Integral-Versions of Some Kinetic Experiments for Determining Large Negative Reactivity of Reactor

Yoshihiko KANEKO

Japan Atomic Energy Research Institute*

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Theoretical treatments, including the methods of source-multiplication, rod-drop and source-jerk, are suggested for dealing with the spatial effects observed in several kinetic experiments for determining large negative reactivity in a reactor. An analysis by means of kinetic eigenfunctions is made on the kinetic behavior of the reactor, when these methods are applied. For each kind of experiment, a new multi-point type formula is established to replace the current single point type expressions, in order to derive the precise reactivity value by utilizing all of the neutron counting data obtained from every part of the reactor core. In the new formulas, the raw neutron counting data are integrated in reference to space and energy, weighted with the product of the static adjoint-neutron density and the static fission spectrum. This integral procedure is effective in eliminating the effects of kinetic distortion and of the spatial harmonics included in the raw counting data. In addition, using the kinetic eigenfunctions, a new formula is also presented for determining the effective intensity of the source neutrons injected in the source-neutron-introduction method for absolute measurements of reactor power.

KEYWORDS: reactors, negative reactivity, pulsed neutrons, source multiplication, rod drop method, source jerk method, source introduction, reactor power, reactor kinetics

I. INTRODUCTION

The experimental techniques most currently used for determining large values of negative reactivity prevailing in a reactor are the pulsed-neutron, source-multiplication, rod-drop and source-jerk methods(1)(2). In these methods, the formulas so far proposed and used to evaluate the reactivity value have been formulated in reference to the observables obtained at only a single spatial point in the reactor. This resulted in the reactivity value thus determined being strongly influenced by the position of this point of measurement. For the pulsed-neutron method, Kosaly et al.(3) and Vandeplas(4) have derived new formulas utilizing all the observables obtained at multiple space-points distributed throughout the reactor core. In their formulas, the effects of either the kinetic distortion of the neutron flux or the spatial harmonics contained in the raw data were eliminated by averaging the observables with respect to space and energy, weighted with the product of the static adjoint-neutron density and the static fission spectrum. It would appear necessary to apply this space-integral approach also to the other experimental methods, to provide the higher accuracies in reactivity determination required today for critical experiments.

It is thus the purpose of the present paper to derive multi-point type formulas for the source-multiplication, rod-drop and source-jerk methods. In the present work a set of kinetic eigenfunctions** are used as the bases

* Tokai-mura, Ibaraki-ken.
** Kinetic eigenfunctions are exponential modes relevant to the reactor system defined by Eq. (5), and include all the prompt- and delayed-neutron modes.
for describing the kinetic behavior of the reactor. The adoption of kinetic eigenfunc-
tions for such purpose has been suggested in Ref. (3) which treated the pulsed-neutron method. In the ensuing Chap. II, the prompt and delayed response of a source-free subcritical reactor to the injection of a pulsed-neutron burst is analyzed in preparation for the analysis undertaken in the succeeding chapters. This is followed in Chap. III by an interpretation of the pulsed-neutron method, which is essentially the same as that of Kosaly et al. (3) The source-multiplication method is taken up in Chap. IV, where the consideration is based on the fact that the total number of neutrons generated by multiplication is essentially the sum of the neutrons deriving from the prompt- and the delayed-neutron modes initiated by the pulsed-neutron bursts. The rod-drop and source-jerk methods, which are closely related to each other, are analyzed in Chap. V with account taken of space-dependence. Finally Chap. VI deals with the response of a critical reactor to the introduction of a continuous neutron source, and a formula is derived for the effective intensity of the source that contributes to the buildup of a rising fundamental mode. The result should prove useful in improving the accuracy of the source introduction method for absolutely measuring the reactor power.

II. RESPONSE TO A SINGLE BURST PULSED NEUTRONS

The response of a source-free subcritical reactor to the injection of a single burst of pulsed neutrons at \( t=0 \) will be analyzed in this chapter in preparation for the succeeding chapters. The response \( N_{\text{pulse}}(t, r, v) \) is obtained as the solution of a time-dependent reactor equation which can be expressed in the symbolic form

\[
\frac{\partial N_{\text{pulse}}(t, r, v)}{\partial t} = M N_{\text{pulse}}(t, r, v),
\]

with the initial condition

\[
N_{\text{pulse}}(0, r, v) = \left( S \delta(v-v_s) \delta(r-r_s) \right),
\]

where \( S, v_s \) and \( r_s \) are respectively the number, the velocity and the position of the injected neutrons. In Eq. (1), \( N_{\text{pulse}}(t, r, v) \) is the state vector of the reactor formed by the neutron density \( n_{\text{pulse}}(t, r, v) \) and the precursor density \( c_{\text{pulse}}(t, r) \):

\[
N_{\text{pulse}}(t, r, v) = \begin{pmatrix} n_{\text{pulse}}(t, r, v) \\ c_{\text{pulse}}(t, r) \end{pmatrix}
\]

The matrix operator \( M \) in Eq. (1) represents the net production of neutrons and precursors in a unit length of time, given by

\[
M = \begin{pmatrix}
(1-\beta)f_p(v) & \int_0^\infty dv' \Sigma_f(r, v)v' + D(r, v)vP^2, \\
-\Sigma_i(r, v)v + \int_0^\infty dv' \Sigma_i(r, v'-v)v', & -\lambda f_d(v)
\end{pmatrix}
\]

where the diffusion approximation and the one delayed neutron group treatment are adopted for the sake of simplicity.

In Eq. (4), \( \lambda \) and \( \beta \) are respectively the decay constant of the precursor and the fraction of the delayed neutrons, while \( f_p(v) \) and \( f_d(v) \) mean the fission spectra for the prompt and the delayed neutrons, respectively. The other notations are as commonly used.

Here, it is assumed that \( N_{\text{pulse}}(t, r, v) \) can be expanded into a set of kinetic eigenfunc-
tions \( N_{\mu}(r, v) \), defined as the solution of the eigenvalue equation

\[
\omega_{\mu} N_{\mu}(r, v) = M N_{\mu}(r, v)
\]

where \( \omega_{\mu} \) is the time eigenvalue associated with the mode \( N_{\mu}(r, v) \). The subscript \( \nu \) distinguishes between the prompt and the delayed modes (\( \nu=p \) and \( \nu=d \), respectively), while the subscript \( \mu \) is used to express the space-energy modes. For example, \( \mu=0 \) de-
notes the fundamental mode in space and energy.

It can be shown\(^\text{(4)}\) that the orthogonal relation holds between the kinetic eigenfunctions \(N_{\mu \nu}(r, v)\) and its adjoint \(N^*_{\mu \nu}(r, v)\):
\[
\langle N_{\mu \nu}(r, v), N^*_{\nu' \mu'}(r, v) \rangle = \begin{cases} 
0 & (\mu \neq \mu' \text{ or } \nu \neq \nu') \\
 l_{\mu \nu} & (\mu = \mu' \text{ and } \nu = \nu')
\end{cases}
\] (7)
where brackets denote inner product. In Eq. (7) the square of the norm is given by
\[
l_{\mu \nu} = \int_r \int_v n_{\mu \nu}(r, v)n^*_{\mu \nu}(r, v)dv' + (\omega_{\mu \nu} + \lambda) \int_r \int_v dv' \Sigma_f(r, v') n_{\mu \nu}(r, v)dv,
\] (8)
which can be approximated reasonably well by
\[
l_{\mu \nu} \approx \int_r \int_v n_{\mu \nu}(r, v)n^*_{\mu \nu}(r, v')dv',
\] (for \(v = p\))
\[
\text{(for } v = d\text{)}
\] (9)
where the subscript \(V\) means that the volume integration is to be performed over the whole reactor\(*\), and \(n^*_{\mu \nu}(r, v)\) is the adjoint neutron density.

It can also be easily shown that, between the precursor and neutron densities, there holds the two important relations
\[
c_{\mu \nu}(r) = \frac{\beta \int_r \int_v \Sigma_f(r, v')v' n_{\mu \nu}(r, v')dv'}{\omega_{\mu \nu} + \lambda},
\] (10)
\[
c^*_{\mu \nu}(r) = \frac{\lambda \int_r \int_v f_d(v)n^*_{\mu \nu}(r, v)dv}{\omega_{\mu \nu} + \lambda},
\] (11)
where \(c^*_{\mu \nu}(r)\) is the adjoint eigen-precursor density.

Using the kinetic eigenfunctions prescribed above, one can represent the response to a pulsed neutron burst \(N_{\text{pulse}}(t, r, v)\) by
\[
N_{\text{pulse}}(t, r, v) = \sum_{\mu \nu} a_{\mu \nu} N_{\mu \nu}(r, v)e^{\omega_{\mu \nu} t},
\] (12)
\[
n_{\text{pulse}}(t, r, v) = \sum_{\mu \nu} a_{\mu \nu} n_{\mu \nu}(r, v)e^{\omega_{\mu \nu} t},
\] (13)
where \(a_{\mu \nu}\) is the expansion coefficient for each mode, and is determined by the source condition.

Substituting Eq. (12) in Eq. (2) one obtains, at \(t=0\),
\[
(S\delta r - r_o) \delta (v - v_o) = \sum_{\mu \nu} a_{\mu \nu} N_{\mu \nu}(r, v).
\] (15)

Multiplying both sides of Eq. (13) by \(N_{\mu \nu}^*(r, v)\), and utilizing the orthogonal relation of Eq. (7), one obtains the expansion coefficient dependent on the source condition:
\[
a_{\mu \nu} = \frac{\sum_{\mu \nu} n_{\mu \nu}^*(r, v_\mu)}{l_{\mu \nu}}
\] (16)

Suppose that before the pulsed-neutron burst there exists no precursor anywhere in reactor core. Using the expression given by Eq. (14) for \(c_{\text{pulse}}(t, r)\), the initial condition represented by Eq. (2) takes the form
\[
c_{\text{pulse}}(0, r) = \sum_{\mu \nu} a_{\mu \nu} c_{\mu \nu}(r) = 0.
\] (17)

Introducing Eq. (10) into Eq. (17), it follows that
\[
\sum_{\mu \nu} a_{\mu \nu} \int_0^\infty \int_v \Sigma_f(r, v')v' n_{\mu \nu}(r, v')dv' = 0.
\] (18)

Since the eigenvalues \(|\omega_{\mu \nu}|\) are much greater than \(\lambda\), Eq. (18) becomes
\[
\sum_{\mu \nu} a_{\mu \nu} \int_0^\infty \int_v \Sigma_f(r, v')v' n_{\mu \nu}(r, v')dv' = 0.
\] (19)

**III. REACTIVITY DETERMINATION BY PULSED-NEUTRON METHOD**

Use of the pulsed-neutron method for determining the negative reactivity of a sub-

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\* When the integrand includes \(\Sigma_f(r, v)\), the volume integration is to be limited within core region.
critical reactor was first suggested by Sjöstrand\(^{(6)}\), who utilized the neutron-counting areas\(^*\) of both prompt and delayed modes.

In his area method, the static reactivity in dollar units is given by

\[
\frac{\rho}{\beta_{\text{eff}}} = -\frac{A_p(r)}{A_d(r)},
\]

(20)

where \(A_p(r)\) and \(A_d(r)\) are the areas of the prompt and the delayed modes, respectively, measured by a fission counter placed at the position \(r\).

In deriving Eq. (20), Sjöstrand did not consider the effect of spatial harmonics. Some improvements were attempted by Gozani\(^{17}\) and by Garelis & Russell\(^{10}\). But the results did not prove very effective for highly subcritical reflected reactors. In the more recent theoretical and experimental work by Masters\(^{9}\) and by Preskitt\(^{10}\), it is remarked that the reactivity obtained with the methods proposed by Gozani and by Garelis-Russell sometimes show a stronger spatial dependence than that by the classical Sjöstrand method. This result is imputed to kinetic distortion of the neutron flux, which is defined as the spatial and spectral differences between the prompt and delayed modes. In order to overcome this difficulty, Kosaly & Fisher\(^{3}\) proposed an integral version of the classical Sjöstrand area method. We shall here briefly review this new version, making reference to the kinetic eigenfunctions introduced in Chap. II.

Using Eq. (13), the counting areas per unit detector volume around \(r\), \(A_p(r)\) and \(A_d(r)\) are given by

\[
A_p(r) = -\frac{\sum_{\mu} g_{pp} \int_{0}^{\infty} d\nu \Psi_f(r, \nu') \nu' n_{pp}(r, \nu') d\nu'}{\sum_{\mu} g_{pp} \int_{0}^{\infty} d\nu \Psi_f(r, \nu') \nu' n_{pp}(r, \nu') d\nu'},
\]

(21)

\[
A_d(r) = -\frac{\sum_{\mu} g_{pp} \int_{0}^{\infty} d\nu \Psi_f(r, \nu') \nu' n_{pp}(r, \nu') d\nu'}{\sum_{\mu} g_{pp} \int_{0}^{\infty} d\nu \Psi_f(r, \nu') \nu' n_{pp}(r, \nu') d\nu'},
\]

(22)

respectively, where the neutron-detection efficiency of the fission counter is assumed to be proportional to \(\Psi_f(r, \nu)\), which is the product of the fission cross section of the reactor core and the fission neutron yield\(^**\). Here for convenience of later treatments, we define also a pseudo counting area by

\[
\tilde{A}_d(r) = \sum_{\mu} \frac{g_{pp}}{\omega_{pp}} \int_{0}^{\infty} d\nu \Psi_f(r, \nu) n_{pp}(r, \nu) d\nu,
\]

(23)

which is approximately equal to the prompt counting area, by virtue of Eq. (19):

\[
A_p(r) \approx \tilde{A}_d(r).
\]

(24)

Let us here define \(\bar{A}_p\) and \(\bar{A}_d\) as the spatially integrated values of \(A_p(r)\) and \(\tilde{A}_d(r)\) weighted with the product of the static adjoint neutron density\(^*\) and the static fission spectrum\(^*\), i.e. \(n_{\alpha}^2(r, \nu) f_s(\nu)\):

\[
\bar{A}_p = \int_V dA_p(r) \int_{0}^{\infty} n_{\alpha}^2(r, \nu) f_s(\nu') d\nu',
\]

(25)

\[
\bar{A}_d = \int_V dA_d(r) \int_{0}^{\infty} n_{\alpha}^2(r, \nu) f_s(\nu') d\nu'.
\]

(26)

Using Eqs. (25) and (26), and the approximate equality (24), the ratio between \(\bar{A}_p\) and \(\bar{A}_d\) is given by

\[
\frac{\bar{A}_p}{\bar{A}_d} = \frac{\int_{0}^{\infty} n_{\alpha}^2(r, \nu) f_s(\nu) d\nu}{\int_{0}^{\infty} n_{\alpha}^2(r, \nu) f_s(\nu') d\nu'}.
\]

(27)

It is known\(^{10}\) that the neutron flux of the delayed mode \(vn_{pp}(r, \nu)\) can be well approximated by the corresponding static neutron flux \(\phi_{pp}(r, \nu)\):

\[
v n_{pp}(r, \nu) \approx \phi_{pp}(r, \nu),
\]

(28)

and it can further be reasonably assumed

\[
\phi_{pp}(r, \nu) = \phi_{pp}(r, \nu'),
\]

and it can further be reasonably assumed

\[
\phi_{pp}(r, \nu) = (1 - \beta) f_p(\nu) + f_d(\nu).
\]

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\* In the present paper, the neutron-counting area is defined as the time-integrated total neutron count obtained with use made of a fission counter in reference to a specified decay mode or kinetic behavior of a reactor. This definition differs from the earlier concepts adopted by Kosaly\(^{3}\) and by Sjöstrand\(^{6}\), who used this term to express the time-integrated neutron flux.

\*\* It is to be noted here that the measurements of the counting areas should be made only within the core region.

\* The static adjoint-neutron density is the fundamental solution of the adjoint equation to the static reactivity eigenvalue equation.

\*\* The static fission spectrum \(f_s(\nu)\) is given by \((1 - \beta) f_p(\nu) + f_d(\nu)\).
here that the same is true of the adjoint neutron density:

$$A_p = \frac{-\sum a_{\mu d}}{\omega_{\mu d} + \lambda} \int \int d\nu \int \int d\nu' \Sigma_f(r, \nu') \phi_{\mu r}(r, \nu') \int_0^\infty n_{\mu d}(r, \nu') f_d(\nu) d\nu$$

$$A_d = \frac{\sum a_{\mu d}}{\omega_{\mu d} + \lambda} \int \int d\nu \int \int d\nu' \Sigma_f(r, \nu') \phi_{\mu r}(r, \nu') \int_0^\infty n_{\mu d}(r, \nu') f_d(\nu) d\nu$$

Here, it is recalled that there holds for a static flux the orthogonal relation

$$\int d\nu \int d\nu' \Sigma_f(r, \nu') \phi_{\mu r}(r, \nu') \int_0^\infty n_{\mu d}(r, \nu') f_d(\nu) d\nu = 0, \quad (\text{for } \mu = \mu') \quad (31)$$

Utilizing the orthogonality of Eq. (31), Eq. (30) yields the reactivity in dollar units:

$$\frac{\rho}{\rho_{\text{eff}}} = \frac{-\omega_{ed}}{\omega_{ed} + \lambda} = \frac{-\rho}{\rho_{\text{eff}}} \quad (32)$$

where use is made of the well-known inhour equation for relating the reactivity $\rho$ to the eigenvalue $\omega_{ed}$.

The validity of Eq. (32) has been examined by Kosaly et al. through numerical calculations simulating the pulsed-neutron experiment down to $-30$ $\$ in several heavily-reflected slab-shaped reactors, with both core and reflector pulsing.

**IV. Reactivity Determination by Source-Multiplication Method**

The source-multiplication method is widely used for determining a large negative reactivity of a subcritical reactor. Earlier examples of application of this method were mostly limited to the initial critical approach. The critical points could be well predicted by plotting the inverse of the neutron counting rate against an increasing number of fuel rods or against increasing water height. For this purpose, a high accuracy was not required for the value of reactivity. Hence, to obtain the static reactivity, satisfactory results could be obtained with the simple space-dependent formula

$$\rho = -\frac{S' \cdot d}{A_i(r)} \quad (33)$$

where $S'$, $A_i(r)$ and $d$ are the intensity of the neutron source, the counting rate of the neutron detector positioned at $r$ and a proportional constant, respectively.

Several years ago it was pointed out by Miyawaki that the source-multiplication method could be applied more precisely by introducing a subcritical constant obtained by integrating the observed neutron flux over the whole volume of the reactor. More recently, several fast critical experiments have been performed with extensive use made of the source-multiplication method for measuring the reactivity worths of control rods.

In this chapter we shall attempt to improve the formula (33), so it could be applied even to a core whose singularity is accentuated by control rod insertion.

The total neutron counting rate $A_i(r)$ is essentially the sum of $A_p(r)$ and $A_d(r)$:

$$A_i(r) = A_p(r) + A_d(r) \quad (34)$$

Substituting Eq. (24) into Eq. (34), Eq. (34) becomes

$$A_i(r) = -\frac{A_p(r) + A_d(r)}{A_i(r)} \quad (35)$$

Using Eqs. (22) and (23), Eq. (35) is rewritten in the form

$$A_i(r) = -\sum a_{\mu d} \int_0^\infty \Sigma_f(r, \nu) v n_{\mu d}(r, \nu) d\nu$$

$$-\sum a_{\mu d} \int_0^\infty \Sigma_f(r, \nu) v n_{\mu d}(r, \nu) d\nu,$$

which leads to

$$A_i(r) = -\sum a_{\mu d} \int_0^\infty \Sigma_f(r, \nu) v n_{\mu d}(r, \nu) d\nu. \quad (36)$$

*In actuality, the source-neutron multiplication experiment is made with a continuous neutron source. However, the application of $A_i(r)$ to such a case is justified by the fact that the operator $M$ is linear. While in the preceding chapters $A_p(r)$, $A_d(r)$ and $S$ have been expressed in units of counts/burst or neutrons/burst, these quantities should henceforth be given in counts/sec or neutrons/sec.
Here, let us define the integrated counting area $\mathcal{A}_t$ by the sum of $\mathcal{A}_p$ and $\mathcal{A}_d$:

$$\mathcal{A}_t = \mathcal{A}_p + \mathcal{A}_d \quad (38)$$

Then, using Eq. (37),

$$\mathcal{A}_t = -\sum \frac{\lambda a_{pd}}{\omega_{pd}(\omega_{pd} + \lambda)} \int \! dr \int_0^\prime dv' \Sigma_f(r, v') \cdot n_{pd}(r, v') f_s(v) dv', \quad (39)$$

and with the approximation of Eq. (28),

$$\mathcal{A}_t = -\sum \frac{\lambda a_{pd}}{\omega_{pd}(\omega_{pd} + \lambda)} \int \! dr \int_0^\prime dv' \Sigma_f(r, v') \cdot n_{pd}(r, v') f_s(v) dv', \quad (40)$$

This becomes, with use made of the orthogonal relation of Eq. (31),

$$\mathcal{A}_t = -\frac{\lambda a_{pd}}{\omega_{pd}(\omega_{pd} + \lambda)} \int \! dr \int_0^\prime dv' \Sigma_f(r, v') \cdot \phi_{pd}(r, v') n_{pd}(r, v') f_s(v) dv'. \quad (41)$$

Utilizing Eqs. (9) and (16), and approximating $\phi_{pd}(r, v)$ and $n_{pd}(r, v)$ by $\phi_{ps}(r, v)$ and $n_{ps}(r, v)$ respectively, one obtains the expansion coefficient for the fundamental delayed-neutron mode $a_{pd}$:

$$a_{pd} \equiv \frac{S n_{ps}(r, v_s)(\omega_{pd} + \lambda)^2}{\beta \lambda \int \! dr \int_0^\prime dv' \Sigma_f(r, v) \cdot \phi_{ps}(r, v) n_{ps}(r, v') f_s(v') dv'} \quad (42)$$

Substituting Eq. (42) into Eq. (41), we obtain

$$\mathcal{A}_t = -\frac{S(\omega_{pd} + \lambda)n_{ps}(r, v_s)}{\beta \omega_{pd} \beta \text{eff}}, \quad (43)$$

where use is made of the following well-known relation between $\beta$ and $\beta \text{eff}$:

$$\frac{\beta \text{eff}}{\beta} = \frac{\int \! dr \int_0^\prime dv' \Sigma_f(r, v) \phi_{ps}(r, v) n_{ps}(r, v') f_s(v) dv}{\int \! dr \int_0^\prime dv' \Sigma_f(r, v) \phi_{ps}(r, v) n_{ps}(r, v') f_s(v) dv}, \quad (44)$$

which, using the latter part of Eq. (32), leads to the expression of the reactivity

$$\rho = -\frac{S n_{ps}(r, v_s)}{\mathcal{A}_t}. \quad (45)$$

The formula (45) will serve as improvement over the currently used Eq. (33) for interpreting the source-multiplication method. It is worth noting that, as indicated by Eq. (43), $\mathcal{A}_t$ is proportional to the adjoint neutron density $n_{ps}(r, v_s)$ of the source neutrons.

V. Reactivity Determination by Rod-Drop and by Source-Jerk Method

The reactivity worths of the control rods used in practical reactor operation are often measured by the rod-drop method. In the one-point reactor-kinetics model, the ratio between the neutron counting rate before the rod-drop and the integrated neutron-count after the rod-drop is related to the reactivity in dollar units:\(^{(13)}\):

$$\rho = \frac{\mathcal{A}_c(r)}{\mathcal{A}_t} \quad (46)$$

where use is made of the following well-known relation between $\beta$ and $\beta \text{eff}$:

$$\rho = \frac{\mathcal{A}_c(r)}{\lambda \mathcal{A}_t(r)}, \quad (46)$$

where $\mathcal{A}_c(r)$ and $\mathcal{A}_t(r)$ are the neutron counting rate before the rod-drop and the integrated neutron-count following the rod-drop, respectively.

Now, it has often been observed that the reactivity obtained by this formula (46) shows strong space-dependence\(^{(2)(14)}\). For this reason we propose an improvement aimed at obtaining the true space-independent static reactivity value.

By using a set of kinetic eigenfunctions, one can assume that the decay of the neutron and precursor densities following the rod-drop can be represented by the summation of all the prompt and delayed neutron modes:

$$n(t, r, v) = \sum_{\mu \nu} a_{\mu \nu} n_{\mu \nu}(r, v) e^{\omega_{\mu \nu} t}, \quad (47)$$

$$c(t, r) = \sum_{\mu \nu} a'_{\mu \nu} c_{\mu \nu}(r) e^{\omega_{\mu \nu} t}, \quad (48)$$

respectively, where $a'_{\mu \nu}$ is the expansion coefficient applicable to this situation.
Since \( c(0, r) \), the initial condition at \( t=0 \), has to be related to \( n(0, r, v) \) by
\[
c(0, r) = \sum_{\nu} \left[ \sum_{\nu'} a_{\nu \nu'}^{0} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(0, r, v) dv \right] \frac{\rho}{\lambda}.
\] (49)

Substituting Eqs. (47) and (48) into the right- and the left-hand sides of Eq. (49) respectively, and utilizing Eq. (10), one obtains
\[
\sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv
= \frac{1}{\lambda} \sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv.
\] (50)

On the other hand, using Eq. (47), we derive the expression for the neutron counts:
\[
\tilde{A}_{c}(r) = \sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(0, r, v) dv dv,
\] (51)
\[
\tilde{A}_{r}(r) = \sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv dv.
\] (52)

We now define the two space-integrated neutron counts
\[
\tilde{A}_{c}(r) = \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(0, r, v) dv dv,
\] (53)
\[
\tilde{A}_{r}(r) = \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv dv.
\] (54)

\[
\sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv
= \frac{1}{\lambda} \sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv.
\] (50)

Which becomes, with the relation of Eq. (50)
\[
\tilde{A}_{c}(r) = \frac{-\lambda \sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(0, r, v) dv dv}{\sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv dv},
\] (57)

Disregarding the contributions of the prompt modes, and with the aid of the assumption expressed by the inequality (61) given hereafter, Eq. (58) is further transformed into
\[
\tilde{A}_{c}(r) = \frac{-\omega_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(0, r, v) dv dv}{\omega_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv dv}.
\] (58)

Which reduces to the form
\[
\tilde{A}_{c}(r) = \frac{-\omega_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(0, r, v) dv dv}{\omega_{\nu \nu}^{0} \int_{0}^{\infty} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv dv}.
\] (59)

where use has been made of Eq. (28) and the orthogonal relation of Eq. (31) as well as the inhour equation.

In deriving Eq. (59) from Eq. (58), we have assumed the inequality
\[
\left| \sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(r, v) dv \right| \leq \left| \sum_{\nu} a_{\nu \nu}^{0} \int_{0}^{\infty} \psi_{\nu}(r, v) n_{\mu \nu}(0, r, v) dv \right|.
\] (61)

This inequality signifies that most of the integrated neutron counts following the rod drop is contributed by neutrons belonging to the delayed modes. The validity of the inequality (61) may easily be confirmed for a one-point reactor kinetics model from the fact that the decay of the delayed modes is much slower than the prompt modes.

Now, formula (60) is also applicable to the case of reactivity determination by the source-jerk method(1). In this method, the extraneous neutron source is abruptly removed from a subcritical reactor which has been at equilibrium with the source. Prior to source removal, that is, in initial condition, the neutron and precursor densities satisfy Eq. (49), and this makes the entire procedure of neutron field analysis from Eq. (50) to (60) valid also for the source-jerk.
VI. Reactor Power Measurement by Source-Introduction Method

Use of the source-introduction method for absolute measurements of reactor power has been suggested by Schultz(15) and Kobayashi et al.(16). The advantage of this method is its simplicity in that it requires only both of the neutron source for the reactor start-up and the neutron detectors that are already provided for normal reactor operation.

Using the one-delayed-neutron-group model, one can express the absolute value of the reactor power \( P \) in units of watts using the equation(15)

\[
P = \frac{S'_{\text{eff}} \lambda I}{\alpha \beta_{\text{eff}}},
\]

(62)

where \( I \) and \( I' \) are respectively the ion chamber current before the source insertion and its slope after the insertion while \( S'_{\text{eff}} \) and \( \alpha \) are the effective intensity of the neutron source inserted per unit time, the average fission neutron yield, and the number of fissions corresponding to 1 W, respectively.

Despite its simplicity, less use has been made this source-introduction method than that of Au-foil activation. This is mainly because no definite estimate can be made of the portion represented by the source neutrons that contribute to the build-up of the rising fundamental mode. And in fact, the neutron-source guide tube is usually designed to terminate outside the reactor core. In what follows, we shall seek an approach to the solution of this problem.

The response \( N_{\text{conti}} = (n_{\text{conti}}(r,v), c_{\text{conti}}(r)) \) of a critical reactor to the introduction of a continuous neutron source with an intensity of \( S' \) (neutrons/sec) can be assumed to be represented by superposing the responses \( n_{\text{single}}(t, r, v) \) to single neutrons, which is none other than \( n_{\text{pulse}}(t, r, v) \) with the coefficients \( a_{\mu v} \) obtained by letting \( S \) equal unity in Eq. (16):

\[
n_{\text{conti}}(t, r, v) = n(r, v) + S' \int_{0}^{t} n_{\text{single}}(\tau, r, v) d\tau
\]

\[
- S' \sum_{\mu v} a_{\mu v} n_{\text{pulse}}(r, v) \frac{(1-e^{\omega_{\mu v}t})}{\omega_{\mu v}},
\]

(\( \mu \geq 2 \), for \( \nu = d \)) (63)*

where, \( n(r, v) \) is the initial neutron density \( n(0, r, v) \).

The reactor power at \( t=0 \) can be related to the neutron density \( n(r, v) \) by the expression

\[
P = \frac{\int r \sum_{\mu \nu} n_{\text{conti}}(r, v) v n(r, v) dv}{\alpha}.
\]

(64)

Similarly, using Eq. (63), one obtains

\[
I(r) = \int s(v) n(r, v) dv,
\]

(65)

\[
I'(r) = S' a_{\text{ad}} \int s(v) n_{\text{ad}}(r, v) dv,
\]

(66)

which, with the help of Eqs. (28), (42) and (44) and using \( \omega_{\text{ad}} = 0 \) for this case, becomes

\[
I'(r) = \frac{S' \lambda n_{\text{ad}}(r, v) \int s(v) n_{\text{ad}}(r, v) dv}{\beta_{\text{ad}} \int r \sum_{\mu \nu} n_{\text{conti}}(r, v) v n_{\text{ad}}(r, v) dv},
\]

(67)

* \( N_{\text{conti}}(t, r, v) \) requires to be the solution of the equation

\[
\frac{\partial N_{\text{conti}}}{\partial t} = MN_{\text{conti}}(t, r, v) + \begin{pmatrix} S' \end{pmatrix},
\]

(a1)

with the initial condition

\[
N_{\text{conti}}(0, r, v) = N(r, v),
\]

(a2)

where \( N(r, v) = (n(r, v), c(r)) \) is the neutron density vector immediately prior to the insertion of the continuous neutron source.

It is clear that the first term of \( n_{\text{conti}}(t, r, v) \), i.e. \( C n(r, v) \), where \( C \) is a proportional constant, is the persisting form of the general solution of the homogeneous equation for Eq. (a1), a particular of the same equation being the second term \( S' \int_{0}^{t} n_{\text{single}}(\tau, r, v) d\tau \). Thus the sum of the two parts constitutes the general solution of Eq. (a1). Then, the proportional constant \( C \) is adjusted to unity so that at \( t=0 \) the sum satisfies the boundary condition of Eq. (a2).
where \( s(v) \) is the neutron-detecting sensitivity of the ion chamber. Combining Eqs. (64)~(67), and upon arrangement to let the initial distribution \( n(r, v) \) equal the static eigenfunction \( n_0^s(r, v) \), one can obtain the reactor power at \( t=0 \):

\[
P = \frac{S' \lambda l(r)n_0^s(r, v)\int v \Sigma_f(r, v)n_0^a(r, v)dv}{\alpha \beta_{\text{eff}} l'(r)\int v \Sigma_f(r, v)n_0^a(r, v')f_s(v)n_0^s(r, v')dv} \tag{68}
\]

Comparing Eq. (68) with the hitherto used formula (62), one derives the formula

\[
S'_{\text{eff}} = S' \frac{\bar{v}n_0^s(r, v)\int v \Sigma_f(r, v)n_0^a(r, v)dv}{\int v \Sigma_f(r, v)n_0^a(r, v')v'n_0^a(r, v')f_s(v)n_0^s(r, v')dv}. \tag{69}
\]

As indicated by Eq. (69), the factor to be applied to \( S' \) to obtain \( S'_{\text{eff}} \) is the ratio of importance between the extraneous neutrons and the core average of the fission neutrons under the fundamental static mode.

The improvement brought to the source introduction method by the formula (68) was demonstrated by actual measurements on the Semi-Homogeneous Critical Assembly SHE(17) by Kitadate & the author(18). Good agreement was seen in the values of reactor power between the results obtained with the improved source-insertion method and with the currently-used Au-foil activation technique.

**VII. CONCLUDING REMARKS**

In the present work, we have taken the stand that added labor in conducting measurements is amply repaid by the resulting improvement in experimental accuracy. This standpoint is justified from the increasingly precision demanded in recent critical experiments. The principal contribution of the present work is the set of new formulas represented by Eqs. (45), (60) and (68), respectively applicable to the methods of source-neutron multiplication, rod-drop (source-jerk) and source-introduction, as summarized in Table 1.

In deriving these formulas, the one-delayed-neutron-group treatment was adopted for simplicity. But, it can be easily proved that Eqs. (32), (45), (60) and (68) are valid also for the case of multi-delayed-neutron-group treatment.

### Table 1

<table>
<thead>
<tr>
<th>Experimental method</th>
<th>One-point</th>
<th>Multi-point</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulsed neutrons</td>
<td>( \rho = -\frac{A_p(r)}{A_d(r)} )</td>
<td>( \frac{\rho}{\beta_{\text{eff}}} = \frac{\tilde{A}_p}{A_d} )</td>
</tr>
<tr>
<td>Source-neutron multiplication</td>
<td>( \rho = -\frac{S \cdot d}{A_i(r)} )</td>
<td>( \frac{\rho}{\beta_{\text{eff}}} = -\frac{\tilde{A}_c}{A_i} )</td>
</tr>
<tr>
<td>Rod-drop (Source-jerk)</td>
<td>( \frac{\rho}{\beta_{\text{eff}}} = \frac{\tilde{A}_c}{A_r} )</td>
<td>( \tilde{A}_c = \frac{A_c(r)}{A_r} )</td>
</tr>
<tr>
<td>Source introduction</td>
<td>( \frac{P}{\alpha \tilde{A}} = \frac{S' \lambda l}{\alpha \beta_{\text{eff}} l'} )</td>
<td>( \frac{P}{\alpha \tilde{A}} = \frac{S'<em>{\text{eff}} \lambda l}{\alpha \beta</em>{\text{eff}} l'} )</td>
</tr>
</tbody>
</table>

Remarks
1. \( \tilde{A}_p = \int_v dr A_p(r) \int v n_0^a(r, v') f_s(v') dv' \)
2. \( \tilde{A}_d = \int_v dr A_d(r) \int v n_0^a(r, v') f_s(v') dv' \)
3. \( n_0^a(r, v) = \text{Value of the adjoint-neutron density for extraneous source neutrons} \)
4. \( \tilde{A}_c = \int_v dr A_c(r) \int v n_0^a(r, v') f_s(v') dv' \)
5. \( \tilde{A}_r = \int_v dr A_r(r) \int v n_0^a(r, v') f_s(v') dv' \)

* This normalization is permissible since neither \( N_0^a(r, v) \) nor \( N_0^a(r, v) \) has previously been subjected to normalization.
ment, upon adoption of the two approximations (1) that all the neutron fluxes of the delayed-neutron modes are represented by the static neutron fluxes, the same being true for all the adjoint neutron densities of the delayed-neutron modes, and (2) that the static reactivity \( \rho \) can well be related to the dynamic reactivity \( \rho_d \) by

\[
\frac{\rho}{\rho_d} \approx \frac{\sigma_d}{\sigma},
\]

the validity of which has been discussed in detail by Gross(19). It is also to be noted here that the average decay constant of the delayed neutron precursor, which appeared in Eqs. (46) and (62), should be evaluated by the equation

\[
\frac{\beta_{i\text{eff}}}{\lambda} = \frac{1}{\lambda} \sum \frac{\beta_{i\text{eff}}}{\lambda_i},
\]

where \( \beta_{i\text{eff}} \) is the effective fraction of the \( i \)-th group delayed neutrons.

In the present analysis, all the time eigenvalues \( \omega_{\mu \nu} \) have been assumed to be discrete. In highly subcritical fast reactors, however, none of the prompt neutron decay constants, including that of the fundamental mode, are discrete. Nevertheless, as discussed in what follows, the final results of the present work, summarized in Eqs. (32), (45) and (60), can be considered valid also for fast reactors. In the present work, the values of \( \omega_{\mu \nu} \) have not been utilized in a manner such that the reactivity should have to be obtained directly from \( \omega_{\mu \nu} \) with use made of the calculated or the measured neutron generation time, an expedient that has been condemned by Simmon & King for the pulsed-neutron method(20); they were utilized only in an indirect way to express either the neutron counting area of the prompt modes \( A_p(r) \) or the contribution of the prompt modes to the build-up of the precursor density in Eqs. (21) and (50) respectively. Both these quantities, actually written in the form of summations with respect to \( \mu \), could be replaced by integrals, if the continuity of \( \omega_{\mu \nu} \) is taken into account. But the former can readily be converted to the pseudo area \( \tilde{A}_d(r) \) through Eq. (24), which is constituted solely of the delayed neutron modes, all of which are discrete. The latter can be readily neglected in Eq. (59) with use made of the inequality (61).

As indicated by Eqs. (32), (45) and (60), the neutron levels must be measured at multiple points throughout the core region of the reactor. But in actuality, this is difficult to realize, and often beyond possibility from practical limitations. Yet, such multi-point observation is indispensable in order to eliminate the effect of kinetic distortion and of the higher harmonics. It is to be noted that \( n_{0d}(r, \nu) \cdot f_s(\nu) \), which is the product of the fundamental adjoint neutron density and the static fission spectrum, has been adopted as weighting function for the spatial integration of the primary observables \( A_p(r), A_d(r), A_c(r), A_s(r) \) and \( A_r(r) \). This measure is truly effective for eliminating the effect of the spatial harmonics induced either by the localized neutron source (in the case of the source-neutron-multiplication experiment) or by the difference in the shape of neutron fluxes before and after the rod-drop (in the case of the rod-drop experiment). Hence, in precise experiments, rigorous application of the formulas (45) and (60) is essential. Even when multi-point measurements are impossible, these two formulas can be useful by employed either for estimating the systematic errors incurred by the one-point measurement or for determining the optimum position of the single detector.

As remarked earlier, the present methods require knowledge of the precise value of \( n_{0d}(r, \nu) \). This applies in particular to the case of cores possessing high singularity, when detailed calculation of \( n_{0d}(r, \nu) \) becomes indispensable, while in the case of a heavily-reflected clean core \( n_{0d}(r, \nu) \) can be assumed constant without impairing the resulting accuracy(3). In this report, the detectors have been assumed to be fission counters, but in ordinary thermal reactors BF3 counters could also be used without affecting the principal features of the present methods. This is because, although the epithermal neutron counts relative to the thermal neutron counts will differ between the two detectors, the difference in the reactivity values derived therefrom will be negligible.
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REFERENCES