Dynamics and Control of Uranium Product Evaporator

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It was established that the dynamics of an evaporator show markedly asymmetric responses. These phenomena are closely related to the response of the liquid level which directly affects the heat capacity. What is more, the time constants of an evaporator are so large that the transfer functions expressing uranium product evaporator are better approximated by a form representing no self-regulation.

Examination of the controllability aspects of three different control algorithms, i.e. control by boiling point raising, cascade control and multivariable control, resulted in the conclusion that the last-mentioned algorithm is superior to the two others for controlling the uranium concentration which is the most important element in uranium product evaporation.

KEYWORDS: dynamics, control, asymmetric response, controllability, control algorithm, uranium product evaporator, multivariable control, optimal control law

I. INTRODUCTION

In the chemical industry, various types of evaporator have long been used, and the principles governing their static design would appear already well established, while their dynamics and control are considered easily manageable due to the simplicity of their operations and their slow responses to disturbances.

In the field of nuclear fuel reprocessing, however, the product materials are very expensive, absolute safety against criticality must be maintained, and completely remote operations are required due to high radioactivity. This makes it necessary to control the operations of process equipment such as extractor and evaporator with much higher stability and precision, and studies on the dynamics and control of processes markedly increase their importance.

Evaporator dynamics have been analyzed by several workers(3)(5)(6), but little has been reported on direct application to the case of uranium product evaporators. These past studies have covered the control algorithms of an evaporator such as control by boiling point raising(3), cascade control(2) and multivariable control(3).

The present paper deals with the dynamics and control of uranium product evaporators. Analysis is undertaken without unreasonable linearization. The three control algorithms described above are compared in respect of their faculty to control the uranium concentration with adequate precision.

II. BASIC EQUATIONS AND TRANSIENT CHARACTERISTICS OF URANIUM PRODUCT EVAPORATORS

Here, natural circulation evaporators such as of thermosiphon and basket types are considered. In such evaporators, the solution boils vigorously so that perfect mixing can be assumed.

Using the nomenclature listed at the end of this paper, the basic equations of evaporator dynamics are generally expressed from Fig. 1 as follows.

Solute mass balance:

\[ A \frac{d}{dt}(\rho \cdot z) = \rho \cdot Q_f - \rho \cdot Q_d - \rho \cdot Q_e \] (1)

UO₂(NO₃)₂ mass balance:

\[ A \frac{d}{dt}(m\rho \cdot z) = m\rho \cdot Q_f - m\rho \cdot Q_d \] (2)

HNO₃ mass balance:

\[ A \frac{d}{dt}(m\rho \cdot z) = m\rho \cdot Q_f - m\rho \cdot Q_d \] (3)

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Fig. 1 Input-output relation in uranium product evaporator

Heat balance:

$$A \frac{d}{dt}(C_i \rho_i \times T_i) = C_f \rho_f Q_f T_f - C_t \rho_t Q_t T_t - (\lambda + C_v T_i) \rho_c Q_c + q - q_L$$ (4)

Boiling point raising:

$$T_i = f_1(m_{i_u}, m_{i_h}) = T_{i_0} + k_1 m_{i_u} + k_2 m_{i_h}$$ (5)

Relation between concentration and density:

$$\rho_i = f_2(m_{i_u}, m_{i_h}) = \rho_{i_0} + k_3 m_{i_u} + k_4 m_{i_h}$$ (6)

In Eq. (5), $T_i$ also depends upon drum pressure and static pressure head, but we assume that the evaporator is open to atmosphere, and hence there is only negligibly small rise of boiling point due to changes in static pressure head. And, further in Eq. (6), $\rho_i$ also depends upon $T_i$, but its effect can be neglected because the deviation of $\rho_i$ with $T_i$ is less than 5.0% in the range $100^\circ C \leq T_i \leq 120^\circ C$.

The values of $k_1$ to $k_4$ in Eqs. (5) and (6) were obtained by least-square fit to data from Ref. (1). The error introduced by this approximation is less than 10% in the range $100^\circ C \leq T_i \leq 120^\circ C$, $1.1 \leq \rho_i \leq 2.0 \text{kg/l}$, $0 \leq m_{i_u} \leq 80 \%$ and $0 \leq m_{i_h} \leq 20 \%$. In Eq. (3), we also neglect the evaporation of HNO3.

Since, by using Eqs. (1) to (6), the two variables selected arbitrarily from $T_i$, $\rho_i$, $m_{i_u}$ and $m_{i_h}$ can represented by the two remaining variables, this evaporator system can be treated as having only four independent state variables, for example, $T_i$, $\rho_i$, $z$ and $Q_v$.

In Fig. 2, we show the transient characteristics of an evaporator at start-up. Data used for the calculation are listed in Table 1. In Fig. 2, it can be seen that $Q_v$, the rate of evaporation, is little affected by $m_{i_u}$ and $T_i$ except during the first 2 or 3 hr. And in Eqs. (1) to (4), the terms that contain $Q_f$ and $q - q_L$ are so dominant over all the others that $Q_v$ is largely determined by $Q_f$ and $q - q_L$. This fact facilitates the start-up operation of an evaporator.

![Fig. 2 Start-up behavior of uranium product evaporator](image)

Table 1 Data used for calculation of uranium product evaporator start-up

<table>
<thead>
<tr>
<th>$Q_f$ (l/hr)</th>
<th>$T_f$ (°C)</th>
<th>$\rho_f$ (kg/l)</th>
<th>$C_f$ (kcal/kg·°C)</th>
<th>$k_1$ (°C/%)</th>
<th>$k_2$ (°C/kg/l)</th>
<th>$\lambda$ (kcal/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>20.0</td>
<td>1.20</td>
<td>0.9</td>
<td>0.90</td>
<td>0.0084</td>
<td>538.8</td>
</tr>
</tbody>
</table>

In Fig. 3 and 4 are shown the calculated responses of $m_{i_u}$, $T_i$ and $z$ for a step change of $Q_f$ and $q - q_L$, respectively. For each case, $m_{i_u}$ and $T_i$ show markedly asymmetric responses. This asymmetry can be explained by the fact that the change in $m_{i_u}$ and $T_i$ is different between the cases of increasing and decreasing heat capacity of the liquid, due to the change in the liquid level $z$. In fact, the responses of $m_{i_u}$ and $T_i$ become almost symmetric when the change in $z$ is small. Thus, if $z$ is not maintained
within certain limits, the transient characteristics of \( m_i^o \) and \( T_i \) may change quite significantly with change in \( z \), and, in such a case, control of the evaporator may become more difficult. This means that a fairly strict control of the liquid level is indispensable for precise control of the evaporator.

### III. COMPARISON OF CONTROL ALGORITHMS

Jobe et al.\(^{(2)}\) have proposed the cascade control system for thermosiphon evaporators. In this system, the steam flow \( q - q_{L} \) to the evaporator is made proportional to the feed rate \( Q_f \) by a magnetic flowmeter-controller, the set point of which is regulated by a density controller. If we apply this algorithm to an evaporator, the dynamics of which is expressed by Eqs. (1)~(6), we have the block diagram shown in Fig. 5. According to Jobe, \( K_C_2 \) in this figure is determined by

\[
K_C_2 = \frac{G_1(s)}{K_C_1} \exp(-Ls), \tag{7}
\]

and \( K_C_1 \) is maintained within the limits dictated by stability requirements.

From Fig. 5, we obtain the open-loop transfer function

\[
G(s) = K_C_2 \cdot K_{IR} \cdot K_C_1 \cdot G_{11}(s) \cdot \exp(-Ls). \tag{8}
\]

We can determine the value of \( K_C_1 \) from Eq. (8) using the Ziegler-Nichols method.

To obtain \( G_{11}(s) \) and \( G_{1i}(s) \), each variable in Eqs. (1)~(6) is expressed by the sum of two parts representing respectively its steady state and its deviation therefrom, and Laplace transformation is applied by neglecting the higher terms than the first order. Their time constants, however, are so large and their responses are so asymmetric (Figs. (3), (4)) that they are better approximated by a form having no self-regulation.
Thus $G_{ij}(s)$ becomes
\[ G_{ij}(s) = \frac{c_{ij}}{s}. \] (9)

Jobe et al. do not give consideration to level control, which, as mentioned a little earlier, is indispensable for ensuring adequate symmetry. We therefore apply an on-off level control with dead zone, as shown in Fig. 6. In further discussions, this control algorithm will be termed of “Algorithm A”.

![Diagram of Algorithm A](image)

**Fig. 6** On-off control of liquid level $z$

The second method of evaporator concentration control is by boiling point raising\(^{[5]}\). By this method, one can control the concentration by detecting the temperature of the evaporating liquid. A block diagram of this method is presented in Fig. 7. Here, $K_{C3}$ can be determined by the Ziegler-Nichols method. One can control the liquid level in the same manner as shown in Fig. 6. We designate this “Algorithm B”.

![Diagram of Algorithm B](image)

**Fig. 7** Block diagram of Algorithm B

As a third “Algorithm C”, we propose a multivariable control system in which we use Dynamic Programming for optimal control\(^{[3][4]}\). This multivariable control system is shown in Fig. 8, and, we obtain the following discretized equations for state changes of the system.

\[
\begin{align*}
\Delta m_i^{U} &\left[ (n+1)\tau \right] \\
= &\Delta m_i^{U}(n\tau) + \tau c'_{11} \Delta q(n-M\cdot\tau) \\
&+ \tau c'_{12} \Delta Q_f(n-M\cdot\tau), \quad (10) \\
\Delta m_i^{H} &\left[ (n+1)\tau \right] \\
= &\Delta m_i^{H}(n\tau) + \tau c'_{21} \Delta q(n-M\cdot\tau) \\
&+ \tau c'_{22} \Delta Q_f(n-M\cdot\tau), \quad (11)
\end{align*}
\]

where $\Delta$ is the deviation from steady state and $\tau$ the sampling time.

**Fig. 8** Block diagram of Algorithm C

The performance index to be minimized can be represented by
\[
J = \sum_{n=1}^{\infty} \left[ q_1(\Delta m_i^{U}(n-1))^2 + q_2(\Delta m_i^{H}(n-1))^2 \\
+ \lambda_1(\Delta q(n))^2 + \lambda_2(\Delta Q_f(n))^2 \right], \quad (12)
\]
where $q_1, q_2, \lambda_1$ and $\lambda_2$ are weighting coefficients.

The optimal control law minimizing the performance index $J$ subject to Eqs. (10) and (11) is
\[
\begin{bmatrix}
\Delta q(n) \\
\Delta Q_f(n)
\end{bmatrix} = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix} \\
\begin{bmatrix}
\Delta m_i^{U}(n) + \tau c'_{11} \sum_{i=1}^{M} \Delta q(n-i) \\
\Delta m_i^{H}(n) + \tau c'_{21} \sum_{i=1}^{M} \Delta q(n-i)
\end{bmatrix}
\]
where $B_{ij}$ are elements of the feedback matrix.
The state variables $m_t^U$ and $m_t^H$ on the right-hand side of Eq. (13) are not easily subject to direct continuous observation, they can nevertheless be determined continuously by using Eqs. (5) and (6) with continuous detection of $\rho_t$ and $T_t$. The control algorithm represented by Eq. (13), is thus practicable. Level control also can be implemented by using $Q_d$ in the same manner, as shown in Fig. 6.

To compare the degrees of controllability obtained by these algorithms, we calculated the responses obtained when the three different control algorithms mentioned above are used to restore the evaporator to its normal state, $m_t^U = 54.9$, $m_t^H = 3.00$ from a disturbed state of $m_t^U = 52.0$, $m_t^H = 3.15$. The results are shown in Fig. 9, the data used for the calculation being as listed in Table 2.

![Graph](image_url)

**Fig. 9** Comparison between three different control algorithms in respect of controllability

It is seen from Fig. 9 that, compared with the case of no control, any of the alternative control algorithms is quite effective in restoring the system variables from disturbed to normal state. Upon detailed individual examination of the three algorithms, the Algorithm B shows good performance for $T_t$ control, but leaves an offset with respect to $m_t^U$ control, which is the primary element of evaporator control.

Algorithm A performs satisfactorily for $\rho_t$ control, but this algorithm also retains an offset with respect to $m_t^U$ control even though almost negligibly small. The cause of these offsets is clear by attributable to the neglect of $m_t^H$ in Eqs. (5) and (6) in these two algorithms. These algorithms cannot therefore be expected to ensure satisfactory evaporator control when the amount of $m_t^H$ is not negligible.

Algorithm C does not neutralize deviations of $\rho_t$ and $T_t$, but it can restore $m_t^U$ to normal level and is free of offset with respect to $m_t^U$, thanks to the multivariable control adopted in this algorithm. Thus it is evident that a multivariable control system is better for precise uranium concentration control of evaporators.

**IV. Conclusion**

In the study described above, we have applied the results obtained from experiments on a K$_2$CO$_3$ evaporator$^{(3)}$ to the case of a uranium product evaporator, treating a solute consisting of two components, UO$_2$(NO$_3$)$_2$ and HNO$_3$. Calculations show that the uranium product evaporator has markedly asymmetric dynamics for uranium concentration control.
concentration and boiling temperature, which agrees with the general behavior observed in our experiment (3).

We can state that these phenomena result from change brought to the liquid level which induces increase or decrease of heat capacity. The dynamics of an evaporator are therefore not self-regulatory, on account of this change in liquid level, nor are the individual controlled variables. Hence, the transfer functions of an uranium product evaporator should be represented by a form having no self-regulation.

One must further note that two more controlled variables, for example, the product density \( \rho_l \) and the boiling temperature \( T_l \), are required for precise control of uranium concentration. In general, the number of the controlled variables must be equal to the number of components of the solute, in order to control an evaporator precisely.

It is thus indispensable for the control algorithm of an uranium product evaporator to embody multivariable control. When the concentration of \( \text{HNO}_3(m_l^p) \) cannot be neglected respect to \( m_l^p \), a multivariable control algorithm such as Algorithm C is required for evaporator operation.

Between the conventional control algorithms A and B, \( \rho_l \) control (Algorithm A) is preferable to \( T_l \) control (Algorithm B) for uranium product evaporators, from the viewpoint of controllability, because \( \rho_l \) is less affected by \( m_l^p \) than \( T_l \) and also, \( \rho_l \) correlates with \( m_l^p \) better than \( T_l \).

(Nomenclature)

\[ A : \text{Cross-sectional area of evaporator vessel (l/cm)} \]
\[ B_{ij} : \text{Element of feedback matrix} \]
\[ C : \text{Specific heat (kcal/kg°C)} \]
\[ c_{ij}, c'_{ij} : \text{Floating speed defined by Eq. (9)} \]
\[ G(s) : \text{Open loop transfer function defined by Eq. (8)} \]
\[ G_{ij}(s) : \text{Transfer function defined by Eq. (9)} \]
\[ K_r : \text{Gain constant of flow transmitter} \]
\[ K_s : \text{Gain constant of setpoint adjustment} \]
\[ KC_i : \text{Proportional gain of the controller} \]
\[ k_1 \sim k_4 : \text{Coefficient defined by Eqs. (5), (6)} \]
\[ L : \text{Dead time (hr)} \]
\[ m_l^p : \text{Concentration of } \text{UO}_2(\text{NO}_3)_2 (\%) \]
\[ m_l^H : \text{Concentration of } \text{HNO}_3 (\%) \]
\[ q : \text{Heat flux (kcal/hr)} \]
\[ q_s : \text{Heat loss (kcal/hr)} \]
\[ q_1, q_2 : \text{Constant used in Eq. (12)} \]
\[ Q : \text{Flow rate (l/hr)} \]
\[ s : \text{Laplacian operator} \]
\[ t : \text{Time of operation (hr)} \]
\[ T : \text{Temperature (°C)} \]
\[ T^0 : \text{Constant defined by Eq. (5)} \]
\[ z : \text{Liquid level (cm)} \]
\[ \rho : \text{Density (kg/l)} \]
\[ \rho_l^0 : \text{Constant defined by Eq. (6)} \]
\[ \lambda : \text{Latent heat of vaporization (kcal/kg)} \]
\[ \lambda_1, \lambda_2 : \text{Constant used by Eq. (12)} \]

(References)