigated with the coupling of 2.5-D and 3-D BEM. Numerical results show that the presence of a pile increases ground vibration around the pile top at relatively low frequencies. In future work, several additional points will be considered. This analysis assumed that boundary conditions defined in Eq. (25) at the soil-pile interface...
Fig. 6  Vertical displacement amplitudes on ground surface with $h = 0$ at frequencies (a) 8, (b) 10, (c) 40, and (d) 50 Hz ($-10 \leq x_1 \leq 10, -10 \leq x_3 \leq 10$).

Fig. 7  Vertical surface displacements scaled with $u^{ref}$ along the (a) $x_1$ axis and (b) $x_3$ axis at several frequencies $\omega = \Omega$ when $h = 20.5$ m.
Fig. 8  Vertical displacement amplitudes on ground surface with $h = 20.5$ at frequencies (a) 8, (b) 10, (c) 40, and (d) 50Hz ($-30.5 \leq x_1 \leq -10.5, -10 \leq x_3 \leq 10$)

$S_3$ in Fig.4-(a) are continuous and there is no wave attenuation in the elastic solid and pile. Therefore, it is expected that vibration amplification around the pile top will be less compared to previous numerical results. In addition, the problem of more than one pile will be considered in detail. Applying the fast multipole method\(^{(12),(13)}\) to 3-D BEM can be effective for such large scale problems.

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