Calculation of the Transmission of Singly Scattered Gamma-Rays through Finite Thin Slabs

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The intensity and number of transmitted singly scattered photons were calculated for 1.0, 2.0, 4.0, 6.0, 8.0 and 10.0 MeV γ-rays normally incident on slabs of lead, iron, concrete and water varying from 1 to 4 mean free paths in thickness.

Energy build-up factors were obtained by integrating them using Simpson's formula. The calculation for composite shields included 1.0, 3.0 and 6.0 MeV γ-rays normally incident on a lead-water shield. The results obtained show good agreement with the values calculated by the Monte Carlo method and the moments method.

I. INTRODUCTION

The interaction of γ-ray with matter has been investigated for a long time, and many theoretical researches have been carried out of recent with the development of electronic computers.

Three methods are most widely used in investigating the penetration and diffusion of scattered γ-rays in matter: (1) moments method; (2) successive scatterings; and (3) Monte Carlo method. Many calculations have been carried out by a large number of authors. The Monte Carlo method is the most effective for the penetration of γ-rays through finite thin slabs, and the moments method is also adequate for an infinite homogeneous medium. However, for the purpose of quickly estimating shielding thickness required to reduce γ-ray intensity, a simpler method than the Monte Carlo is desirable for practical purposes.

Peebles and Plesset calculated the number of scattering events in finite lead slabs with incident γ-ray energy of 2.55 MeV. It is recognized that most of transmitted scattered γ-rays are singly scattered for a slab with thickness less than 4 mean free paths.

In this paper, a very simple calculating method by singly scattered γ-ray approximation and its application to a composite shield are described. The results obtained by this method are compared with the values calculated by the Monte Carlo method and the moments method.

II. PROCEDURE OF CALCULATION

The geometry selected for the investigation is shown in Fig. 1. Photons are normally incident on a finitely thick slab of infinite extent with plane parallel faces. Some of the singly scattered photons emerge from the rear face of the slab without a second interaction with matter.

The probability that a photon will be transmitted through the slab after undergoing single scattering with an emergent angle θ per unit solid angle, $P(E_0, θ)$ is given by

$$P(E_0, θ) = \int_{-\infty}^{R'} \exp(-\mu_0 x) \frac{M d\sigma}{d\Omega} \cdot \exp(-\mu (t-x) \sec θ) dx$$

$$= \frac{M d\sigma}{\mu \sec θ - \mu_0} \{ \exp(-\mu_0 t) - \exp(-\mu t \sec θ) \},$$

(1)

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where \( \frac{d\sigma}{d\Omega} = \frac{r_s^2}{r_s^2 + 1 + \alpha (1 - \cos \theta)} \)
\[ \left( 1 + \cos^2 \theta + \frac{\alpha \theta(1 - \cos \theta)^2}{1 + \alpha (1 - \cos \theta)} \right). \] (2)

The number and energy flux of singly scattered photons in an emergent angle between \( \theta \) and \( \theta + d\theta \) are given by
\[ 2\pi N(\theta) \sin \theta \frac{d\theta}{\cos \theta} = 2\pi N_0 P(E_0, \theta) \sin \theta \frac{d\theta}{\cos \theta} \] (3)
\[ 2\pi I(\theta) \sin \theta \frac{d\theta}{\cos \theta} = 2\pi I_0 P(E_0, \theta) \sin \theta \frac{d\theta}{\cos \theta} = \frac{2\pi N_0 E}{E_0} \frac{P(E_0, \theta) \sin \theta}{\cos \theta} d\theta, \] (4)

where \( \frac{E}{E_0} = \frac{1}{1 + \alpha (1 - \cos \theta)} \). (5)

The total number and energy flux of radiation transmitted through the slab after undergoing single scattering are
\[ N = 2\pi \int_0^{\pi/2} N(\theta) \sin \theta \frac{d\theta}{\cos \theta} = 2\pi N_0 \int_0^{\pi/2} P(E_0, \theta) \sin \theta \frac{d\theta}{\cos \theta}, \] (6)
\[ I = 2\pi \int_0^{\pi/2} I(\theta) \sin \theta \frac{d\theta}{\cos \theta} = 2\pi I_0 \int_0^{\pi/2} P(E_0, \theta) \sin \theta \frac{d\theta}{\cos \theta}. \] (7)

The number and energy build-up factors for singly scattered photons are
\[ B_n = 1 + \frac{N/N_0}{\exp(-\mu \delta)}, \] (8)
\[ B_e = 1 + \frac{I/I_0}{\exp(-\mu \delta)}. \] (9)

It is convenient to refer to published tables and graphs for the calculation. Several methods of treating the build-up factor for composite shields have been suggested. Some of these may be adopted as follows:

1. For a light material followed by a heavy material, use the build-up factor of the heavy material only.
2. For a heavy material followed by a light material, use the product of individual build-up factors. This has also been suggested for more than 2 materials, but in many cases this may lead to a very conservative answer.

\[ B = \prod B_i \quad (i=1, 2, 3 \cdots n) \] (10)

For instance, the calculation for a lead-water shield can be performed as follows.
\[ B(Pb) \cdot \exp(-\mu_{Pb} f(Pb)) \cdot B(H2O) \cdot \exp(-\mu_{H2O} f(H2O)) \]
\[ = B(Pb) \cdot B(H2O) \cdot \exp[-(\mu_{Pb} f(Pb) + \mu_{H2O} f(H2O))]. \] (11)

The transmitted singly scattered photons through the lead slab are obliquely incident on the water slab. In the calculations, however, the photons were assumed to be normally incident on the water slab, and without energy attenuation. The singly scattered photons thus entering the water slab were then treated in a manner similar to the case of one material slab discussed earlier. The foregoing assumption leads to an overestimated result for the singly scattered photons, but the result may be considered reasonably acceptable considering the neglect of the contribution by multiple scattered photons.

### III. RESULTS AND DISCUSSION

The calculation was carried out for slabs of lead, iron, concrete and water. They are the most commonly used materials for radiation shielding.

Some of the calculated values for the probabilities of the transmission of singly scattered photons, \( P(E_0, \theta) \), are shown in Fig. 2.

Energy build-up factors were obtained by this method using Simpson's formula. The energy build-up factors for 1.0, 3.0 and 6.0 MeV \( \gamma \)-rays normally incident on the one material slab shield are shown in Fig. 3 (a)~(c). The values for 4.0 MeV \( \gamma \)-rays incident on concrete are similar to those of water, and the values for 2.0 and 1.0 MeV \( \gamma \)-rays incident on concrete and water are similar to those of iron. The calculation for composite shields was carried out for a lead-water shield. The thickness of the lead slab is 11.58 cm and that of the water slab is 35.81 cm. The energy build-up factors for 1.0, 3.0 and 6.0 MeV \( \gamma \)-rays normally incident on the shield are shown in Fig. 4 (a)~(c).
Transmission probability of the singly scattered photon, \( P(E_0, \theta) \), as a function of slab thickness (mean free paths) in the case of 1.0 MeV \( \gamma \)-rays normally incident on an iron slab.

Fig. 2

The values obtained by the moments method\(^{(5)}\) and the Monte Carlo method\(^{(6)(7)}\) are also plotted in these figures for the purpose of comparison. Considering the simplicity of the foregoing method, the results obtained show good agreement with the values calculated by the more elaborate methods.

The approximation can be used for the case of \( \gamma \)-rays above 2.0 MeV incident on slabs less than 3 mean free paths in thickness. For \( \gamma \)-rays under 1.0 MeV, it can be also used for iron and lead slabs less than 1 mean free path in thickness.

**Symbols**

- \( t \) : slab thickness (cm)
- \( \mu_0 \) : linear absorption coefficient of \( \gamma \)-ray with energy \( E_0 \) (cm\(^{-1}\))
- \( x \) : path length of an incident photon in the slab before scattering (cm)
- \( M \) : number of electrons per unit volume of a slab (electrons/cm\(^3\))
- \( d\sigma/d\Omega \) : differential Klein-Nishina cross section for scattering of a photon in the direction \( \theta \) per unit solid angle (cm\(^2\)/steradian)
- \( \theta \) : emergent angle between the direction of an incident photon and that of the emergent photon (degrees)
- \( N(\theta) \) : number of transmitted singly scattered photons in the direction \( \theta \) per unit solid angle
- \( N_0 \) : number of incident photons
- \( I(\theta) = EI(\theta) \) : intensity of singly scattered radiation in the direction \( \theta \) per unit solid angle
- \( L_0 = E_0 N_0 \) : intensity of the incident photon beam
- \( E \) : scattered photon energy (MeV)
- \( E_0 \) : incident photon energy (MeV)
- \( r_0 \) : electron radius (cm)
- \( \alpha = E_0/mc^2 \) : energy of incident photon in \( mc^2 \)
- \( m \) : electron rest mass
- \( c \) : velocity of electromagnetic radiation in vacuum
- \( B_N \) : number build-up factor
- \( B_N^* \) : energy build-up factor

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**References**

Fig. 3(a)~(f) Energy build-up factors as a function of the thickness (mean free paths) of a slab shield. The marks plotted in the figures show the values obtained by the moments method\(^5\) and the Monte Carlo method\(^7\) as follows.

<table>
<thead>
<tr>
<th>Material</th>
<th>Present method</th>
<th>Moment method</th>
<th>Monte Carlo Method</th>
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<td>○</td>
<td>△</td>
<td></td>
</tr>
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<td>●</td>
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<tr>
<td>water</td>
<td>○</td>
<td>●</td>
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Fig. 4 (a)~(c) Energy build-up factors as a function of the thickness (cm) of a finite lead-water slab shield. The thickness of the lead slab is 11.58 cm and that of the water slab is 35.81 cm. Circles indicate the calculated values obtained by the present method, and the line represents the Monte Carlo calculation\(^9\).