Void Fraction Correlation for Boiling and Non-Boiling Vertical Two-Phase Flows in Tube

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Applicability of previously proposed void fraction correlation was studied on the boiling steam-water flows in tube. The correlation was found to be applicable to the data of Sekoguchi (d=13.55 mm) and Bartolomei (d=25 mm), i.e. the value of parameter K in the correlation equals to unity. The average value of K for pipes of smaller diameter (d=6.1, 7.7 mm) was 0.57 for both adiabatic and diabatic steam-water flows. These values of K are not dependent on flow regime, heat flux and superficial water velocity within the investigated ranges. A criterion was established with a dimensionless group $E_0\lambda$ which determines the K-value as: $K=1.0$ for $E_0\lambda \geq 2 \times 10^{-6}$ and $K=0.57$ for $E_0\lambda < 2 \times 10^{-6}$.

A comparison between the predicted values by this method and experimental values at pressures from atmospheric to 80 kg/cm² abs., heat fluxes 0~1.5 $\times 10^6$ kcal/m²·hr, pipe diameters 5~76.2 mm and gas-water volumetric flow ratios 0.06~404, showed that the present correlation is adequate within ±15% of deviation.

**KEYWORDS:** two-phase flow, void fraction, general correlation, boiling, high pressure, water, steam, accuracy, errors, flow rates, heat flux

I. INTRODUCTION

In two-phase flow phenomena precise knowledge of average void fraction, the fraction of an element of volume which is occupied by gas, is of considerable importance to nuclear reactor technology relating to mean density, stability and circulation characteristics in boiling channels. The average void fraction would be determined by the geometry of the flow channel, flow direction, flow rate of each phase, the system pressure, heat generation at the wall, the physical properties of each phase, e.g. the density, the viscosity and the surface tension. From different point of view, the average void fraction is obtained from the void fraction profile in the flow channel and it is, in turn, determined from velocity profile of each phase.

Zuber & Findlay(27) presented a general expression to predict the average volumetric concentration (void fraction) taking into account both the effect of flow and concentration profiles as well as the effect of the local relative velocity between the phases. “Modified Bankoff’s equation(9)” which is an extension of Bankoff’s empirical correlation(2) is often used in the design calculation of nuclear reactor but it lacks experimental foundation. Curves presented by Martinelli & Nelson(14) for prediction of the void fraction of two-phase steam-water flow are also widely used in nuclear engineering, but they are mainly comprised of the data in horizontal tubes by Lockhart & Martinelli(11) and they are not based on the experiments at higher pressures. In fact, it is not accurate at higher...
pressures\textsuperscript{(26)}. In addition to these references, many other void fraction correlations have been reported\textsuperscript{(8)(10)(12)(15)(23)(26)}.

One of the authors proposed the following simple and sufficiently accurate empirical correlation of the void fraction which could be applied to all flow regimes of gas-water systems in vertical tubes\textsuperscript{(17)(24)}. Defining a parameter \( K \) by

\[
K = \frac{w_g - w_l}{w_{go}},
\]

the relation between the void fraction \( a \) and the volumetric flow concentration of gas \( \beta \), and quality \( x \) is expressed from mass balance as follows:

\[
\frac{\alpha}{(1-\alpha)(1-K\alpha)} = \frac{\beta}{1-\beta},
\]

or

\[
\frac{\alpha}{(1-\alpha)(1-K\alpha)} = \frac{\rho_l \cdot x}{\rho_g \cdot (1-x)}.
\]

The only unknown in these equations is the parameter \( K \). The value of \( K \) determined for gas-water and non-boiling steam-water systems in vertical upward flow for wide range of experimental variables was equal to unity well within the scatter of the experiments\textsuperscript{(17)(24)(25)}. For air-water flow, Inoue \& Aoki\textsuperscript{(17)} presented an empirical equation corresponding to Eq. (2), where \( K=1 \).

In this paper the possibility of application of the correlation to boiling systems will be investigated using recent experimental data\textsuperscript{(3)(16)(19);(22)}.

\section*{II. VALUE OF K FOR BOILING STEAM-WATER SYSTEM IN PIPES}

\subsection*{1. Nature of Parameter K}

Figure 1 shows the relation between \( \alpha \) and \( \beta \) calculated from Eq. (2) when the value of parameter \( K \) is varied from 0 to 1.25. Such a flow of \( K=0 \), that is \( \alpha=\beta \), referred to as the homogeneous flow. The void fraction line departs from the homogeneous flow line with increasing \( K \)-value. But, for \( K \)-values larger than unity, the void fraction \( \alpha \) does not approach to unity as \( \beta \) approaches to unity. If the flow regime is thermally in equilibrium, it should be that \( \alpha \) is less than or equal to \( \beta \), also \( \alpha=0 \) for \( \beta=0 \) and \( \alpha=1 \) for \( \beta=1 \). If \( K \) is not a function of \( \beta \) but takes constant value, the range of the value of \( K \) which satisfies these conditions is \( 0 \leq K \leq 1 \). From the definition of slip ratio between the two phases and a little manipulation of Eq. (2) or (3), it will be known that \( K=1.0 \) is the condition which gives maximum value of slip ratio. Another characteristic of \( K \) is that the constant deviation line with the void fraction from the line of given \( K \)-value is not symmetric with respect to \( \beta \) as shown in Figs. 2 and 4.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Fig1.png}
\caption{Nature of parameter \( K \)}
\end{figure}

\subsection*{2. Value of Parameter \( K \) for Boiling Water System}

The measurement of the void fraction for boiling system in vertical tubes is rather difficult, and there exist few experimental works until recent time.

Recently, the experimental results\textsuperscript{(3)(16)(19)(22)} of the void fraction were reported under the condition of uniform heating along the axis of the tubes. Here, parameter \( K \) will be examined on the data only belong to the bulk boiling region.

Sekoguchi \& coworkers\textsuperscript{(22)} reported some results of the average void fraction calculated from the void fraction profiles measured by a needle probe, across the tube of 13.55 mm at pressures 2~16 kg/cm\textsuperscript{2} abs. Figure 2 shows the relation \( K \) vs. \( \beta \) from these data having various inlet velocities at the test section, 0.3~2.0 m/sec, at heat flux of 0.2x10\textsuperscript{4} kcal
/m²·hr. No definite tendency of inlet water velocity or heat flux was found on the void fraction within investigated range. The agreement between the value of $K=1.0$ and these data is within 15% deviation. In Fig. 3 experimental values of void fraction and volumetric flow concentration of vapor from Refs. (3) and (22) are plotted with a line of Eq. (2) at $K=1.0$. Dotted lines in the figure show $±15\%$ deviation of the void fraction from the line of $K=1.0$. Figures 2 and 3 show that the void fraction correlation, Eq. (2), is applicable to the average void fraction for bulk boiling.

Several authors\(^{4,15}\) have discussed about the considerable effects of mass velocity and/or of heat flux on the void fraction in annuli or rectangular channels. The disagreement with the result of their discussion and the result in this report may be due to the difference in channel geometry.

### 3. Value of $K$ for Pipes of Smaller Diameter

Figure 4 shows the experimental results of Miropol’skii & Schneerova\(^{16}\) for boiling and non-boiling steam-water flow through a 7.7 mm pipe at pressures 20~70 kg/cm\(^2\) abs. In Fig. 5 are plotted the data reported by Rouhani & Becker\(^{19}\) for heavy water-steam mixture flowing through a pipe (6.1 mm I.D.) over the range of pressures 7~60 bars and inlet water velocities 0.66~1.9 m/sec. The average value of $K$ for the above experiments is 0.57. On the other hand, the void fraction data of air-water flows in pipes of smaller diameter (5.0 and 9.0 mm) by Inoue & Aoki\(^7\) yield $K=1.0$ for whole range of their experiments. Assuming those data all valid, we shall investigate the effects of diameter and other parameters on the value...
of $K$. As mathematical analysis to determine the $K$ is difficult at present, a dimensional analysis will be performed.

Assuming the relative velocity between the phases is determined under the influences of gravitational, inertial, viscous and interfacial forces, a set of dimensionless groups was chosen as follows:

$$f_i \left( \frac{w_{lo} + w_{go}}{w_g - w_t}, We, Fr, Re \right) = 0.$$ 

The superficial velocity $(w_{lo} + w_{go})$ was selected for the arbitrary velocity in these groups, because it characterizes the flow and it is simple and also convenient in correlation of the $K$-value.

From the definitions of $K$ and $\beta$, Eq. (4) becomes

$$f_i \left( \frac{1}{K\beta}, We, Fr, Re \right) = 0.$$ 

Considering the $K$-value is not affected with superficial velocity $(w_{lo} + w_{go})$ the following equation is derived from Eq. (5).

$$K = f(\beta, E_o, \lambda),$$

where $E_o = \Delta \rho g d^2/\sigma$, $\lambda = v_t^2 \rho_t/\sigma$.

The dimensionless group $E_o$, Eötvös number, represents the ratio of gravitational to interfacial forces and $\lambda$ the ratio of Weber number to Reynolds number squared. The volumetric concentration of gas $\beta$ almost has no effect on the value of $K$, as was shown by the experimental results (Figs. 2~5). Accordingly from Eq. (6), the value of $K$ may be a function of dimensionless groups $E_o$ and $\lambda$.

To develop the relation among $K$, $E_o$ and $\lambda$, the values of $K$ were plotted against $E_o$ and $\lambda$. The $K$-value was fixed as $K=1.0$ following to the previous reports$^{[24,25]}$, in case the void fraction was predicted within the accuracy $\pm 15\%$. And for other cases (the data belong to this group were boiling and non-boiling steam-water flows in pipes of smaller diameter less than 10 mm I.D.), the arithmetic average value of $K$ was calculated for the all data in Refs. (16) and (19). $K$-value seems to be a function of $\beta$, but the numbers and accuracy of the data were not enough for this purpose. Experimental works used for these purposes are given in Table 1. The relation between the $K$-value and $E_o$ showed a branch for smaller diameter data (Fig. 6(a)). And in a figure of $K$ vs. $\lambda$, $K$ is equal to unity with an exception of narrow range for the value of $\lambda$. When $E_o \lambda$ was chosen as abscissa in Fig. 6(b), a better correlation was obtained. The dimensionless group $E_o \lambda$ is determined from the properties of fluid and the diameter of test section. In Fig. 6(b) the values of $K$ change from unity to 0.57 or vice versa at $E_o \lambda = 2 \times 10^{-6}$, irrespective of the system components.
III. GENERALIZED VOID FRACTION CORRELATION

It can be seen from Figs. 2~5 and 6(b) that the average values of parameter $K$ are nearly unaffected by system components, heat fluxes, inlet water velocities and flow regimes within the range of the experimental variables and are expressed as:

$$K = 1.0 \quad \text{for} \quad E_0 < 2 \times 10^{-6}$$

$$K = 0.57 \quad \text{for} \quad E_0 > 2 \times 10^{-6} \quad (7)$$

From this criterion for $K$, the void fraction can be calculated by Eq. (2). The void fractions thus obtained $\alpha_p$ are shown with the experimental data $\alpha_m$ in Fig. 7, and the prediction is seen to be well within $\pm 15\%$ for a large majority of the points, which are scattered almost uniformly.

The empirical equation may be applied to adiabatic and to diabatic systems in tubes, and heat fluxes $0 \sim 1.5 \times 10^6$ kcal/m$^2$·hr, diameters 5~76.2 mm and gas to water volumetric flow

![Fig. 7](image-url) Comparison between predicted and measured void fraction

at pressures atmospheric $\sim 80$ kg/cm$^2$·abs., and heat fluxes $0 \sim 1.5 \times 10^6$ kcal/m$^2$·hr, diameters 5~76.2 mm and gas to water volumetric flow.
ratios $0.06 \sim 10^4$. It may be applied at pressures atmospheric $\sim 140 \text{ kg/cm}^2\text{abs.}$ for adiabatic steam-water flows$^{(25)}$.

To compare the present work with the other correlations of the void fraction, the present correlation when $K = 1.0$ and $K = 0.57$ in the form of Eq. (3) is plotted together with the correlations by Martinelli-Nelson$^{(14)}$, Madsen$^{(12)}$, Thom$^{(25)}$ and Jones' modified Bankoff equation$^{(9)}$, which do not include the effects of heat flux and superficial liquid velocity. The values at atmospheric pressure are shown in Fig. 8(a) and at pressure 70 kg/cm$^2$ abs. are in Fig. 8(b). The empirical correlation proposed is in better agreement to other studies at wide range of steam quality. Proposed equation is more simple, including no experimental constant for $E\lambda \geq 2 \times 10^{-6}$.

![Graph](image1)

Hatched region denotes the data range at $E\lambda \geq 2 \times 10^{-6}$.

![Graph](image2)

Hatched region denotes the data range at $E\lambda \leq 2 \times 10^{-6}$.

**Fig. 8(a)** $\alpha$ vs. $\alpha$ at atmospheric pressure

**Fig. 8(b)** $\alpha$ vs. $\alpha$ at pressure 70 kg/cm$^2$ abs.

**IV. CONCLUSIONS**

The applicability of the previously proposed void fraction correlation was investigated on the recent data of boiling steam-water flows in tubes. The value of $K$ is expressed by the properties of fluid and the diameter of the pipe as expressed in Eq. (7), and this gives a generalized correlation for void fraction. The correlation was compared with the data listed in Table 1 and the following conclusions were obtained.

1. The void fraction can be predicted within $\pm 15\%$ accuracy, from Eq. (2) and the criterion shown in Eq. (7).

2. The effects of superficial liquid velocity and heat flux on the void fraction fall within the range of $\pm 15\%$ and no definite tendency was found within the range.

**NOMENCLATURE**

- $d$: Diameter of tube
- $Eo = \Delta \rho / \sigma$
- $Fr = \rho (w^2 + w_0^2) / \Delta \rho / d$
- $g$: Gravitational acceleration
- $K$: Parameter defined by Eq. (1)
- $\rho$: Pressure
- $q$: Heat flux
- $Re = (w_0^2 + w_{00}^2) / \nu_1$
- $V_1$: Inlet water velocity
- $We = \rho d (w_{00}^2 + w_0^2) / \sigma$
- $W_1$: Mass velocity of water
- $w_0$: True gas velocity
- $w_1$: True water velocity
- $w_{00}$: Superficial gas velocity
- $w_{10}$: Superficial water velocity
- $\epsilon$: Void fraction
- $\beta$: Volumetric flow concentration of gas
- $d\rho = p_1 - \rho_0$
- $\lambda = \nu_1 / \rho_1$
- $\nu_1$: Kinetic viscosity of water
- $\rho_0$: Density of gas
- $\rho_1$: Density of water
- $\sigma$: Surface tension

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