Considerations are presented on the wall protection of inertial confinement fusion reactors by means of a falling cylindrical curtain of liquid metal that is intermittently constricted into a string of closed envelopes by the action of a series of cusped fields generated around the falling liquid cylinder. The formation of a liquid metal envelope in free space is discussed and the motion of the liquid curtain during constriction is numerically analyzed under the assumption of infinitesimally small curtain thickness and infinitely large conductivity. The single-turn cusp-field driver coils are assumed to have a circular cross section. The calculations indicate that the energy required for the driving field is quite small compared with the thermonuclear energy produced per pulse, and that an adequate electromagnetic force is generated by the coils for a 1 m radius cylindrical curtain of liquid lithium if it is constricted into closed envelopes within an interval of 30 ms.

KEYWORDS: wall protection, fluidized wall concept, falling lithium curtain, cusp field, constriction, liquid metal envelope, numerical simulation, driving energy, inertial confinement fusion reactor

I. INTRODUCTION

In the design study of inertial confinement fusion reactors, much discussion has centered around the various possible means of protecting the reactor wall against the energy released from the exploding pellets of energetic neutrons, charged particles, pellet debris and X-rays. Kulcinski(1) has reviewed the many earlier concepts proposed for wall protection, including Blascon, wetted wall, fluidized wall, rotating drum, dry sacrificial wall, magnetic protection and gas protection. Which of these concepts should provide the best solution is a difficult question to answer at this time, since all of them involve more or less serious problems that call for further experimental study.

The present paper takes up the fluidized wall concept. The principal purpose of a fluidized wall is to protect the primary structural wall of the reactor cavity from direct exposure to thermonuclear explosion. The wall formed by liquid lithium also has functions as heat transfer medium and as blanket to absorb the major part of the thermonuclear energy and also to breed tritium. The systems proposed in the past for the fluidized wall concept are: (1) liquid lithium waterfall(2), which features a thick continuous annular fall of liquid lithium, (2) HYLIFE(3) which uses multiple liquid lithium streams, i.e. an array...
of cylindrical liquid jets, and (3) magnetically guided lithium flow\(\textsuperscript{(4)}\), where a thick lithium layer flows down through a static magnetic field (\textit{e.g.} cusp field) along the spherical structural wall.

In these fluidized wall concepts, particular attention has been given to imparting a closed configuration to the falling liquid lithium, with the aim of completely confining within it the spherical energy release\(\textsuperscript{(2)}\) and of protecting the reactor components from shock waves\(\textsuperscript{(2)(3)}\). Bearing these considerations in mind, the present paper proposes a system of reactor wall protection provided by intermittently creating a string of liquid metal envelopes.

**II. FORMATION OF LIQUID METAL ENVELOPE**

As shown in Fig. 1, a cylindrical curtain of falling liquid metal is introduced into the reactor cavity. Along the length of its fall through the reactor, the curtain is girded at intervals by single-turn cusp-field driver coils of circular cross section, and which are connected to capacitor banks. Each time the electrical circuit is closed, the liquid curtain near the coils is locally constricted by the magnetic pressure of the cusp-field. In this manner, instantaneously before each shot, a liquid metal envelope is formed in free space to shield the primary wall from the energy (especially of short-range deposition, \textit{e.g.} energies of charged particles, and of pellet debris) released from an exploding pellet contained therein.

**III. BASIC EQUATIONS**

In numerically simulating the motion of the liquid metal curtain during its constriction, it is assumed for simplicity that the curtain is infinitely thin, infinitely long and has infinite conductivity. The displacement of the falling liquid curtain is further neglected for the interval of time taken for the constriction. The cusp-field driver coils with circular cross section (major and minor radii \(a, r\), respectively) are placed at \(z=\pm b\) in the cylindrical coordinates \((r, \theta, z)\) as shown in Fig. 2.

Let \(r(t)=(r(t), z(t))\) be the position of a fluid element at time \(t\). With the longitudinal cross section of the curtain expressed by \(r=f(z)\), the Lagrangian variable \(Z\) is defined in virtue of the mass conservation law by

\[
Z=(R\sigma)\int_0^z f(z')\sigma(z')[1+(df/dz')^2]^{1/2}dz', \tag{1}
\]

where \(R\) is the initial radius of the curtain and \(\sigma\) the surface mass density, while the suffix \(i\) denotes initial value. From Eq. (1),

\[
\sigma(t, Z)/\sigma_i=(R/r)[(\partial r/\partial z)^2+(\partial z/\partial Z)^2]^{-1/2}. \tag{2}
\]
The equation of motion of the curtain is

$$\sigma d^2\mathbf{r}/dt^2 = -\mathbf{n} P,$$  \hspace{1cm} (3)

where $\mathbf{n}$ and $P$ are respectively the unit normal vector and the magnetic pressure on the outer surface of the curtain, the initial condition is $\mathbf{r}(0, Z) = (R, Z)$. The unit normal vector is found to be

$$\mathbf{n} = [(1 + (df/dz)^2)^{-1/2}(1, -df/dz)] = [(\partial r/\partial Z)^2 + (\partial z/\partial Z)^2]^{-1/2}(\partial z/\partial Z, -\partial r/\partial Z).$$  \hspace{1cm} (4)

The magnetic pressure $P(Z)$ is expressed by $P = (2\mu_0)^{-1}B^2$, where $\mu_0$ is the free-space magnetic permeability, and $B(Z)$ is the magnetic flux density at the curtain surface.

The induced current density $J(Z)$ per unit length of the curtain is found from the boundary condition of the free-space magnetic field. Having assumed an infinitely conducting liquid metal curtain, the boundary condition is $\psi(Z) = 0$ ($\psi$: magnetic flux) at its outer surface. If the driver coils are circular loops of wire,

$$\psi(Z) = F(r(Z), a^+) - F(r(Z), a^-) I$$

$$+ \int_{r(Z)}^{r(Z')} [F(r(Z), r(Z')) - F(r(Z), r(-Z'))] J(Z') dZ' = 0,$$  \hspace{1cm} (5)

where $I(t)$ is the driver coil current and $a^\pm$ signifies $(a, \pm b)$. The function $F(r(Z), r(Z'))$ gives the flux at $r(Z) = (r, z)$ produced by a unit ring current located at $r(Z') = (r', z')$:

$$F = (2\mu_0/k)(rr')^{1/2}[(1-k^2/2)K(k) - E(k)],$$

where $K(k)$ and $E(k)$ respectively are the complete elliptic integrals of the first and second kinds, and $k^2 = 4rr' / [(r+r')^2 + (z-z')^2] < 1$.

It should be noted that in Eq. (5) the current density $J(Z)$ is an odd function on account of the cusp field that is applied.

The coil current $I(t)$, of initial value $I_i$, is determined from the conservation of the magnetic flux $\psi_c$ trapped between the surfaces of the driver coil and the liquid curtain:

$$I(t) = I_i \psi_c(0) / \psi_c(t),$$

$$\psi_c = \psi_c / (\mu_0 I(t) R),$$

where $\psi_c$ is given by an equation analogous to Eq. (5). In Eq. (6), $\psi_c(t) / \psi_c(0)$ means the ratio of system inductance between those at time $t$ and at incipience, and depends only on the geometrical configuration of the system.

Once the current density $J(Z)$ and the coil current $I$ are found respectively from Eqs. (5) and (6), the magnetic flux density $B(Z)$ at the curtain surface can be obtained by using the Biot-Savart law (see APPENDIX). It should be noted that the assumption of flux conservation is valid only when $\pi[(a-\rho)^2 - R^2] / (\mu_0 \eta) / T \gg 1$, where $\eta$ is the resistivity of the liquid curtain and $T$ the constriction time.

**IV. NUMERICAL TREATMENT**

The equation of curtain motion is solved in space by finite difference technique, and in time by fourth-order Runge-Kutta Gill method. The mesh points in the $Z$ coordinate are chosen in such manner that $Z_{j+1/2} = (j + \frac{1}{2}) \Delta Z$, $(j=0, \pm 1, \pm 2, \cdots)$, where $\Delta Z$ is the increment of $Z$.

Then, the derivative $\partial r/\partial Z$ in Eq. (3) is replaced by

$$\frac{\partial r}{\partial Z}_{j+1/2} = [r(Z_{j+1/2}) - r(Z_{j-1/2})] / 2\Delta Z$$
and the boundary conditions adopted are \( \frac{∂r}{∂Z} \bigg|_{Z=N} = \frac{∂r}{∂Z} \bigg|_{Z=0} = 0 \), where \( Z = \frac{r}{R} \).

The current density \( J(r_j) \) per unit length of the curtain is obtained by solving the system

\[
\sum_{l=1}^{N-1} [F(r_{j+1/2}, r_j) - F(r_{j+1/2}, r_{j-1/2})] f(r_j) dZ = -\frac{∂}{∂r} \left[ F(r_{j+1/2}, a^+) - F(r_{j-1/2}, a^-) \right] I,
\]

where \( r_{j+1/2} = r(Z_{j+1/2}) \) and \( r_j = (r_{j+1/2} + r_{j-1/2})/2 \). Furthermore, the flux density \( B \) at \( r_{j+1/2} \) is found from

\[
B(r_{j+1/2}) = G(r_{j+1/2}, a^+) - G(r_{j+1/2}, a^-) I + \sum_{l=1}^{N-1} [G(r_{j+1/2}, r_l) - G(r_{j+1/2}, r_{j-1})] J(r_i) dZ,
\]

where \( G \) is as defined in APPENDIX.

The magnetic lines of force near the driver ring current are approximately described by circles in the \((r, z)\)-plane. This permits us to treat the case of driver coils possessing finite thickness by replacing the magnetic surface of radius \( \rho \) with the conductor surface, as indicated in Fig. 3. It should be noted that the position of the center of a circular field line differs in general from that of the ring current, and that it displaces with lapse of time on account of the motion of the liquid curtain. The displacement of the magnetic surface—of radius \( \rho \)—is compensated in the numerical analysis by letting the ring current follow the movement, with its position determined from the condition

\[
\psi_c(a + \rho, b) = \psi_c(a - \rho, b) = \text{const.},
\]

the displacement in the axial direction being neglected.

**V. RESULTS AND DISCUSSIONS**

The non-dimensional quantities are defined by

\[
\hat{t} = t/r, \quad \hat{Z} = Z/R, \quad \hat{P} = P/P_0 \quad \text{and} \quad \hat{t} = t/t_f,
\]

where \( P_0 = (2\mu_0)^{-1}(\mu_0 I_i/2\pi R)^z \) and

\[
\tau = \sqrt{\sigma_1 R/P_0} = (8\pi^2 \sigma_1 R^3/\mu_0 I_i)^{1/2}.
\]

The equation of motion—Eq. (3)—then becomes

\[
d^2\hat{t}/d\hat{Z}^2 = -\frac{∂\hat{Z}/∂\hat{Z}}{h} \hat{P}, \quad d^2\hat{z}/d\hat{t}^2 = \frac{∂\hat{r}/∂\hat{Z}}{h} \hat{P}.
\]

This system depends solely on the driver-coil parameters, i.e. \( \tilde{a} = (a/R, ± b/R) \) and \( \tilde{p} = \rho/R \). Figure 4 shows profiles of the liquid curtain in the \((r, z)\)-plane at various lapse of time \( \hat{t} \) in the course of constriction.

Upon solving Eq. (9) to find the time \( \hat{t} = t_f \) at which the curtain becomes closed into envelopes \( \tau \) of Eq. (8) is determined by

\[
\tau = T/\hat{t}_f,
\]

- 4 -
where $T$ is the constriction time, i.e., the time required to form the envelope in an actual reactor. Thus, from Eqs. (8) and (10), for given $T$, the current

$$I_i = \left(8\pi^2 \sigma_1 R^4 / \mu_0 \right) \frac{1}{2} t_f / T,$$

where $t_f$ is seen from Fig. 5 to be of the order of unity.

Since the purpose of forming the liquid metal envelope is to protect the first wall in an inertial confinement fusion reactor, it is necessary to satisfy the requirements of:

1. Small driving cusp-field energy compared with the thermonuclear energy per pulse.
2. Integrity of the liquid curtain during constriction until pellet explosion.

At $t=0$, cusp-field energy $e = 2/\sqrt{2} I_i \phi_e$.

From Eqs. (7), (8) and (10), the required driving field energy for a given $T$ is calculated by

$$e = 8\pi^2 \sigma_1 R^4 / T^2 \cdot e(\delta, \hat{\delta}, \rho), \quad (\hat{\delta} = \delta / \rho).$$

Figure 6 shows that the normalized field energy $\hat{e}$ changes from $\sim 1$ to $\sim 10$ in the present range of the coil parameters. With increasing $\delta$ and decreasing $\hat{\delta}$, the distance between surfaces of the coil and the curtain increases, and the magnetic flux density on the curtain diminishes for a given coil current $I_i$. This is the reason why $t_f$ is an increasing function in reference to $\delta$. An increase in this distance $\delta$ also tends to enhance the initial inductance of the system. This makes it advisable to have the coil surface positioned as close as possible to the initial curtain, in order to reduce the required energy for the driving field. The computed results also reveal negligible dependence of $\hat{\delta}$ and $t_f$ on $\hat{\delta}$ for $\hat{\delta} \gt 1$. 

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**Fig. 4(a),(b)** Profiles of liquid curtain during constriction in $(r, z)$-plane

**Fig. 5** Time $t_f$ at which constricted liquid curtain closes completely to form envelopes.
As regards liquid integrity during constriction, the outer surface of the curtain is subjected to Rayleigh-Taylor instability due to the inward acceleration. If the instability is of the flute-type, where the deformation wave vector is perpendicular to the lines of force, the growth rate of the perturbation amplitude can be roughly estimated by

\[ \left[ m \frac{d^2 r}{dt^2} / R \right]^{1/2}, \]

where \( m \) is the mode number of the deformation wave in the azimuthal direction.

Hence, from linear theory, the initial perturbation amplitude is increased by a factor \( \exp(\sqrt{mI})^{(6)} \),

where \( I'(Z) = \int_0^{Z_f} \sqrt{d^2 F(Z)/dt^2} d\bar{t} \).

It is found upon computation that the maximum value of \( I'(Z) \) is of the order of unity in the present range of parameters. The instability would be liable to cause curtain rupture when the perturbation amplitude grows to a value comparable with the curtain thickness during the period of displacement \(^{(7)}\). This means that the required protection can be ensured only by sufficiently increasing the initial curtain thickness. Figure 7 shows the normalized thickness \( \sigma(Z)/\sigma_i \) of the liquid curtain at time \( t \) for the cases of \( a/R = 1.2 \) and 1.5. When the curtain is constricted toward the center axis, its mean thickness should increase in virtue of mass conservation. From Figs. 4 and 7, however, it is seen that for a small major radius of the driver coil, the liquid curtain is constricted so locally that the thickness actually diminishes in a certain zone on account of the rapid motion in the axial direction. This phenomena is unfavorable for maintaining the soundness of the curtain during constriction.

Hence, to ensure curtain integrity, the major radius of the coil is not made too small, and the required driving field energy is limited. Figures 6 and 7(c) would suggest that reduction of \( \dot{e} \) should be achieved rather by increasing \( \dot{\rho} \).
Table 1 shows an example of design parameters for the present system. Liquid lithium has been taken up as curtain material on account of its favorable properties for tritium breeding. A lithium curtain 5 cm thick will absorb $\sim 40\%$ of the energy released from an exploding pellet including all the energy of short-range deposition\(^{(2)}\). When the length of the initial shell (= height of reactor cavity) is greater than 5 m and the thermonuclear energy $E_f$ per pulse is $\sim 1,000$ MJ\(^{(3)}\), the initial curtain radius should be smaller than 1 m to obtain a mean temperature rise of curtain (i.e. the difference $\Delta T$ between inlet and outlet temperatures of liquid lithium) that is greater than 100°C (cf. $\Delta T \approx 100\sim 200$°C for the primary coolant in FBR designs\(^{(8)}\)). For a pulse repetition rate of $\sim 1$ Hz\(^{(2)}\), the constriction time $T$ of the liquid curtain must be very much smaller than 1 s, because most of the time interval between each shot will be consumed in re-establishing the liquid curtain. For $T=30$ ms, the required driving energy and the initial coil current are found respectively from Eqs. (11) and (12) to be $e=2.20eMJ$ and $I_i=1.6f MA$. If $\alpha=1.5$, $\beta=1.2$ and $\rho=0.3$ are chosen for the coil parameters, we obtain $e=4.4 MJ$ and $I_i=1.9 MA$, where the energy of 4.4 MJ is quite small compared with $E_f$ ($\sim 1,000$ MJ). The initial hoop stress $P_\phi$ acting on each driver coil is calculated by

$$P_\phi = (\pi \rho^2)^{-1} \cdot \frac{1}{2} I_i \frac{\partial \phi}{\partial a} = 4 \pi \sigma_i R (I_i / \rho T) \frac{\partial \phi}{\partial a},$$

where $\partial \phi / \partial a \geq 5$ in the present range of coil parameters, and we obtain $P_\phi \geq 25$ MPa for $\rho=0.3$. This value of hoop stress will be acceptable if stainless steel (yield stress of AISI 316 $= 500$ MPa) is used for the driver coils. It should be noted that the repulsive force between the coils is small compared with the hoop force, since $(\partial \phi / \partial a) (\partial \phi / \partial a)$ is estimated to be $\sim 10^{-2}$ for $\beta \sim 1$.

VI. CONCLUSIONS

A discussion has been presented on the formation of a liquid metal envelope in free space for the purpose of protecting the wall of an inertial confinement fusion reactor, particularly against the energy released from short-range deposition. Under the assumption of infinitely thin liquid curtain and infinite conductivity, the motion of the curtain constricted by a cusp field was numerically analyzed, and the required driving energy and the initial coil current obtained for a given constriction time.

It has been found that, within the range considered of the system parameters, the required driving energy is quite small compared with the thermonuclear energy generated per pulse and that the electromagnetic force acting on the driver coils is acceptable. The numerical results have also indicated that driver coils with small major radius are unfavorable from the viewpoint of curtain integrity against the Rayleigh-Taylor instability. For this reason, the reduction of the required driving energy should be realized rather by
increasing the minor radius of coil than by decreasing its major radius.

**[NOMENCLATURE]**

- $a$: Major radius of driver coil
- $a^*= (a, \pm b)$
- $b$: Axial position of driver coil
- $B = (B_r, B_z)$: Magnetic flux density
- $\epsilon$: Cusp-field energy, $\tilde{\epsilon} = \frac{1}{2} \phi_e$
- $F$: Magnetic flux produced by unit ring current
- $G = (G_r, G_z)$: Magnetic flux density produced by unit ring current
- $I$: Driver coil current
- $I_i$: Initial value of $I$
- $J$: Current density per unit length of liquid metal curtain
- $n$: Unit normal vector on outer surface of liquid metal curtain
- $P$: Magnetic pressure, $\tilde{P} = P/P_0$
- $P_0 = (2\pi \mu_0)^{-1}(\mu_0 I_i/2\pi R)^2$
- $r = (r, z)$: Position of fluid element
- $\hat{r} = r/R = (\hat{r}, \hat{z})$
- $t$: Time, $\tilde{t} = t/\tau$
- $\hat{t} = t_f/\tau$, where $t_f$ represents the instant of complete constriction of liquid curtain into closed envelope
- $T$: Constriction time
- $Z$: Lagrangian variable; also initial value of $z$
- $\rho$: Minor radius of driver coil
- $\rho_c$: Magnetic flux at driver coil surface
- $\phi_c = \phi_e / (\mu_0 I_i R)$
- $\tau = \sqrt{\sigma_c R / \rho_0}$
- $\sigma_0$: Surface mass density of liquid metal curtain
- $\sigma_i$: Initial value of $\sigma$

(Text edited grammatically by Mr. M. Yoshida.)

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**REFERENCES**


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**APPENDIX**

The magnetic flux density $B(Z)$ at the curtain surface is obtained from

$$B(Z) = [G(r(Z), a^*) - G(r(Z), a^-)] I \left[1 + \int_0^Z [G(r(Z), r(Z')) - G(r(Z), r(-Z'))] J(Z') dZ'\right],$$

where the function $G(r(Z), r(Z')) = (G_r, G_z)$ gives the flux density at $r(Z) = (r, z)$ produced by a unit ring current located at $r(Z') = (r', z')$, and is made up of the components

$$G_r = \frac{\mu_0}{2\pi r} \left[\frac{z - z'}{(r + r')^2 + (z - z')^2} \right]^{1/2} \left[ - K(k) + \frac{r^2 + r'^2 + (z - z')^2}{(r - r')^2 + (z - z')^2} E(k) \right],$$

$$G_z = \frac{\mu_0}{2\pi r} \left[\frac{z - z'}{(r + r')^2 + (z - z')^2} \right]^{1/2} \left[ K(k) + \frac{r^2 + r'^2 + (z - z')^2}{(r - r')^2 + (z - z')^2} E(k) \right],$$

where $K(k)$ and $E(k)$ are as defined in Chap. III.