Dynamic Performance of Subway Vehicle with Linear Induction Motor System*

Pingbo WU**, Ren LUO**, Yan HU** and Jing ZENG**

**Traction Power State Key Laboratory,
Southwest Jiaotong University, Chengdu, China
E-mail: wupingbo@163.com

Abstract
The light rail vehicle with Linear Induction Motor (LIM) bogie, which is a new type of urban rail traffic tool, has the advantages of low costs, wide applicability, low noise, simple maintenance and better dynamic behavior. This kind of vehicle, supported and guided by the wheel and rail, is not driven by the wheel/rail adhesion force, but driven by the electromagnetic force between LIM and reaction plate. In this paper, three different types of suspensions and their characteristic are discussed with considering the interactions both between wheel and rail and between LIM and reaction plate. A nonlinear mathematical model of the vehicle with LIM bogie is set up by using the software SIMPACK, and the electromechanical model is also set up on Simulink roof. Then the running behavior of the LIM vehicle is simulated, and the influence of suspension on the vehicle dynamic performance is investigated.

Key words: Subway Vehicle, Linear Induction Motor, Electromechanical Model, Dynamic Performance

1. Introduction

Compared with the traditional subway, the LIM subway can greatly reduce the required tunnel cross-section size and the project cost, and meanwhile, it can improve the vehicle climbing and curving ability. Thus, the LIM subway is more suitable for the complex urban terrain and environment. In 1986, the first linear motor subway was built in Vancouver, Canada. Then Toronto, Detroit, New York, Kobe, Osaka, Fukuoka, Yokohama, Kuala Lumpur and other cities have also built LIM subways [1, 2, 3, 4]. The LIM subways have been also applied in Guangzhou metro line 4 and Beijing airport line in China. It can say that the LIM subway has been widely received because of its low cost, flexible circuit design, good ride comfort, and will be one of the best urban transport systems [5, 6, 7].

The LIM is a flat plate-like shape formed by partially cutting a conventional AC rotary induction motor and moves linearly. It has the advantage of small space size because of its rotor installed in the track and stator installed in the vehicle, and can significantly reduce the wheel diameter and height of the underside without power transmission devices such as gear box and coupling. If the low-floor technology is used, the vehicle overall height can reduce about 1m compared to the conventional wheel-rail vehicles, and the tunnel cross-section area can be reduced by about 40%. Thus the project cost can be significantly reduced [7, 8, 9]. As for the LIM vehicle, its wheels only posses the function of supporting and guiding, the traction and braking forces are generated by the electromagnetic induction working on lines arising board [10, 11, 12]. It can be known that the LIM bogie is different from conventional bogie in structure and the longitudinal load transmission.

Because the linear motor mainly hangs on the frame and the gap between the LIM and reaction plate should maintain the same value, special requirement is given for the design of the bogie and the dynamic model is more complex. In this paper, focusing on the
characteristics of LIM vehicle, the dynamic model is set up using the software SIMPACK and the LIM electromechanical coupling model is set up in Simulink tools. The simulation for the vehicle is undertaken by using SIMPACK combined with Simulink.

There are three types of motor suspensions; axle-mounted, bogie frame-mounted and auxiliary frame-mounted [6, 7, 13]. The axle-mounted suspension was used in the early years in Japan, and is rarely used now because of the linear motor entirely belonging to un-sprung weight, and larger wheel-rail force and wheelset radial shaking caused by traction through axle-box transmission may seriously affect the wheel-rail dynamics [9, 10, 11]. The second one is that the motor hangs on the bogie frame. The disadvantages such as the larger air-gap between linear motor (coil) and reaction plate and the air-gap changing with the vibration may reduce the motor efficiency 70% compared with the rotary motor [14, 15]. The larger air-gap change, lower motor efficiency and larger energy consumption will influence the vehicle driving and braking performance. It will also affect the passenger comfort if using larger axle-box vertical stiffness. The third one is using the auxiliary framework between axle-boxes as motor suspension support beam, such as type BM3000 bogie used by Guangzhou Metro Line 4, and type MK II bogie of Canada [14, 15, 16]. Similar to axle mounted suspension, this type of suspension eliminates the effect of primary spring to air-gap, but will increase the wheel-rail interaction force, running noise and wheel wear due to the increase of un-sprung mass [17, 18].

The LIM efficiency is inversely proportional to power factor with the air gap. The larger the motor air gap, the less the motor efficiency will be. Thus it is necessary to improve the air-gap of the LIM vehicle by using effective method. In this paper, the bogie frame mounted suspension for LIM with larger primary suspension stiffness is used. In order to adapt to track twist friendly, the framework with two side frames interpolating is utilized.

The nonlinear mathematical model of the vehicle with LIM bogie is set up by adopting the software SIMPACK, and the electromechanical model is established by using the Simulink tool. The running of the LIM vehicle is simulated and the effect of the suspension is studied. The characteristics of the vehicle and the bogie structure with LIM are discussed. Based on the vehicle and track structure applied in Guangzhou metro line 4, the nonlinear critical speed of the LIM vehicle, curving performance, carbody response, and effect of air gap change on the performance of LIM are carefully studied. The vehicle system dynamic performance are simulated in time domain and the statistic results of rail/wheel force and LIM reaction plate based on the simulation results are given. The suitable structure and parameters of the vehicle with LIM bogie are also suggested.

2. Vehicle Dynamic Model

The LIM vehicle system is a complex multi-body system. In order to accurately simulate the dynamic performance of the vehicle system, a mathematical model with horizontal and vertical movements coupling is set up, and the nonlinearities in the vehicle system are taken into account, such as the nonlinear wheel-rail contact geometry, nonlinear wheel-rail interaction forces and nonlinear suspension parameters.

The following assumptions are adopted when building up the mathematical model of the vehicle system,

(1) The flexibility of wheelset, frame, carbody and other parts are much smaller than the elastic suspensions, the vehicle is considered as a multiple rigid body system composed of wheelsets, bogie frames, carbody, primary and secondary suspensions.

(2) The interactions between vehicles are weak, thus single vehicle model is considered.

The schematic diagram of the LIM vehicle system is shown as Fig.1. The total number of degrees of freedom of the vehicle system is 66, in which each body in the system such as the wheelset; frame, stator structure and carbody are considered to have six degrees of
freedom. The track elasticities in the lateral and vertical directions are also considered in the model.

The simulation is conducted by using SIMPACK, a multi-body dynamics simulation software, which is able to automatically formulate the motion equations of various types of mechanical systems, and makes calculations in frequency or time domain through different types of solvers. The software has a good input and output interface, a powerful post-processing function, an open programming environment, powerful interface module so that it can combine with other well-known design, control and finite element softwares to deal with a variety of mixed problems.

3. Linear Motor Model

Because the stator and rotor are in a vibration system, the vertical vibration of the vehicle directly results in air-gap changes. This change also leads to the changes of electromagnetic force (vertical attractive force) and horizontal force (traction) [18, 19]. In order to fully consider the relationship between the linear motor air-gap and the hoisting parameters, the LIM model must be set up.

3.1 LIM equivalent gap calculation

The LIM has an open slot along the vertical for the installation of coil, its air-gap magnetic field along the vertical distribution is non-homogeneity[20, 21]. With the analysis model of rotating electrical, equivalent air-gap without open slot is in place of the actual air-gap. The equivalent air-gap $g_e$ is given by the following form.

$$g_e = (1+k_s) k_l k_c g_0$$

(1)

where: $k_s$ is the saturation factor, $k_l$ is the coil magnetic flux leakage coefficient, $k_c$ is the Carter coefficient, $g_0$ is the electromagnetic air-gap.

3.2 LIM horizontal end effect

The first horizontal end effect is caused by the no-load air-gap magnetic field along the horizontal non-homogeneity distribution, which can be considered by air-gap coefficient. But the coefficient is so small that it can be ignored because the secondary conductor plate width of rail transit linear motor is greater than the primary core laminate.

The second horizontal end effect is the impact of secondary conductor plate to air-gap magnetic field along horizontal, and can be considered by coefficient when current flows through secondary conductor plate.

The compensation factor to conductive layer is given by the following form

$$K_a = 1 - \frac{\tanh(\pi w_p / t)}{(\pi w_p / t)[1 + \tanh(\pi w_p / t) \tanh(\pi c / t)]}$$

(2)

The compensation factor to introduction magnetosphere is expressed by the expression below.
\[ K_j = \left(1 - \frac{\tanh(\frac{\pi w_{pe}}{t})}{\pi w_{pe}/t}\right)^{-1} \]  

where \( t \) is the pole span, \( c \) is the width of secondary conductor unilaterally extending the primary core, \( w_p \) is half thick of primary core and \( w_{pe} \) is half width of equivalent primary core.

### 3.3 Nonlinear characteristics of induction plates

In order to obtain better speed-thrust characteristics and reduce costs, usually aluminum-iron composite reaction plate in the area of the track traction is adopted. Magnetic properties of iron are nonlinear, and its permeability is different. Flux penetration depth of iron is also nonlinear. With changing of slip frequency, material permeability and conductivity, its nonlinear change is given. Then the penetration depth \( \delta \) is given by

\[ \delta = \text{Re} \left[ \frac{1}{(\beta^2 + j s \omega u_1 \sigma_{e1})^{1/2}} \right] \]  

where \( \beta = \frac{\pi}{t}; s \) is slip rate; \( \omega \) is electric frequency; \( u_1 \) is permeability; \( \sigma_{e1} \) is equivalent gap.

Permeability also changes nonlinearly, the penetration depth and permeability through the iterative approach can be determined. If vertical and horizontal end effects are considered, under the given frequency, speed, slip rate, voltage and linear motor parameters, conductive plate thickness of reaction plate and magnetic induction intensity of magnetic field by the iterative approach of electromagnetic field parameters can be obtained. Finally, by the magnetic field analysis, electromagnetic force and vertical force can be obtained, as well as the linear induction motor equivalent circuit parameters.

### 3.4 Calculations on normal force and traction force of linear motor

According to electromagnetic field theory, magnetic strength \( H_a \) can be obtained along the vertical distribution. Suppose \( L \) is the linear motor length, then the electric force is calculated by

\[ F_z = \mu_0 w_p \int_{-L/2}^{L/2} \left| H_a(x) \right|^2 \, dx \]  

Because of longitudinal end effect, the magnetic field strength is different between front and back part of motor. So it is non-homogeneity along normal force distribution. The motor torque along nodding can be calculated by the following form

\[ M_y = \mu_0 w_p \int_{-L/2}^{L/2} \left| H_a(x) \right|^2 \, x \, dx \]  

The magnetic intensity distribution along the vertical changes with nodding, then \( M_y \) and \( F_z \) are changing. The electromagnetic thrust is given by the following form

\[ F_x = 0.5 w_{pe} \int_{-L/2}^{L/2} \text{Re} \left[ A_{my} B_{mc} \right] \, dx \]  

where \( A_{my} \) is line current density and \( B_{mc} \) is equivalent traveling wave magnetic field, and at the speed of \( v \).

To consider the longitudinal end effect, a vertical braking force is as follow:

\[ F_{z2} = 0.5 w_{pe} \int_{-L/2}^{L/2} \text{Re} \left[ A_{my} \exp[j(\omega t - \pi v/t)] \cdot B_{mc} \cdot \exp[-\pi v/a_1] \exp[j(\omega t - \pi v/t + \Theta)] \right] \, dx \]  

where \( \Theta \) is the angle between traveling wave magnetic fields and primary current, \( v \), and \( a_1 \) are coefficients.

In order to verify the correctness of the above proposed model, the electromagnetic mechanics model and finite element model of LIM were established.
The forces calculated by electromagnetic field theory and finite element method are shown in the Fig.2. It can be seen from the results that the proposed model is correct. Comparing with the finite element model, the electromagnetic mechanics model has the feature of less computer time and fast calculation speed, and can meet the requirement of real-time dynamic simulation.

The role of equivalent circuit is to solve the electric current, but the current also determines the magnetic field. Thus equivalent circuit parameters can be obtained by using iterative method to solve this closed-loop circuit.

Under a certain speed, because the current changes slightly, and a shorter step integral length, a given boundary condition (including current) is considered to determine magnetic field, equivalent circuit parameters and magnetic field strength, and to calculate equivalent current (as calculation conditions in next step).

4. Simulation Results

The aim of simulation is to obtain reasonable parameters for ensuring passenger
comfort and improving linear motor efficiency at the same time. In order to better simulate the dynamic responses of the vehicle system, let the linear metro vehicle running on an enough long random track, and the accelerations of the carbody are sampled after the vehicle is operated a certain distance. Under the condition of speed 80km/h, the increase of primary vertical stiffness and damping, secondary vertical stiffness and framework hinged stiffness have major influence on the LIM air-gap.

The increase of primary vertical stiffness has little influence on the ride quality, but will greatly influence the changes of air-gap. As primary vertical stiffness increases, the average maximum air-gap change gradually decreases. When the vertical stiffness larger than 2.5MN/m, the influence tends to small, which is shown in Fig. 5. With the increase of primary vertical damping, the air-gap change gradually decreases, the lateral ride quality also improves, as shown in Fig.6. It is found that large damping is necessary for the LIM bogie.

Framework hinged stiffness has large influence on air-gap. Smaller hinged stiffness makes larger average maximum air gap. If the stiffness is about 10MNm/rad, minimum of air-gap can be obtained. Then with stiffness increased, the average maximum air-gap change gradually increases, as shown in Fig.7. With the increasing of the second vertical stiffness, the air-gap increases gradually, as shown in Fig.8.

![Fig.5 Influence of primary vertical stiffness on induction plate air-gap](image)

![Fig.6 Influence of primary vertical damping on induction plate air-gap](image)

![Fig.7 Influence of framework hinged stiffness on induction plate air-gap](image)
5. Conclusions

(1) The air gap of LIM is an important indicator to reflect the linear motor vehicle transmission efficiency and operating quality. The model of vehicle with LIM bogie is the foundation for the research of constant-gap control and linear metro vehicle system dynamics.

(2) The primary vertical stiffness and damping, second vertical stiffness and hinged stiffness of framework affect air-gap significantly. It is important to study carefully how to reduce the air-gap variation and ensure the passenger comfort.

(3) It is necessary to research the constant-gap control because of the contradiction between reducing air-gap variation and ensuring passenger comfort.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (No. 50675181), the Traction Power Laboratory Foundation (No.2008TPL-T02) and the New Century Excellent Talent Foundation (NCET-07-0717). The ‘TIANFU Excellent youth’ Science Innovation Foundation (NO.2).

References

(9) Murakami Akinobu. Trends in urbanization and patterns of land use in the Asian mega


