Measurement of Neutron Life in a D₂O-System by Neutron Fluctuation

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The neutron life has been determined in a D₂O system by combining observation of the inherent fluctuations which occur in the neutron population and reactivity measurements by period method.

The ratio of variance to mean number of counts was measured as a function of counting intervals. Short counting-time intervals were chosen in the experiment to eliminate contribution of delayed neutrons. Also, the variance was measured in the subcritical state, to eliminate the counting loss due to the dead time of the detector.

The decay constant for the critical state has been determined by extrapolation of the observed relation between the decay constant and reactivity.

INTRODUCTION

The pile-oscillator and the pulsed neutron methods are the most commonly used in determining the kinetic parameters of a reactor. These methods give precise values but require complicated apparatus.

There is a time correlation between the fission neutrons in a reactor. Analysis of the correlated neutrons is important for obtaining the reactor kinetic parameters. One of the methods of analysis, called the Rossi-α method, which analyzes the time intervals between successive neutrons, is suitable for a system in which the fission rate is very low. For high fission rates, frequency analysis of the output current of an ion chamber is preferable. In this case, the detector system must be calibrated by random noise. The radiation from long-lived isotopes such as Co, is useful for this calibration. Fission rates in the intermediate range are best analyzed from the counts in time intervals, using statistical techniques of stochastic processes. Originally this method was used to obtain the fission neutron dispersion $\nu$.

Statistical analysis of the number of neutrons detected in short time intervals has been applied to measurements of the decay constant $\alpha$ in the D₂O thermal system. The experiment was performed in several sub-critical states. Also, the reactivity was measured by the period method. A combination of these techniques was used to determine the neutron life in the Aqueous Homogeneous Critical Facility.

THEORY

Any two neutrons detected at times $t_1$ and $t_2$ in a reactor can be classified into either correlated or accidental pair, depending on whether or not they have a common ancestor. The total number of the two classes of pairs detected during intervals $dt_1$ and $dt_2$ is:

$$F^2 \varepsilon^2 dt_1 dt_2 + \frac{(\nu^2 - \nu)}{2\alpha \tau^2} F^2 \varepsilon^2 e^{-\alpha (t_1 - t_2)} dt_1 dt_2,$$

where $F$: Fission rate
$\varepsilon$: Number of counts per fission
$\tau$: Generating time
$\alpha$: Decay constant.

The number of pairs in a time interval $t$, namely $C(C-1)/2$, where $C$ is the counts during $t$, can be obtained by integrating Eq. (1) over the range $t_1 = 0$ to $t_2 = t$.

It is more convenient to write the above result in the form:

$$\frac{C^2 - (C^2)}{C} = 1 + \frac{\varepsilon (\nu^2 - \nu)}{\alpha^2 \tau^2} \left[ 1 - \frac{1 - e^{-\alpha t}}{\alpha t} \right].$$

When there is no correlation with time, the distribution of $C$ becomes a pure poisson distribution, and $(C^2 - (C^2))/C = 1$. Experimentally this relation can be verified by counting the radiation from a long-life...
radioisotope during a suitable time interval.

Equation (2) is not correct for a time interval in which there is contribution of delayed neutrons. The necessary correction was given by Bennet(6) and the contribution of delayed neutrons was observed experimentally by Albrecht(7).

\[
\frac{C_{t}^2 - (C_{0})^2}{C} = 1 + \sum_{i} K_{i} \left[ 1 - \left( 1 - e^{-\omega_{i}t} \right) \right],
\]

where \(-\omega_{i}\) are the roots of the Inhour equation.

**EXPERIMENT**

An experiment was performed in the Aqueous Homogeneous Critical Facility to measure the ratio of the variance to the mean number of counts as a function of counting time interval. The system structure is shown in Fig. 1, and details of the cores used are given in Table 1. The spherical reflector of 1,500 mm inner diameter surrounds the spherical core tank filled with heavy water solution of 20% enriched uranyl sulfate.

![Fig.1 Structure of D2O System](image)

A BF3 counter (Nuclear Chicago Model NC-207) was inserted near the surface of the spherical core through the neutron source sleeve. The detecting system is quite orthodox, as shown in Fig. 2. The timer controls the counting interval. A neutron source, is not required for the experiment, the counts in this D2O-system being sufficiently provided by spontaneous fissions and photo-neutrons. The counts were punched on IBM cards. For each time interval, about 1,500 counting data were used. The ratios of the variance to the mean number of counts were calculated with an IBM 650.

![Fig.2](image)

**Table 1 Details of the Core Systems**

<table>
<thead>
<tr>
<th></th>
<th>A-core</th>
<th>B-core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core-radius (cm)</td>
<td>40.0</td>
<td>33.0</td>
</tr>
<tr>
<td>Reflector thickness (cm)</td>
<td>35.0</td>
<td>42.0</td>
</tr>
<tr>
<td>Fuel concentration in the core (^{235}\text{U} \text{g/l})</td>
<td>7.04</td>
<td>13.44</td>
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<tr>
<td>Boron concentration in the reflector (\text{B mg/l})</td>
<td>182.6</td>
<td>69.0</td>
</tr>
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</table>

These ratios obtained at various rod positions in the A- and B-core systems whose core radius are 40.0 and 33.0 cm respectively, are shown in Figs. 3 and 4 respectively.

The values of \(a\) and \(\varepsilon (\bar{p} - \bar{D})/\tau^2\) were determined at rod positions 54 and 50 in the A- and B-core systems respectively by superposing the theoretical curve on the experimental. The other \(a\) values were determined from their heights, i.e. \(\varepsilon (\bar{p} - \bar{D})/a^2 \tau^2\). The decay constants determined by the saturated heights are approximate, because the space-energy independent theory is not sufficiently accurate for the reactor with reflector(9). The delayed neutrons do not contribute in the short intervals of the present case namely, 10~250 ms.

![Fig.3 Variance/Mean -1, in A-core System](image)
There is the following approximate relation between the decay constant and reactivity.

\[ \alpha = \alpha_e \left( 1 - \beta \frac{\rho}{\beta} \right) \left( 1 - \frac{1}{\rho} \sum \frac{\beta_i}{\beta} \lambda_i \right) \]

\[ \approx \alpha_e \left( 1 - \beta \frac{\rho}{\beta} \right), \tag{4} \]

The experimental relation between the observed decay constant and reactivity determined by the period method is shown in Figs. 5 and 6 for the A- and B-core systems, respectively. The constant \( \alpha_e \) can be determined by extrapolating the decay constant to \( \rho/\beta = 0 \).

The experimental relation in Figs. 5 and 6 is linear but there is a difference of gradient between the experimental curve and that from Eq. (4).

By assuming \( \beta_{\text{att}} = 0.00875 \) which takes account of the photo-neutron effect, the following values for neutron life in the A- and B-core systems were obtained.

\[ l_A = 1.19 \pm 0.31 \text{ ms} \]
\[ l_B = 0.621 \pm 0.093 \text{ ms} \]

The error comes mainly from the decay constant rather than from the reactivity.

**RESULT AND DISCUSSION**

Neutron life in the D_2O-system was determined by measurement of the ratio of the variance to mean number of counts as a function of counting time interval, combined with reactivity measurement by the period method. High count rates were observed even in sub-critical state due to the presence of photo-neutrons. High background makes it difficult to measure the decay constant in a homogeneous D_2O-system by the pulsed neutron method. In efforts to eliminate the counting loss due to detector dead time, the experiment was performed in sub-critical state without neutron source.

The decay constant was easily determined for small \( \alpha \) and for other decay constants through the application of theoretical relations. Neutron life was determined by extrapolating the relation obtained between decay constant and reactivity. This method is useful for thermal reactors where neutron lives are long.

Even in highly sub-critical states, the decay constants will still be obtained if the detector...
is placed in a position of high neutron importance. Experimental precision will be improved by recording the neutron pulses and will further permit measuring the contribution of delayed neutrons.

Stribel(10) reported that the space-energy independent theory of neutron fluctuation is not sufficiently accurate for a reactor with reflector. The product of the measured quantities $\alpha$ and $\langle \epsilon (\bar{\nu}^2 - \bar{\nu}) \rangle / 2\alpha \tau^2$ was not a constant in his experiment. But it is assumed in the present analysis that the space-energy independent theory is valid in approximation. Both decay constants and saturated heights, namely $\langle \epsilon (\bar{\nu}^2 - \bar{\nu}) \rangle / \alpha^2 \tau^2$, being determined separately, it is desirable to compare these values with those determined from the simple space-energy independent theory.

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References