Indication Lowering of Average Magnitude Type Campbelling System in Low Pulse Rate Region

Dependence of Characteristic on Product of Pulse Rate and Pulse Width

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In the nuclear power calibration process of JOYO, the first experimental fast breeder reactor in Japan, the indication of the Intermediate Range Monitoring System (IRMS), employing Average Magnitude type Campbelling System (AMCS), was calibrated with the reactor power at 45.82 kW. The reactor power was then decreased and the nonlinear indication lowering of the IRMS was observed. In this paper, we present a derivation of an equation representing the AMCS characteristic and show that the indication lowering occurs when the product of mean arrival rate of pulses and their width is small. The computed values based on the derived equation agreed very well with the observed ones in the JOYO IRMS below 45.82 kW, and it was confirmed that the evaluation method was applicable. Furthermore, it was evident from the evaluation that the indication lowering of the JOYO IRMS above 45.82 kW was negligibly small, and did not affect the reactor power ascension testing. Also, it was proved in a following thermal calibration test above 50 kW that the indication lowering was allowably small indeed in this power region.

KEYWORDS: nuclear instrumentation, nuclear monitoring, Campbelling system, average magnitude, statistical fluctuation, JOYO reactor, calibration, nuclear power

I. INTRODUCTION

Recently, Campbelling systems based upon the statistical fluctuation method have been used to monitor reactor power or neutron flux. There are two types of Campbelling systems. They are (1) the average magnitude type and (2) true mean squared type. The Average Magnitude type Campbelling System (AMCS) is based on the principle that the mean value of the rectified half wave of the chamber current fluctuation is proportional to the square root value of the neutron flux.

Because the AMCS has the advantage being able to reduce the dynamic range of the measuring circuit to 1/2 of the mean squared type, the AMCS is employed in Nuclear

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Instrumentation System (NIS) of “JOYO”, the first experimental fast breeder in Japan\(^{(1)}\).

In the calibration process of the JOYO NIS below the 50 kW power level, we experienced a nonlinear indication lowering of the Intermediate Range Monitoring System (IRMS) employing the AMCS. It was considered that the indication lowering effect was caused by the deviation of the amplitude distribution of the chamber current fluctuation from normal distribution, because of the shortage of the number of arrival pulses\(^{(2)}\). Hence, it was required that the calibration error of the IRMS caused by the indication lowering effect be evaluated before conducting the initial power ascension test.

In this paper, the results of the evaluations of the indication lowering and the calibration error are described. We first derive an equation representing the AMCS characteristic, or the mean value of the rectified half wave of the current fluctuation caused by superposition of chamber pulses, assuming the arrival rate of the pulses obeys a Poisson distribution and consider the characteristic of the AMCS. Next, we evaluate the relation between the pulse arrival rate and the indication lowering in the range of low neutron flux based on the derived equation, and the indication lowering observed in the JOYO IRMS. Finally, we establish an index from the results at which measurements without correction can be made.

II. Expression of AMCS Characteristic

1. Derivation of Representative Equation

In the following, we derive an expression for the AMCS characteristic or the mean value of the rectified half wave of the current fluctuation caused by superposition of chamber pulses.

The probability distribution \(p(i)\) of the current fluctuation is given as follows by Ref. (3) based on the assumptions that the arrival probability of chamber pulses obeys a Poisson distribution and the pulse height is constant, though actual pulse height from ionization chamber is distributed in a certain range:

\[
p(i) = \frac{1}{\sigma} \phi^{(0)}(z) + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\lambda_n}{\sigma^n} \phi^{(n)}(z) + \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\lambda_n}{n(n-1)!} \frac{\lambda_n \lambda_m m}{m!(n-1-m)!} \frac{1}{\sigma} \phi^{(n)}(z),
\]

\[\text{where } \sigma^2 = \nu \int_{-\infty}^{\infty} f^2(t) \, dt, \quad \phi^{(n)}(z) = \frac{1}{\sqrt{2\pi}} \frac{dn}{dz} e^{-z^2/2}, \quad z = i/\sigma, \quad \lambda_n = \nu \int_{-\infty}^{\infty} f^2(t) \, dt,
\]

for mean arrival rate of pulses \(\nu\), and functional expression of a pulse wave-form \(f(t)\).

The mean value \(\langle i \rangle\) of the rectified half wave of the current fluctuation is represented by

\[
\langle i \rangle = \int_{0}^{\infty} ip(i) \, di.
\]

Substituting Eq. (1) into Eq. (2), the following equation is obtained as shown in APPENDIX:

\[
\langle i \rangle = \frac{\sigma}{\sqrt{2\pi}} \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{\lambda_n \sigma^{2n}}{2n-k} \right) \sum_{k=0}^{n-k} \frac{(n-k)! 2^{n-k}}{k! [2(n-k)]!} \right] - \frac{\sigma}{\sqrt{2\pi}} \left( \frac{\lambda_n \sigma^{2n+1}}{2n+1-k} \right) \sum_{m=0}^{n+1} \frac{(n+1-k)! 2^{n-k}}{k! [2(n+1-k)]!} \right].
\]

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The above equation will be changed into more intuitive expression to consider the effects of pulse height and pulse width on the AMCS characteristic. Describe the normalized function of $f(t)$ by $g(t)$ such that

$$g(t) = \frac{f(t)}{h},$$

(4)

where $h$ is peak value of $f(t)$, and pulse width of $f(t)$ by $a$. Next, perform a variable transformation from $t$ to $\tau$ using the equation,

$$\tau = \frac{x}{a} t,$$

(5)

for $x$ which is determined as satisfying the following equations:

$$\int_{-\infty}^{\infty} g^k(t) dt = \frac{a}{x^k}.$$

(6)

Then, the following relationships are established:

$$\lambda_{2n} \sigma^{-2n} = \frac{x^{n-1}}{(\nu a)^{n-1}} \Gamma_{2n},$$

(7)

$$\lambda_{m} \lambda_{2(n+1)-m} \sigma^{-(2n+1)} = \frac{x^{m-1}}{(\nu a)^{m-1}} \Gamma_{m} \Gamma_{2(n+1)-m},$$

(8)

where $\Gamma_{n} = \int_{-\infty}^{\infty} g^n \left( \frac{a}{x} \right) d\tau$.

Then, Eq. (3) can be transformed as follows:

$$\langle i \rangle = \frac{h}{\sqrt{2\pi x}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{x^{n-1}}{(\nu a)^{n-1}} \Gamma_{2n} \xi(n) \left( \sum_{m=0}^{2n+1} \frac{(2n+1)! \Gamma_{m} \Gamma_{2(n+1)-m}}{m! (2(n+1)-m-1)!} \right) \xi(n+1) \right],$$

(9)

where

$$\xi(n) = \sum_{k=0}^{n} \left( -\frac{1}{2} \right)^{k} \frac{(n-k)! \Gamma}{k! \Gamma(n-k)!}.$$

The above Eq. (9) represents the characteristic of AMCS. In the following section, we consider the effects of $n$, $a$, $\gamma_n$, $x$ and $h$ in Eq. (9) on the output characteristic of AMCS.

2. Consideration on AMCS Characteristic

First, we investigate the effects of $\nu$ and $a$. The summation terms with respect to $n$ in Eq. (9) represent the deviation from the true indication value. It is clear from Eq. (9) that the terms representing the deviation are negligibly small compared to the first term in the brackets if $\nu a$ is large enough. Then Eq. (9) can be written as follows and statistical fluctuation method can be applied:

$$\langle i \rangle = \frac{h}{\sqrt{2\pi x}} \sqrt{\nu a}.$$

(10)

Conversely, as $\nu a$ becomes small, the summation of the terms representing the deviation can not be neglected compared to the first terms, and indication lowering takes place as explained in detail in a later section.

It must be noticed that $\nu$ always appears in the form of a product with $a$, instead of appearing solely as $\nu$ itself in Eq. (9). This means that the product, $\nu a$ plays a substantial
role in the statistical fluctuation method. In other words, the mean value of the rectified half wave of the chamber current fluctuation is proportional to $\sqrt{\nu a}$ rather than $\sqrt{\nu}$ in the high neutron flux region, and the indication lowering is dominated by $\nu a$ rather than $\nu$ in the low neutron flux region. To make the pulse width wide is effective in making the indication lowering small. For instance, if the pulse rate becomes $1/N$ of planned value due to the detector sensitivity or configuration, we can increase the pulse width $N$ times to keep resulting $\langle i \rangle$ invariant.

Next, we consider effects of $g_n$, $x$ and $h$. These variables depend on the pulse shape. Notice that $g_n$ and $x$ are independently defined from $h$. It is evident that $h$ does not affect the indication fall, but only absolute magnitude of $\langle i \rangle$, as $h$ does not appear in the brackets of Eq. (9). To evaluate the indication lowering dependence on $g_n$ and $x$, the pulse shape must be given. We will evaluate the effects of $g_n$ and $x$ for various pulse shapes in the subsequent section.

### III. Evaluation of Indication Lowering

1. Numerical Computation of $\langle i \rangle$ for Some Pulse Shapes

Calculating $g_n$ and $x$, we determined the specific expression of $\langle i \rangle$ for some expected pulse shapes. The results are shown in Table 1. To perform the calculations of $g_n$ and $x$, the pulse width $a$ must be given. The pulse width is determined such that the pulse charge, or area of the pulse, are identical in order to enable comparison of the different pulse shapes.

We also numerically computed $\langle i \rangle$ based on the expression for the pulse shapes. The results, which indicate the ratio of $\langle i \rangle$ to the true value, are shown in Table 2. It is evident from the results that these deviations bring on indication lowering. The lowering ratio is obtained by subtracting the value in Table 2 from unity.

In the numerical computation, there were difficulties in the calculations due to the addition and subtraction of very large and very small numerical values. Also, it was found that the number of terms $n$ to converge $\langle i \rangle$ to desired value increases as $\nu a$ decreases. Therefore, results were only obtained in the limited range of about one or two decades after the beginning of the fall as shown in Table 2. However, the difference due to pulse shape is at most 3% of the value in the range.

2. Evaluation of Indication Lowering of JOYO IRMS

In the following, we evaluate the indicated lowering of the JOYO IRMS, which employs AMCS as shown in Fig. 1. Where, the current fluctuation signal from the fission counter is converted to the voltage signal at the first stage amplifier.

In the JOYO nuclear power calibration process, after the indication of the IRMS was calibrated with the reactor power at 45.82 kW ($\nu a=2.0$), which is the upper limit that can be calibrated by nuclear method, the reactor power was decreased in steps to 12.6 kW ($\nu a=0.535$) and 1.21 kW ($\nu a=0.05$). Then, the nonlinear indication lowerings of the IRMS were observed, and it was necessary to evaluate the lowering above 45.82 kW, at which the calibration of the IRMS was made, before conducting the initial power ascension test.

To evaluate the lowering, the pulse shape was approximated by a triangular one having 5 $\mu s$ width (based on observation of the actual pulse at the input point of the wide-range rectifier). The pulse rate used in the evaluation was the actually measured one.

According to the results of the numerical computation, as shown Table 1, based on the equation representing the AMCS characteristic, the indication lowering should be 0.8%
Table 1: Expression of \( <i> \) some expected pulse shapes

<table>
<thead>
<tr>
<th>Function ( f(t) )</th>
<th>Graph</th>
<th>( x, y_{2n} )</th>
<th>( &lt;i&gt; ) where ( \xi(n) = \sum_{k=0}^{n} \left( \frac{-1}{2} \right)^k \frac{1}{k!} \frac{n-k}{2^{n-k}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangular pulse</strong></td>
<td>( h ) for ( 0 \leq t \leq a )</td>
<td>( x = \frac{1}{h^2} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \frac{1}{2^{n+1}} \right] )</td>
</tr>
<tr>
<td>for ( t &lt; 0 ), ( t &gt; a )</td>
<td>( y_{2n} = x^{-(n-1)} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
<td></td>
</tr>
<tr>
<td><strong>Triangular pulse</strong></td>
<td>( -h \left( \frac{t}{2a} - 1 \right) ) for ( 0 \leq t \leq 2a )</td>
<td>( x = \frac{3}{h^2} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
</tr>
<tr>
<td>for ( t &lt; 0 ), ( t &gt; 2a )</td>
<td>( y_{2n} = \frac{3^n}{2n+1} x^{-(n-1)} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
<td></td>
</tr>
<tr>
<td>( -\frac{h}{a}</td>
<td>t</td>
<td>-1 ) for (</td>
<td>t</td>
</tr>
<tr>
<td>for (</td>
<td>t</td>
<td>&gt; a )</td>
<td>( y_{2n} = \frac{2^n}{2n} x^{-(n-1)} )</td>
</tr>
<tr>
<td><strong>Exponential pulse</strong></td>
<td>( he^{-t/a} ) for ( t \geq 0 )</td>
<td>( x = \frac{2}{h^2} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
</tr>
<tr>
<td>for ( t &lt; 0 )</td>
<td>( y_{2n} = \frac{2^n}{2n} x^{-(n-1)} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
<td></td>
</tr>
<tr>
<td>( he^{-(2t/a)^2} ) for ( t \geq 0 )</td>
<td>( x = \frac{2}{h^2} \sqrt{\frac{2}{\pi}} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
<td></td>
</tr>
<tr>
<td>for ( t &lt; 0 )</td>
<td>( y_{2n} = \frac{2^n}{2n} x^{-(n-1)} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
<td></td>
</tr>
<tr>
<td><strong>Gaussian pulse</strong></td>
<td>( \frac{1}{\sqrt{2\pi}} e^{-x^2/2} ) for ( t \geq 0 )</td>
<td>( x = \frac{2}{h^2} \sqrt{\frac{2}{\pi}} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
</tr>
<tr>
<td>for ( t &lt; 0 )</td>
<td>( y_{2n} = \frac{2^n}{2n} x^{-(n-1)} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
<td></td>
</tr>
<tr>
<td><strong>Cosine pulse</strong></td>
<td>( h \cos \frac{t}{a} ) for ( 0 \leq t \leq \frac{\pi a}{2} )</td>
<td>( x = \frac{4}{\pi} \frac{1}{h^2} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
</tr>
<tr>
<td>for ( t &lt; 0 ), ( t &gt; \frac{\pi a}{2} )</td>
<td>( y_{2n} = \frac{2^n}{2n} x^{-(n-1)} )</td>
<td>( &lt;i&gt; = \frac{h}{\sqrt{2\pi}} \sqrt{\nu a} \left[ 1 + \sum_{n=1}^{\infty} \frac{1}{\pi \nu a} \frac{n-1}{2^n} \right] )</td>
<td></td>
</tr>
<tr>
<td>( h \cos \frac{2t}{a} ) for (</td>
<td>t</td>
<td>\leq \frac{\pi a}{4} )</td>
<td>( x = \frac{4}{\pi} \frac{1}{h^2} )</td>
</tr>
<tr>
<td>for (</td>
<td>t</td>
<td>&gt; \frac{\pi a}{4} )</td>
<td>( y_{2n} = \frac{2^n}{2n} x^{-(n-1)} )</td>
</tr>
</tbody>
</table>
Table 2 Ratio of indication value to true one

<table>
<thead>
<tr>
<th>Pulse rate × Pulse width (νa)</th>
<th>Rectangular</th>
<th>Triangular</th>
<th>Exponential</th>
<th>Gaussian</th>
<th>Cosine</th>
<th>Max. – Min.</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.999</td>
<td>0.999</td>
<td>0.998</td>
<td>0.998</td>
<td>0.995</td>
<td>0.004</td>
</tr>
<tr>
<td>4.0</td>
<td>0.998</td>
<td>0.998</td>
<td>0.997</td>
<td>0.998</td>
<td>0.994</td>
<td>0.004</td>
</tr>
<tr>
<td>3.0</td>
<td>0.997</td>
<td>0.997</td>
<td>0.996</td>
<td>0.996</td>
<td>0.991</td>
<td>0.006</td>
</tr>
<tr>
<td>2.0</td>
<td>0.994</td>
<td>0.994</td>
<td>0.992</td>
<td>0.993</td>
<td>0.986</td>
<td>0.008</td>
</tr>
<tr>
<td>1.5</td>
<td>0.990</td>
<td>0.990</td>
<td>0.988</td>
<td>0.989</td>
<td>0.981</td>
<td>0.009</td>
</tr>
<tr>
<td>1.0</td>
<td>0.980</td>
<td>0.982</td>
<td>0.978</td>
<td>0.979</td>
<td>0.969</td>
<td>0.013</td>
</tr>
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<td>0.9</td>
<td>0.976</td>
<td>0.979</td>
<td>0.974</td>
<td>0.975</td>
<td>0.965</td>
<td>0.014</td>
</tr>
<tr>
<td>0.8</td>
<td>0.972</td>
<td>0.975</td>
<td>0.969</td>
<td>0.970</td>
<td>0.960</td>
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<tr>
<td>0.7</td>
<td>0.965</td>
<td>0.969</td>
<td>0.963</td>
<td>0.964</td>
<td>0.954</td>
<td>0.015</td>
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<tr>
<td>0.6</td>
<td>0.956</td>
<td>0.962</td>
<td>0.955</td>
<td>0.956</td>
<td>0.945</td>
<td>0.017</td>
</tr>
<tr>
<td>0.5</td>
<td>0.943</td>
<td>0.951</td>
<td>0.944</td>
<td>0.943</td>
<td>0.933</td>
<td>0.018</td>
</tr>
<tr>
<td>0.4</td>
<td>0.924</td>
<td>0.935</td>
<td>0.927</td>
<td>0.926</td>
<td>0.915</td>
<td>0.020</td>
</tr>
<tr>
<td>0.3</td>
<td>0.893</td>
<td>0.909</td>
<td>0.902</td>
<td>0.897</td>
<td>0.887</td>
<td>0.022</td>
</tr>
<tr>
<td>0.2</td>
<td>0.840</td>
<td>0.864</td>
<td>0.858</td>
<td>0.849</td>
<td>0.840</td>
<td>0.024</td>
</tr>
<tr>
<td>0.15</td>
<td>0.797</td>
<td>0.827</td>
<td>0.824</td>
<td>0.811</td>
<td>0.803</td>
<td>0.030</td>
</tr>
<tr>
<td>0.1</td>
<td>0.736</td>
<td>0.772</td>
<td></td>
<td>0.756</td>
<td>0.748</td>
<td>0.036</td>
</tr>
<tr>
<td>0.09</td>
<td>0.721</td>
<td>0.758</td>
<td></td>
<td>0.741</td>
<td>0.734</td>
<td>0.037</td>
</tr>
<tr>
<td>0.08</td>
<td>0.704</td>
<td>0.718</td>
<td></td>
<td>0.718</td>
<td>0.718</td>
<td></td>
</tr>
<tr>
<td>0.07</td>
<td>0.686</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

for the triangular pulse at νa = 2.0 (45.82 kW). Therefore, we assume the output voltage of the wide-range rectifier in Fig. 1 equals to 0.992 νa at the power level 45.82 kW (i.e. the output voltage is linearly transformed to coincide with the value of νa, which is normalized to <i> = √νa for large enough νa). Note that at 45.82 kW, νa = 2.0.

The measured output voltages are indicated in Fig. 2 by the mark c. It is seen from the figure that the indication lowerings at 1.21 and 12.6 kW were about 24 and 5%, respectively.

Also, in Fig. 2, the results of numerical computation corresponding to the experiment are shown by the mark o. Comparing measured values with computed ones, it can be seen that the experiment agrees with the calculation within 2% at 12.6 kW (νa = 0.535). However, the observed lowering is smaller than calculated one at 1.21 kW (νa = 0.05). This is believed to be caused by the dependence of the fluctuation component on the α-ray emission of the fission counter itself or circuitry noise. Eliminating the effect of the α-noise of fission counter, it can be considered that the evaluation agrees very well with the experiment and the derived equation is suitable for the evaluation.

As above mentioned, the indication lowering of the JOYO IRMS was 0.8% at νa = 2.0 based on the derived equation. The error of the IRMS calibration at 45.82 kW (νa = 2.0)
was too small to interfere with the power-up tests of the reactor. Also, it was confirmed in a following thermal calibration test above 50 kW that the error was allowably small indeed in this power region.

Based on the results of the numerical computation, the indication lowering for $\nu a = 1.0$ is about 2%, and there is remarkable lowering below $\nu a = 1.0$. Therefore, the power level at which $\nu a = 1.0$ can be considered as the limit at which measurements without correction can be made.

**IV. CONCLUSION**

An equation representing the AMCS characteristic, or the mean value $\langle i \rangle$ rectified half wave of the current fluctuation caused by the superposition of pulses, was derived assuming that the arrival probability obeys a Poisson distribution. Indication lowering of AMCS in the low pulse rate region were investigated analytically and by comparison with JOYO test results. The following results were obtained.

1. In the statistical fluctuation method, the product of the mean arrival rate, $\nu$ of pulses and their width, $a$ plays a substantial role. In other words, in the expression for the indication lowering, the mean arrival rate of the pulses always appears in the form of a product with the pulse width instead of appearing solely as the rate itself. Therefore, in the case of no appearance of lowering, $\langle i \rangle$ is proportional to $\sqrt{\nu a}$, and the lowering value is a function of $\nu a$ in low neutron flux region.

2. In the strict sense, the lowering value also depends on pulse shape but $\nu a$. Expressions for $\langle i \rangle$ for some common pulse shapes were derived, and their value were numerically computed. From these results, it was found that the difference in indication reduction due to pulse shapes was at most a few percent in the range of about one or two decades below the beginning of the lowering.

3. Comparison of the computed values with experimental ones measured in the nuclear power calibration process of the JOYO IRMS below 50 kW power level were made. The results agreed closely with each other, although noise due $\alpha$-ray emission in the fission chamber itself or other circuitry noise caused some discrepancies, and the derived equation is suitable for the evaluation of the indication lowering.

4. It was evident from the numerical computation of the derived equation representing the AMCS characteristic above 45.82 kW power level that the indication lowering of the JOYO IRMS was negligibly small and the calibration error was allowably small. Therefore, the reactor power ascension test results were not affected. This was also confirmed in the subsequent thermal calibration test.

5. Generally speaking, $\nu a = 1.0$ can be considered as the limit at which measurements without correction can be made. It is desirable, however, that the measuring range be above $\nu a = 1.0$.

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APPENDIX

Derivation of Eq. (3)

Using Hermite polynomials $\phi^{(n)}(z)$ is expressed by

$$
\phi^{(n)}(z) = \left[ (-1)^n \sum_{k=0}^{n/2} \frac{(-1)^k}{2^k k! (n-2k)!} z^{n-2k} \right] \phi^{(0)}(z). \tag{A1}
$$

Substituting Eq. (A1) into Eq. (1), we have the following equation:

$$
p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-z^2/2\sigma^2} + \left[ \sum_{n=3}^{\infty} \frac{\lambda_n}{n!} \left\{ \sum_{k=0}^{n/2} \frac{(-1)^k}{2^k k! (n-2k)!} \frac{n!}{(n-m-1)! \sigma^n} \right\} \right] \frac{1}{\sqrt{2\pi}\sigma} e^{-z^2/2\sigma^2}. \tag{A2}
$$

From Eqs. (2) and (A2), $\langle i \rangle$ is expressed as follows:

$$
\langle i \rangle = \int_{-\infty}^{\infty} i p(z) dz = \int_{-\infty}^{\infty} z p(z) dz = \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-z^2/2\sigma^2} dz + \frac{1}{\sqrt{2\pi}} \left[ \sum_{n=3}^{\infty} \frac{\lambda_n}{n!} \left( \sum_{k=0}^{n/2} \frac{(-1)^k}{2^k k! (n-2k)!} \int_{-\infty}^{\infty} z^{n+1-2k} e^{-z^2/2\sigma^2} dz \right) \right] \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-z^2/2\sigma^2} dz + \frac{1}{\sqrt{2\pi}} \left\{ \sum_{n=3}^{\infty} \frac{\lambda_n}{n!} \left( \sum_{k=0}^{n/2} \frac{(-1)^k}{2^k k! (n-2k)!} \int_{-\infty}^{\infty} z^{n+1-2k} e^{-z^2/2\sigma^2} dz \right) \right\}. \tag{A3}
$$

Calculating the integration in the brace in the above equation, the polynomial in the brace, i.e.

$$
\alpha_n = \sum_{k=0}^{n/2} \frac{(-1)^k}{2^k k! (n-2k)!} \int_{-\infty}^{\infty} z^{n+1-2k} e^{-z^2/2\sigma^2} dz \tag{A4}
$$

is transformed into the following equation for odd numbered $(2n-1)$ and even numbered $(2n)$ terms, respectively:

$$
\alpha_{2n-1} = \frac{\sqrt{2\pi}}{2n}, \quad \frac{2n-1)!}{(n-1)!} \sum_{k=0}^{n-1} (-1)^k \frac{(n-1)!}{k! (n-k-1)!}, \tag{A5}
$$

$$
\alpha_{2n} = 2^n \frac{(2n)!}{(n-1)!} \sum_{k=0}^{n-1} (-1)^k \frac{(n-k)!}{2^k k! [2(n-k)]!}, \tag{A6}
$$

where \( \sum_{k=0}^{n-1} (-1)^k \frac{(n-1)!}{k! (n-1-k)!} = 0 \).

Therefore,

$$
\alpha_{2n-1} = 0. \tag{A7}
$$

Substituting Eqs. (A6) and (A7) into Eq. (A3), Eq. (3) is obtained.