Estimation of Neutron Yield from Individual Fragment in Medium-Excitation Fission

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A method of calculation is described to estimate the average number of neutrons emitted per fragment in medium-excitation fission from published experimental data on neutron emission in thermal-neutron induced fission, average total kinetic energy as a function of fragment mass and mass yield in low- and medium-excitation fission reactions. Use is made of a relation of fragment excitation energy with internal excitation and deformation energies, and the difference in kinetic energy between the fission reactions at two-excitation energies. A tentative calculation is made for the fission of $^{238}$U induced by 12 MeV protons. The results are in good agreement with experimental data.

The method developed in the present work may make it possible to predict the average number of neutrons emitted from individual fragment in medium-excitation fission which has not yet been measured experimentally.

**KEYWORDS**: uranium 238, meV range 10-100, protons, fission fragments, excitation energy, neutron emission yield, medium excitation fission, calculations

I. INTRODUCTION

Information on the dependence of the number of neutrons emitted per fragment on the excitation energy of a fissioning nucleus provides a basis for further understanding of the fission process, in particular the mechanism by which fragment formation and scission occur. The average number of emitted neutrons $\nu_i(m_i)$ as a function of pre-neutron-emission fragment mass $m_i$ has, however, been obtained mainly for low-excitation fission, such as spontaneous fission and thermal-neutron induced fission. (Hereafter the subscript $i$ refers to the $i$-th fragment, $i=1,2$.) The experimental data on $\nu_i(m_i)$ for medium- and high-excitation fission are scanty and have been given almost for charged-particle induced fission(7)–(9).

Though the values of $\nu_i(m_i)$ in fast-neutron induced fission are very important in nuclear engineering, these data have not been reported except our previous work(7) because of experimental difficulties caused by inherent high background and low yield. So far as we know, only one statistical estimation method has been proposed for medium-excitation fission by Sharma et al.(8), who have calculated neutron evaporation, together with isotopic and charge distributions and independent yields of individual fragments in fission of $^{235}$U induced by 2 and 14.7 MeV neutrons. Unfortunately, this estimation method of $\nu_i(m_i)$ cannot be applied to a general case, because the precise data are not

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always available on the mass yield and charge distribution parameters of fission products to be used as the basic input data. In addition, the contribution from higher-order-chance fission and the dependence of shell effect on the excitation energy of fissioning nucleus are not taken into account in this method.

In the present paper, a method is described to calculate \( \nu_i(m_i) \) in medium-excitation fission, using a relation of fragment excitation energy with internal excitation and deformation energies, and the difference in kinetic energy between the fission reactions at two-excitation energies. As basic input data use is made of published experimental results for \( \nu_i(m_i) \) in thermal-neutron induced fission, average total kinetic energy as a function of fragment mass and mass yield in \( {}^{238}\text{U}(p, f) \) and neutron-induced fission of \( {}^{235}\text{U} \) and \( {}^{238}\text{U} \). A tentative calculation is carried out for \( \nu_i(m_i) \) in \( {}^{238}\text{U}(p, f) \) reaction at an incident-proton energy \( E_p \) of 12 MeV. The results are then compared with published experimental data.

II. PROCEDURE OF CALCULATION

1. Excitation Energy of Fragment and \( \nu_i(m_i) \)

The present calculation method is based on the assumption that the average excitation energy \( E_X(m_i) \) of fragment in medium-excitation fission is given by the sum of three terms: one of them is \( E_X(m_i) \) in low-excitation fission and other two are the difference in average internal excitation energy \( E_I(m_i) \) and that in average deformation energy \( V_D(m_i) \) of fragment between low- and medium-excitation fission reactions. Namely, \( E_X(m_i) \) in medium-excitation fission is expressed by

\[
E_X^{[j]}(m_i) = E_X^{[j]}(m_i) + \Delta E_I(m_i) + \Delta V_D(m_i), \quad (i=1, 2),
\]

where the superscripts \( L \) and \([j]\) denote the low-excitation fission and the \( j \)-th-chance fission, respectively, and \( \Delta \) the increment in each quantity in medium-excitation fission over that in low-excitation fission.

The value of \( E_X^{[j]}(m_i) \) is calculated from the average number of neutrons \( \nu_i^{[j]}(m_i) \) and average energy of \( \gamma \)-rays \( E_l^{[j]}(m_i) \) emitted from \( i \)-th fragment in low-excitation fission by the following equation

\[
E_X^{[j]}(m_i) = \{\eta_i^{[j]}(m_i) + B_i^{[j]}(m_i)\} \nu_i^{[j]}(m_i) + E_l^{[j]}(m_i) , \quad (i=1, 2),
\]

where \( \eta_i(m_i) \) and \( B_i(m_i) \) are the average center-of-mass kinetic energy and average binding energy of neutron, respectively. The \( \nu_i^{[j]}(m_i) \) value is estimated by the equation \( \eta_i^{[j]}(m_i) = 0.653\sqrt{\nu_i^{[j]}(m_i)+\nu_i^{[j]}(m_2)+1} \) (MeV) formulated by Terrell(9) on the basis of evaporation theory. The value of \( B_i^{[j]}(m_i) \) is evaluated by averaging the neutron binding energies in the cascade of neutron emission, which are obtained from the mass table compiled by Seeger(10) on the assumption that the charge distribution is Gaussian along isobaric line. The \( E_l^{[j]}(m_i) \) value is assumed to be one-half of the binding energy of the most loosely bound neutron, or the \( \nu_i^{[j]}(m_i)+1 \)-th neutron, and to be averaged with an appropriate weight for nonintegral value of \( \nu_i^{[j]}(m_i) \).

The data on \( \nu_i^{[j]}(m_i) \) are not always available for the fissioning systems under consideration. We can, here, recall a general dependence of neutron yield on fragment mass in low-excitation fission described by Terrell(9). The generality must, however, be somewhat qualified, because in the region of \( m_i = 105 \sim 120 \) amu the value of \( \nu_i^{[j]}(m_i) \)
for each fissioning nucleus is different according as the complementary fragment is deformed or undeformed\(^{(12)}\). In the present work, the \(\nu_{ij}(m_i)\) value is, therefore, derived by interpolating the results obtained by Apalin et al.\(^{(13)}\) for \(^{239}\)U\((n_{th}, f)\) and \(^{239}\)Pu\((n_{th}, f)\) reactions. The excitation energy \(E_F\) of these fissioning nuclei is low and estimated to be about 6.6 MeV for both reactions from the mass table of Seeger\(^{(10)}\). Except the region of \(m_i=105\sim120\) amu, the value of \(\nu_{ij}(m_i)\) used in the present work agrees approximately with Terrell's general curve of neutron yield.

A simple picture of energy division at scission has been proposed by Burnett et al.\(^{(4)}\) to interpret the difference in energetics between medium-excitation fission reactions of \(^{233}\)U by 8.5 and 13 MeV protons. It is based on a static scission configuration as follows: an increase in \(E_F\) causes only the heavy fragment to soften somewhat, thereby increasing \(V_{Di}(m_i)\) and correspondingly decreasing the average mutual Coulomb potential energy \(V_C(m_i)\) (and therefore also the average total fragment kinetic energy \(E_K(m_i)\)) as a function of mass of one nucleus of a fragment pair. The increase in \(E_F\) results in an equal increase in the sum of \(E_{1i}(m_i)\). In the present work, this picture is extended to the case of a large difference in \(E_F\) such as that between low- and medium-excitation fission reactions. The following equations are assumed to hold:

\[
\Delta V_{Pi}^{ij}(m_i) = -\Delta V_{Di}^{ij}(m_i) = -\Delta E_{K}^{ij}(m_i), \quad (i=1, 2) \tag{3a}
\]

for heavy fragment mass region,

\[
\Delta V_{Pi}^{ij}(m_i) = 0, \quad (i=1, 2) \tag{3b}
\]

for light fragment mass region and

\[
\Delta E_{K}^{ij} = \Delta E_{K}^{ij}(m_i) + \Delta E_{K}^{ij}(m_2) \tag{4}
\]

for the increment in \(E_F\).

If the nuclear temperature is assumed to be uniform throughout the fissioning nucleus at scission, \(\Delta E_{K}\) is divided between the fragments in proportion to the respective mass. That is:

\[
\Delta E_{K}^{ij}(m_i) = \Delta E_{K}^{ij}(m_i) \frac{m_i}{A^{ij}}, \quad (i=1, 2) \tag{5}
\]

where \(A^{ij}\) is the mass of \(j\)-th-chance fissioning nucleus. The value of \(\Delta E_{K}^{ij}\) is easily obtained from the kinetic energy of incident particles which induce fission reactions. The value of \(\Delta E_{K}^{ij}\) is given by the equation

\[
\Delta E_{K}^{ij} = \Delta E_{K}^{ij} - B_{n} - \eta_{n},
\]

where \(B_{n}\) is the binding energy of neutron emitted from fissioning nucleus and \(\eta_{n}\) the kinetic energy of the emitted neutron. Usually, about 1.6 MeV is taken as \(\eta_{n}\).

The method to estimate the value of \(\Delta E_{K}^{ij}(m_i)\) will be discussed in the next section. If this value is obtained, \(E_{K}^{ij}(m_i)\) in medium-excitation fission can be evaluated by substituting Eqs. (2), (3a), (3b) and (5) into Eq. (1). Then, \(\nu_{ij}(m_i)\) in \(j\)-th-chance fission is derived from the equation

\[
\nu_{ij}^{ij}(m_i) = \frac{E_{K}^{ij}(m_i) - E_{K}^{ij}(m_i)}{\eta_{K}^{ij}(m_i) + B_{n}^{ij}(m_i)}, \quad (i=1, 2). \tag{6}
\]
Since $E^{ij}_{K}(m_i)$, $B^{ij}(m_i)$ and $\eta^{ij}_{K}(m_i)$ depend on $\nu^{ij}_{K}(m_i)$, $\nu^{ij}_{K}(m_i)$ is taken at first as $\nu^{ij}_{K}(m_i)$-value and other three values are obtained by the same method as that described below Eq. (2). An iterative procedure is used until satisfactory convergence is obtained for the value of $\nu^{ij}_{K}(m_i)$. The total number of emitted neutrons $\nu^T(m_i)$ is given by a superposition of $\nu^{ij}_{K}(m_i)$. That is:

$$\nu^T(m_i) = \sum_j \nu^{ij}_{K}(m_i) \alpha^{ij}(m_i), \quad (i=1, 2),$$

(7)

where $\alpha^{ij}(m_i)$ is the probability of formation of fragment $m_i$ in $j$-th-chance fission and defined by

$$\alpha^{ij}(m_i) = \frac{Y^{ij}(m_i) R^{ij}(m_i)}{Y(m_i)}, \quad (i=1, 2),$$

(8)

and

$$\sum_j Y^{ij}(m_i) R^{ij}(m_i) = Y(m_i), \quad (i=1, 2).$$

(9)

Here $Y^{ij}(m_i)$ and $Y(m_i)$ are the fragment mass yields in $j$-th-chance fission and total fission, respectively, and $R^{ij}(m_i)$ the ratio of $j$-th-chance fission to that of total fission. The values of $Y^{ij}(m_i)$ and $\alpha^{ij}(m_i)$ will be obtained by the method described in the next section.

2. Derivation of $\Delta E^{ij}_{K}(m_i)$, $Y^{ij}(m_i)$ and $\alpha^{ij}(m_i)$ from Published Data

To obtain $\Delta E^{ij}_{K}(m_i)$, $Y^{ij}(m_i)$ and $\alpha^{ij}(m_i)$, the data should be given on $E_{K}(m_i)$, $Y(m_i)$ and $R^{ij}$ in the fission at the original compound-nucleus excitation energy $E_F$ equal to $E^{ij}_{K}$ and also in the low-excitation fission. Such data are, however, not always available at present. The data in proton-induced fission of $^{238}$U have been obtained by Ferguson et al. (14) for relatively wide range of $E_p$ (8~13 MeV). Lower-energy proton experiment is very difficult due to Coulomb barrier repulsion, but the data in fission reactions induced by lower energy neutrons have been obtained systematically for $^{235}$U and $^{238}$U (15)(16). The following procedure is, therefore, adopted to derive the values of $\Delta E^{ij}_{K}(m_i)$ and $Y^{ij}(m_i)$ from these published data on the assumption that at a given excitation energy the increment in $E_{K}(m_i)$ does not change with fissioning system and the fragment-mass yield varies linearly with the mass of fissioning nucleus.

Now, suppose $E_{KA}(m_i)$ and $E_{KB}(m_i)$ are the values of $E_{K}(m_i)$ in fission of $^{235}$U at the proton energy $E_p=\varepsilon_A$ and $E_p=\varepsilon_B$, respectively. Since the threshold proton energy is about 7 MeV for second-chance fission of $^{238}$U, both the first- and second-chance fission reactions are energetically possible in the proton energy range of 8~13 MeV. Then, the difference between $E_{KA}(m_i)$ and $E_{KB}(m_i)$ is expressed by the relation

$$E_{KA}(m_i) - E_{KB}(m_i) = \sum_{j=1}^{2} \left\{ \Delta E^{ij}_{K}(m_i) \alpha^{ij}(m_i) - \Delta E^{ij}_{K}(m_i) \alpha^{ij}(m_i) \right\}, \quad (i=1, 2),$$

(10)

where subscripts A and B mean the quantities at $E_p=\varepsilon_A$ and $E_p=\varepsilon_B$, respectively. We take 8 MeV as $\varepsilon_A$ and the value of $E_p$, at which $E_p$ is equal to that of the fissioning compound nucleus under consideration, as $\varepsilon_B$. Then, $E_{KA}(m_i)$, $Y_{A}(m_i)$ and $R^{ij}_{A}$ are given by the values at $E_p=8$ MeV and $E_{KB}(m_i)$, $Y_{B}(m_i)$ and $R^{ij}_{B}$ at $E_p=\varepsilon_B$ from interpolation of the results of Ferguson et al.

The values of $E^{ij}_{K}$ are 13.6 and 5.6 MeV for $j=1$ and 2 in the fission of $^{238}$U induced by 8 MeV protons, respectively. Therefore, $Y^{ij}_{K}(m_i)$ is replaced by the value interpolated between the fragment mass yields in fission reactions of $^{238}$U induced by thermal neu-
trons and $^{238}$U induced by 1.3 MeV neutrons ($E_F = 6.6$ MeV in both cases, which is nearly equal to $E_F^{(2)}$) and $Y_{Y}^{(m)}(m_i)$ is obtained similarly from the data in the neutron-induced fission reactions of $^{235}$U and $^{238}$U at the value of $E_F^{(2)}$. Then, the values of $Y_{Y}^{(m)}(m_i)$ and $Y_{Y}^{(m)}(m_i)$ are obtained from Eq. (9) and those of $\alpha_{Y}^{(m)}(m_i)$ and $\alpha_{Y}^{(m)}(m_i)$ from Eq. (8).

While, $DE_{1}^{(A)}(m_i)$ is replaced by the difference in $E_{K}(m_i)$ between the first-chance fission induced by 7 MeV neutrons and thermal-neutron induced fission of $^{235}$U ($E_F = 13.6$ MeV which is equal to $E_F^{(1)}$ and $=6.6$ MeV, respectively) and $DE_{1}^{(B)}(m_i)$ by the difference in $E_{K}(m_i)$ between fission reactions of $^{238}$U at $E_F$-values of $E_F^{(2)}$ and 6.6 MeV. The total kinetic energy $E_{K}^{(m)}(m_i)$ in the first-chance fission of $^{235}$U induced by 7 MeV neutrons is related with those $[E_{K}^{(m)}(m_i) \text{ and } E_{K}(m_i)]$ in the second-chance fission ($E_F \approx 5.6$ MeV) and the total fission by the equation

$$E_{K}(m_i) = E_{(m_i)}^{(1)} + DE_{1}^{(A)}(m_i) + DE_{1}^{(B)}(m_i) + DE_{2}^{(m_i)},$$

(11)

where $E_{K}^{(m)}(m_i)$ is given by the value of $E_{K}(m_i)$ in the thermal-neutron fission of $^{238}$U, because $E_F$ in the second-chance fission of $^{235}$U induced by 7 MeV neutrons is approximately equal to that in the thermal-neutron fission. In calculation of $\alpha_{Y}^{(m)}(m_i)$ from Eq. (8), use is made of the value of $R^{(j)}$ reported by Howerton (17) and that of $Y_{Y}^{(m)}(m_i)$ derived similarly as $Y_{Y}^{(A)}(m_i)$ and $Y_{Y}^{(B)}(m_i)$. The value of $E_{K}^{(m)}(m_i)$ is given by $E_{K}^{(m)}(m_i)$ derived from Eq. (11) and $DE_{1}^{(A)}(m_i)$ is obtained. Then, $DE_{1}^{(B)}(m_i)$ is calculated from Eq. (10), taking null as $DE_{2}^{(A)}(m_i)$ because the value of $E_{K}^{(m)}(m_i)$ is nearly equal to that of $E_{K}$ in fission of $^{238}$U induced by thermal neutrons.

When the fission under consideration is $^{238}$U($p, f$), the values of $DE_{1}^{(A)}(m_i)$ and $\alpha_{Y}^{(A)}(m_i)$ are given by $DE_{1}^{(A)}(m_i)$ and $\alpha_{Y}^{(A)}(m_i)$, respectively. For other fission reactions, $DE_{1}^{(m)}(m_i)$ is replaced by $DE_{1}^{(m)}(m_i)$ and $DE_{1}^{(m)}(m_i)$ by the difference in $E_{K}(m_i)$ between fission reactions of $^{235}$U at $E_F$-values of $E_F^{(2)}$ and 6.6 MeV. The value of $\alpha_{Y}^{(m)}(m_i)$ is given by the following method. The value of $Y_{Y}^{(m)}(m_i)$ is obtained by the procedure similar to that in the case of $Y_{Y}^{(m)}(m_i)$ and $Y_{Y}^{(m)}(m_i)$. The values of $R^{(j)}$ for the proton-induced fission of $^{235}$U and $^{238}$U are given by the results of Ferguson et al. (14) and those for the neutron-induced fission of $^{235}$U, $^{238}$U and $^{239}$Pu by the results of Howerton (17). In general, $R^{(j)}$ may be estimated also from the method described by Huizenga & Vandenbosch (18). The value of $\alpha_{Y}^{(m)}(m_i)$ in Eq. (7) is evaluated by substituting the values of $Y_{Y}^{(m)}(m_i)$ and $R^{(j)}$ into Eq. (8).

III. APPLICATION TO CALCULATION OF $\nu_{(m)}$ IN FISSON OF $^{238}$U INDUCED BY 12 MEV PROTONS

To see the validity of the present calculation method, it is applied to the fission of $^{238}$U induced by 12 MeV protons, where some results (4)~(5) for $\nu_{(m)}$ have been obtained experimentally. The fissioning nuclei in the first- and second-chance fission reactions are $^{239}$Np ($E_F = 17.6$ MeV) and $^{238}$Np ($E_F \approx 9.4$ MeV), respectively.

The values of $E_{K}^{(m)}(m_i)$ are derived from Eq. (2) by taking $\nu_{(m)}^{(J)}(m_i)$ for $A^{(j)}=239$ and 238 amu as described in Sec. II-1. As the value of $\varepsilon_{B}$ is 12 MeV, the increments $DE_{J}^{(P)}$ are about 11.0 and 2.8 MeV for $j=1$ and 2, respectively, from which $DE_{1}^{(m)}(m_i)$ is obtained by Eq. (5). The value of $DE_{2}^{(m)}(m_i)$ for heavy fragment mass range can be given by Eq. (3a), $-DE_{2}^{(m)}(m_i)$ being estimated as described in Sec. II-2.

Substituting the values of $E_{K}^{(m)}(m_i)$, $DE_{1}^{(m)}(m_i)$ and $DE_{2}^{(m)}(m_i)$ thus obtained into Eq. (8),
(1), the value of \( E_{ij}^{\mu}(m_i) \) is evaluated, from which \( \nu_{ij}^{\mu}(m_i) \) can be calculated by Eq. (6). Then, \( \nu_{ij}^{\mu}(m_i) \) is given by Eq. (7) using the values of \( a_{ij}^{\mu}(m_i) \) obtained in Sec. II-2.

**IV. RESULTS AND DISCUSSIONS**

Figure 1 shows the values of \( \nu_{ij}^{\mu}(m_i) \) in first-chance (dotted curve) and second-chance (dot-dashed curve) fission reactions. The increments \( \Delta \nu_{ij}^{\mu}(m_i) \) in \( \nu_{ij}^{\mu}(m_i) \) \((j=1, 2)\) in medium-excitation fission over that in low-excitation fission are shown in Fig. 2 by dotted and dot-dashed curves, respectively. The \( \Delta \nu_{ij}^{\mu}(m_i) \) values are larger in heavy fragment region than those in light fragment region and show a maximum at \( m_i \approx 132 \text{ amu} \) in both cases. These facts are consistent with the results observed experimentally(4)(6), which may be attributed to the thermal equilibrium in the nascent fragments and the diminution or disappearance of shell effects at higher temperature. In the present calculation, the former can be ascribed to the assumption of Eq. (5) for \( \Delta E_{ij}^{\mu}(m_i) \) and the latter to the introduction of \( \Delta V_{ij}^{\mu}(m_i) \) in Eq. (1).

![Graph showing number of neutrons as function of fragment mass](image)

**Fig. 1** Number of neutrons as function of fragment mass for \(^{238}\text{U} (p, f)\) at \( E_p = 12.0 \text{ MeV} \)

In Fig. 3 the \( \nu_{ij}^{\mu}(m_i) \) values calculated from Eq. (7) are compared with the experimental results for \(^{238}\text{U} (p, f)\) at \( E_p = 12 \text{ MeV} \) obtained by Cheifetz & Fraenkel(5) (open circles), Bishop et al.(6) (open triangles) and Burnett et al.(4) (closed circles). The agreement with those of Bishop et al. is better, though the \( \nu_{ij}^{\mu}(m_i) \) curve derived in the present work lies between the experimental values except around the maximum and reproduces the general trend of the three experimental results. The maximum in \( \nu_{ij}^{\mu}(m_i) \) occurs in the region of \( m_i = 112 \sim 117 \text{ amu} \) in all four cases, but the absolute values of the present work are somewhat higher. Probable errors \( \delta \nu_{ij}^{\mu}(m_i) \) in \( \nu_{ij}^{\mu}(m_i) \) are large in
Fig. 2 Increment in number of neutrons as function of fragment mass in medium-excitation fission over that in low-excitation fission.

Fig. 3 Comparison of present calculated results with experimental data on number of neutrons as function of fragment mass for $^{238}$U \((p, f)\) at $E_p = 12.0$ MeV.
this region, for example about $\pm 0.35$ neutron around $m_1=115$ amu, because of the uncertainties in the input data $\nu^{LJ}(m_1)$. While $\delta \nu^{LJ}(m_1)$ is less than $\pm 0.05$ neutron in the regions of both peaks in mass distribution (at $m_1=101$ and 138 amu). The calculated $\nu^{LJ}(m_1)$ curve shows the minimum around $m_1=132$ amu corresponding to the double-closed-shell configuration ($Z=50, N=82$).

These experimental results represent the sum of prefission- and postfission-neutrons. Though the input data on $\nu^{LJ}(m_1)$ include the number of prefission-neutrons $\nu_p$, the increment $\Delta \nu_p$ is not taken into account in the present calculation. It appears from the results of Cheifetz & Fraenkel's work that the value of $\Delta \nu_p$ is mainly due to the neutrons evaporated from the compound nucleus in competition with fission, the average number of which has been estimated to be 0.25 to 0.42 neutron per fission. Even if the half of this value is added to each $\nu^{LJ}(m_1)$ value, the agreement between the present calculation and the experiments remains satisfactory.

It is found that the method developed in the present work has made it possible to predict well the average number of emitted neutrons as a function of fragment mass in fission of $^{238}$U induced by 12 MeV protons. For a fission, in which $\nu(m_i)$ is to be obtained, the experimental values of $\nu^{LJ}(m_i)$, $E_K(m_i)$ and $Y(m_i)$ required as input data of calculation have not always been reported. These data, however, can be replaced by the values calculated from available experimental data as described in Chap. II. The procedure to calculate $\nu^{LJ}(m_i)$ can be used in the fissioning nucleus mass range of $A=236-240$ amu. For $A$ less than 236 amu such as in the case of $^{232}$Th(n, f), $\nu^{LJ}(m_i)$ are derived by interpolating or extrapolating the results obtained by Apalin et al. for $^{233}$U(n$_{th}$, f) and $^{235}$U(n$_{th}$, f) reactions. The calculation procedures of $E_K(m_i)$ and $Y(m_i)$ can be used for excitation energies of fissioning nucleus lower than about 18 MeV, which correspond approximately to the energies of incident particles (neutrons or protons) less than about 13 MeV. It is, therefore, expected that the present estimation method of $\nu(m_i)$ can be applied to any medium-excitation fission in the above mentioned regions of $A$ and $E_F$.

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