Derivation of BWR Core Dynamics Models and Analysis of JPDR Core Transient Tests

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The purpose of the present work is to establish a core dynamics model for the JPDR plant (natural circulation boiling water reactor plant). First, upon surveying the analytical model with the so-called distributed parameters in reference to the experimental results obtained during the JPDR initial power-up test, it was decided to develop the core dynamics model with lumped rather than distributed parameters, and to further modify it to obtain the highest possible accuracy. Three equations for mass, energy and momentum balances were used, and particular care was taken in the determination of the core void fraction and recirculation flow, the development of the thermo-hydrodynamic equation in the saturated region, and the choice of void sweep time as well various numerical constants.

The derived dynamics model was then applied to the analysis of the JPDR transient test involving Bypass Regulator and Initial Pressure Regulator operations, as well as to that of the Bypass Regulator oscillation test.

The calculated results by this model agreed well with data from experiment. A comparison was then made between the model and other core dynamics models. It was concluded that this model intrinsically has good accuracy and is easy to handle; it should prove very useful for core dynamics analysis of any BWR, in their relation to the out-of-core controlling system.

I. INTRODUCTION

It is intended in this paper to develop a new core dynamics model not only applicable to the transient and dynamic analysis of the Japan Power Demonstration Reactor (JPDR) plant, but which should prove useful also for the design of boiling water reactor plants with respect to their reactor dynamics and control\(^{(1)}\).

Various models have in the past been developed for analyzing the core-dynamics of natural-circulation boiling-water reactors. Some of these have been applied to investigation of the JPDR core dynamics\(^{(2)(3)}\).

In the present paper, a recently developed model is first studied, namely a model with so-called distributed parameters, using partial differential equations in terms of space and time for expressing the core thermal hydro-dynamics; in particular, the assumptions employed in the model are examined.

The experimental data obtained by JPDR power operation and tests, led to the belief that the assumptions currently used should be revised, and it was decided to establish another model with so-called lumped parameters, to replace the one based on distributed parameters, and to aim at the highest possible accuracy through detailed parametric survey conducted beforehand on steady-state core characteristics.

II. ANALYTICAL MODELS FOR THE CORE DYNAMICS

1. Fundamentals to Study Two-phase Flow Dynamics

There are three fundamental equations for analyzing the reactor core dynamic characteristics, which are complicated by two-phase coolant flow.

The equations are expressed as follows, in terms of the unit area per unit distance in the heated region:

a) Mass balance equation:

\[
\frac{\partial}{\partial t} \sum \rho_i f_i + \frac{\partial}{\partial z} \sum \rho_i f_i u_i = 0, \tag{1}
\]

where \(\sum f_i = 1\).

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b) Energy balance equation:
\[ \frac{\partial}{\partial t} \sum \rho_i f_i u_i + \frac{\partial}{\partial z} \sum \rho_i f_i w_i = Q, \]  \( \text{(2)} \)
where \( u_i = -\rho_i / f_i \).

c) Momentum balance equation:
\[ \frac{\partial}{\partial t} \sum \rho_i f_i u_i + \frac{\partial}{\partial z} \left( \rho_i f_i w_i \right) = -\frac{\partial p}{\partial z} + \frac{F}{A} - (\text{loss}). \]  \( \text{(3)} \)

Of the terms in the above equations, \( \rho_i, v_i \) and \( i_i \) are functions of \( p \). In order to determine \( f_i, w_i \) and \( p \) as functions of \( z \) and \( t \), two equations, namely, for the determination of the pressure and of the core coolant flow, are required.

It is quite difficult to solve the above five equations rigorously and simultaneously. The usual expedient is to substitute the momentum balance equation by another equation to determine the slip ratio.

The slip ratio equation is easier to handle than the momentum equation, if it is assumed that the slip ratio is constant both timewise and spacewise. In most cases, the value is assumed to be unity. It should be noted that this assumption is not valid for actual reactors, an actual instance being the tests with JPDR.

2. Solution with Distributed Parameters

Solving Eqs. (1) and (2) with respect to \( f_i \), in general, \( f \), the following equation can be derived:
\[ \frac{\partial f}{\partial t} + U \frac{\partial f}{\partial z} = q, \]  \( \text{(4)} \)

where
\[ U = F_s(f) \left( w_0 + \int_a^z \left( Q + \frac{1}{f} + F_1(f) \frac{dp}{dt} \right) \right) + P_s(f) \frac{dp}{dt}, \]
where \( \omega_0 = v_i - v_w \) and \( di = i_i - i_w \), and
\[ q = (1/\rho_i di) F_s(f) \left( Q + \frac{1}{f} + F_1(f) \frac{dp}{dt} \right) \]
\[ + P_s(f) \frac{dp}{dt}. \]

Actually \( (1/f) \ll F_1(f) \), so that \( (1/f) \) is neglected. \( U \) is the apparent velocity of the void, and \( q \) is the apparent force applied to the void. \( F_s(f) \) through \( F_5(f) \) are listed in Table 1 for two cases of slip ratio, i.e., unity and non-unity.

<table>
<thead>
<tr>
<th>( \gamma \neq 1 )</th>
<th>( \gamma = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1(f) )</td>
<td>( \left{ \rho_w (1-f) \frac{\partial \rho_i}{\partial p} + \rho_i f \frac{\partial \rho_i}{\partial p} \right} )</td>
</tr>
<tr>
<td>( F_2(f) )</td>
<td>( \left{ \frac{1-f}{\rho_w} \frac{\partial \rho_i}{\partial p} + f \frac{\partial \rho_i}{\partial p} \right} )</td>
</tr>
<tr>
<td>( F_3(f) )</td>
<td>( \frac{\gamma + f (1-f) \partial \rho_i}{\partial p} ) ( \frac{(1-f) \gamma}{(1+f) \gamma} )</td>
</tr>
<tr>
<td>( F_4(f) )</td>
<td>( \frac{1-f+\gamma P_i \rho_w}{1-f+\gamma f} )</td>
</tr>
<tr>
<td>( F_5(f) )</td>
<td>( \frac{(1-f)}{1+f+\gamma f} \frac{\gamma \partial \rho_i}{\rho_w} \left( \frac{1}{\rho_i} \right) \left( \frac{\partial \rho_i}{\partial p} \right) - \frac{1}{\rho_i} \left( \frac{\partial \rho_i}{\partial p} \right) )</td>
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We differentiate the above equation so as to express small deviations around the rated value, and then convert it into a linear equation. After Laplace transformation, we integrate it from the boiling boundary to the point under consideration; and we obtain the linear equation
\[ df = \frac{F_1}{F_s} \exp \left( -\int U dx \right) \left( \int_{t_1}^{T} \frac{P_0}{U} \exp \left( \int \frac{dp}{dt} dt \right) \right), \]  \( \text{(5)} \)
where
\[ \varphi = q \frac{\delta Q}{Q} + \left( F_s \left( \frac{1}{\rho_i di} \right) \right) \left( F_1 + F_2 \right) \frac{dp}{dt} \]
\[ + Q' F_2 \delta \left( \frac{1}{\rho_i di} \right) - q' F_1 \left( \frac{dp}{dt} \right) \int_{t_1}^{T} \delta Q dx \]
Thus the total average void fraction is obtained simply by integrating Eq.(5), then taking the average value over the whole core length. In this case, the external disturbances are the power fractional change $\delta Q/Q$, the inlet flow velocity change $\delta w_0$, the pressure change $\delta p$, the rate of pressure change $dp/dt$ and boiling boundary change, $z_1$.

In order to solve Eq.(5), the following assumptions must be used:

a) Slip ratio is unity, i.e., $\gamma = 1$

b) The reactor power distribution through the core axis is constant, i.e.,

$$Q(z) = \bar{Q} = \text{const.}$$

With these assumptions, many terms in Eq.(4) can be simplified, and $U$ becomes equal to either the steam or water velocity, the latter two quantities taking furthermore the same value; $U$ is then a linear function of location in space;

$$U(z) = w_s(z) = w_w(z) = w_0 + \bar{Q} \left( \frac{dw}{dt} \right) (z - z_1).$$

(6)

Thus, the equation is solved. Its application to JPDR transient analysis has already been reported.

3. Experimental Data on the Slip Ratio and Power Distribution

It was found through the JPDR initial start-up test that the assumptions for the slip ratio and power distribution adopted in the distributed parameter model, are not necessarily justified for the following two reasons.

a) Slip ratio would be far larger than unity.

Various calculational and experimental results are shown in Table 2. From the figures in Table 2, the slip ratio may be said to be much larger than unity.

b) Power distribution through the core axis is rather more distorted than expected.

<table>
<thead>
<tr>
<th>Calculation from</th>
<th>Value of slip ratio</th>
<th>Remarks, values used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lottzes-Flinn(6)</td>
<td>1~2.3</td>
<td>Coolant flow adopted: 3 to 4 ft./sec</td>
</tr>
<tr>
<td>Lottzes-Flinn(6)</td>
<td>1.6~1.7</td>
<td>Coolant flow adopted: 3 to 4 ft./sec</td>
</tr>
<tr>
<td>Bankoff(2)</td>
<td>Average exit 1.64</td>
<td>Exit steam quality adopted: 0.060</td>
</tr>
<tr>
<td>JPDR experiment</td>
<td>Average exit 1.51~1.61</td>
<td>Subcooled temperature was measured, from which slip ratio was calculated assuming carry-under of 20% total core steam flow</td>
</tr>
<tr>
<td>GE report 3869</td>
<td>Average exit 1.51</td>
<td>Adopted $\gamma = \frac{p_w}{p_g} \frac{1-f}{f} \frac{1-x}{1-x}$</td>
</tr>
</tbody>
</table>

The power distribution was carefully measured by irradiating wires in JPDR at various power levels and also by decay-gamma scanning each fuel assembly in half the core after 100 hr full power operation.

An example of decay-gamma scanning plot is shown in Fig. 1.

![Fig. 1 Neutron Flux Distribution (43.5 MWt)](image)

It is seen in the figure that the peaking factor is fairly large, being more than 1.4, even for measured values of power distribution curve chopped at both top and bottom. This is worse than with a chopped sine curve, that is, a typical axial distribution curve. Thus the distribution through the core axis could not be considered flat.

Consequently it was concluded that the two assumptions employed in the calculation with distributed parameters did not validly represent reality. It was necessary to consider...
whether more accurate conditions or assumptions could be incorporated into that calculation, or, if not, whether the calculation model itself might better be modified.

It would appear difficult to rigorously perform the calculation with distributed parameters. This is because $U$ in Eq. (4) is a function involving $Q(x)$ which is a transcendental function, and $\delta \tau$, the over-all core void fraction, is a triple-integral of the reciprocal of $U$. So in this study, it was decided to adopt a dynamic model with lumped rather than distributed parameters, but with due consideration was given to choosing those that reflect such factors as the slip ratio variation and power distribution along the core axis.

Hogle's analysis is one example related to what is generally termed lumped parameter. While his calculation is fairly accurate, there are several items that require further examination, such as in the determination of the void fraction, of the recirculation flow, in the development of the hydrodynamic calculation within the core, and in the evaluation of the void sweep time as well as of various parameters used in the calculation.

In the present model, the above items have been reviewed, with the view to a more accurate analysis.

The first step is precise parametric calculation of steady-state, nuclear, thermo-hydraulic characteristics.

III. DEVELOPMENT OF A NEW MODEL FOR BWR DYNAMICS ANALYSIS

1. Parametric Study of the Steady-state Characteristics of the JPDR Core from Nuclear, Thermal and Hydraulic Aspects

A precise parametric study has been made on steady-state nuclear, thermal and hydraulic characteristics of the JPDR core, in order to obtain sufficient data for determining both the initial conditions and the change rate of various parameters in the case of transient operations.

A one dimensional computer code for IBM 7044, KYNAK, was first prepared, then applied to the JPDR core calculation.

Details of this code and various computed results have been given in other papers. There appeared, as expected, some discrepancies between calculated results and data actually measured. In such cases, the measured data were adopted in the dynamics calculation.

2. Assumptions for Starting the Calculation

(a) A model with 6 delayed neutron groups of $^{239}$U is adopted.

(b) Decay heat is 7% of the prompt reactor power.

(c) 3% of the prompt neutron energy is consumed within the moderator.

(d) Pressure in the core is the same as in the pressure vessel dome.

(e) The subcooled temperature is 3.7°C (actually measured).

(f) Carry-under is assumed to be 20% of the core steam flow, or approximately 1% of the total recirculation flow.
(g) Average neutron temperature is 1.8 times the physical average fuel temperature. Doppler effect is proportional to the average neutron temperature.

(h) The different variables maintain their distribution existing under normal core condition.

(i) The reactor is bare with one group of energy.

(j) The flow from the core into the demineralizer system, and that from the control rod seal coolant system into the core are both neglected.

(k) Moderator temperature coefficient is neglected.

(l) Reactor water level control is neglected.

3. Kinetic Equation and Heat Transmission into Coolant

The kinetic equation for fundamental mode is expressed in Laplace transformation:

\[ \frac{df}{\phi^* + \delta \phi} = \frac{\Delta \text{matt} / \beta_{\text{att}}}{s \beta_{\text{att}} + \sum \beta_{\text{att}} / (rt, s + 1)} \]  

As discussed elsewhere, the decay heat is estimated to be 7% of the total heat generation and its decrease with time is approximated by the Laplace transformation

\[ 0.03 \phi^* + 0.03 \left( \phi^* + \delta \phi \right). \]  

Then heat generated in the fuel,

\[ \delta n_e = (0.97 + 1.32 s) \delta \phi. \]  

The heat transfer from fuel to cladding surface can be derived in Hankel transformation. It is the sum of the infinite series of single time delay transfer functions. For the JPDR, it is approximately,

\[ \delta Q = 0.853 \frac{\phi^*}{1 + 10.5 s} + 0.147 \frac{\phi^*}{1 + 2.0 s}. \]  

4. Hydrodynamic Equation (See Fig. 2)

(1) Core

Integrating Eqs. (1) and (2), over the saturated region in the core, the following thermo-hydrodynamic equations will be obtained.

\[ \frac{d}{dt} (M_{w,c} + M_{e,c}) = w_0 - w_{w_2} - w_{s_2} \]  

\[ + \rho_w \frac{\partial V_{c,mat}}{\partial t}, \]  

\[ \frac{\partial}{\partial t} \left( M_{w,c} \left( \frac{i_s - \rho w}{f} \right) + M_{w,c,mat} \left( i_{w} - \rho w \right) \right) = Q_{1} + w_{iw} - w_{w_2 i w} - w_{s_2 i w}, \]  

where

\[ V_{c,mat} = v_{e,c} + \rho w \cdot M_{w,c,mat}. \]  

Similar equations can be derived for the subcooled region in the core, where \( w_{w,sub} \) and \( i_{w,sub} \) can be approximated by \( v_w \) and \( i_w \) respectively, with an error of less than 1%. Here \( Q_{e,c} \) is used instead of \( Q_{e,c} \), and also \( \rho_w \) instead of \( \rho_w \) since it indicates the boiling boundary change.

Noting that the core volume \( V_{e,c} \) the volumetric sum of the saturated and subcooled regions in the core, is invariable, we differentiate the above three equations with respect to time, take small deviations around the steady value, make linear equations by neglecting second or higher order of derivatives, and delete \( \delta M_{w,c} \) and \( \delta M_{e,c} \). Then the following equation will result:

\[ \delta Q - (i_s - i_t) \delta W_s - w_{w_2} \delta i_{w} - w_{s_2} \left( \frac{\partial \psi}{\partial p} \right) \delta p - i_t \delta W_{s_2} = D_i \delta \phi, \]  

where

\[ D_i \equiv M_{e,c} \left( \frac{\partial \psi}{\partial p} \right) - \frac{V_{e,c}}{f} \]  

\[ - \left( \frac{i_t}{\partial p} \right) \left[ M_{e,c} \left( \frac{\partial \psi}{\partial p} \right) + M_{w,c} \left( \frac{\partial \psi}{\partial p} \right) \right]. \]  

In the above calculation, the following two assumptions are employed:

\[ \frac{(i_s - i_t) v_{w_2}}{(v_{w_2} - v_{s_2})} \equiv i_t \]  

\[ \left( \frac{i_t}{\partial p} \right) \equiv 0. \]  

The above equation is in a form that should determine the steam flow \( \delta W_{s_2} \) as a function of various inputs \( \delta Q, \delta W_s, \delta i_s, \delta p \) and \( \delta \phi \).

Actually, the steam flow leaving the top of the core \( \delta W_{s_2} \), is delayed by a certain period of time, compared with the calculated flow \( \delta W_{s_2} \). This time delay is taken into consideration in the following manner.

\[ \delta W_{s_2} = \delta W_{s_2} - \frac{1}{1 + \tau_{s_2}} \]  

\[ \tau_s \]  

is approximately equal to the time constant for the void sweep through the core. In the case of JPDR, it is about 1 sec.

(2) Chimney, Top Plenum and
Upper Dome

For the regions of chimney, top plenum and upper dome, equations can be similarly derived, with attention to the excess flow entering the chimney region from the inter-channel gaps in the core, and estimating the steam carry-under to be about 20% of the total core steam flow. Calculation results in,

$$
\Delta t^* (0.8 \delta W_{st} - \delta W_{st}) = D_b \delta \rho,
$$

where

$$
D_b = M_{st} \tau_p \left( \frac{\partial \rho}{\partial \rho} \right)_p - \left( \frac{\partial \rho}{\partial \rho} \right)_p \cdot V_F + \frac{1}{f} \left( \frac{\partial \rho}{\partial \rho} \right)_p - \frac{1}{f} \left( \frac{\partial \rho}{\partial \rho} \right)_p.
$$

This equation gives the rate of pressure change with time as a function of the balance of the steam flow in and out of the region considered.

The above equation indicates that for a given balance of the above steam flow, the rate of pressure change with time becomes smaller with increase of either the saturated water mass, $M_{st}$, or of the steam mass, $M_{st}$, or with decrease of $\Delta t^*$, i.e., increase of the operating pressure.

(3) Downcomer, Lower Dome and Lower Plenum

Both mass and energy balance equations hold also in the regions of downcomer, lower plenum and lower dome:

$$
\delta W_{st} = \delta W_{st} + \delta W_{st} + 0.2 \delta W_{st},
$$

$$
\delta W_{st} + (W_{st} + \delta W_{st}) \delta \rho = \delta \rho \delta W_{st} + W_{st} \delta \rho + 0.2 \delta W_{st} + 1 \delta W_{st} + W_{st} \left( \frac{\partial \rho}{\partial \rho} \right)_p \delta \rho,
$$

where $\delta W_{st}$ is neglected, assuming $\delta W_{st} \ll \delta W_{st}$.

These equations give the change of inlet enthalpy as a function of various inputs, such as $\delta W_{st}$, $\delta W_{st}$, $\delta W_{st}$, $\delta W_{st}$ and $\delta \rho$.

Taking into consideration the time delay of subcooled enthalpy, due to the recirculating flow-down along the downcomer, as well as its dispersing time within the lower dome and lower plenum, the enthalpy change is, by Laplace transformation,

$$
\delta \rho = \delta \rho \exp(-12s) \frac{1}{1+7s}.
$$

The time constants are determined by referring to the experimental data, obtained during the feedwater transient tests.

5. Equation of Momentum Change or Force

Consider the case where Eq. (3) is integrated within a certain region, with no physical external force (such as pump discharge head) applied to the region,

$$
\frac{\partial}{\partial t} \int_{in}^{out} \rho_f \frac{p}{g} ds = \frac{1}{2} \int_{in}^{out} \sum \rho_f \frac{p}{g} ds - \int_{in}^{out} \sum \rho_f g ds - \int_{in}^{out} \Sigma (F.P.D) g * ds.
$$

This equation is applied to the individual regions, i.e., core, chimney, upper plenum, downcomer, lower dome and lower plenum.

It may be noted that:

(a) The term on the left of Eq. (21) is the momentum change rate.

(b) It can be converted to

$$
\frac{\partial}{\partial t} \int_{in}^{out} \rho_f \frac{p}{g} ds = \frac{1}{2} \int_{in}^{out} \sum \rho_f \frac{p}{g} ds - \int_{in}^{out} \sum \rho_f g ds - \int_{in}^{out} \Sigma (F.P.D) g * ds.
$$

Summing the terms over the whole individual regions, it will become

$$
\frac{\partial}{\partial t} \int_0^{\text{Total region}} \rho_f \frac{p}{g} ds = \frac{1}{2} \int_{in}^{out} \sum \rho_f \frac{p}{g} ds - \int_{in}^{out} \sum \rho_f g ds - \int_{in}^{out} \Sigma (F.P.D) g * ds.
$$

where the "effective length of the recirculation loop" means the sum of the flow lengths of all the individual regions, each length being weighted by ratio between the flow area of the region considered and the area in the active core region.

(b) The first term on the right hand is the momentum flow-in minus the momentum flow-out, or the net momentum flow-in. Summing the terms for the whole loop, the result will be nil. The momentum flow-out at the free surface of reactor water and the momentum flow-in at the feed water sparger ring may be neglected since they are small and their momentum vectors are normal to the main loop vector considered.

(c) The second term on the right hand is the momentum change, due to the change of void fraction, which is typical of
boiling water reactor cores. This term need not be taken into consideration in any region, except the core, since the void fraction change along the flow path outside the core can be neglected. The term for the core region can be approximated by \( \frac{1}{2} K_1 f W_0 \). It may be regarded as a kind of pressure drop term.

(d) The third term is the gravitational pressure. This effect can be neglected in such regions as the upper and lower plenums, and lower dome. The gravitational force depends upon the void fraction, thus also pressure.

(e) The last term represents the frictional and geometrical pressure drops, and is expressed by \( RW_0^2 \), a function proportional to the square of the mass flow. \( R \) is a function of the Fanning factor (for single phase flow), density, hydraulic dimension and flow cross section, while in the case of two-phase flow, \( R \) is determined predominantly by the steam quality or void fraction, and thus represented approximately by \( R = K f \). It is known that the geometrical and frictional pressure drops in the core are the predominant contributions to the total pressure drop through the whole system. Therefore the term can be expressed by \( K f W_0^2 \).

Summing the second and last terms, it will be \( K f W_0^2 \). In the actual reactor, however, as \( W_0 \) increases, the amount of voids carried with the coolant out of the core also increases; this results in a reduction of void fraction, so that the effective pressure drop is nearly proportional to the mass flow rate, not to the square thereof. It is consequently expressed by \( K_4 W_0 \).

The constant \( K_4 \) is determined from the steady-state condition; thus, \( K_4 W_0 = \text{Gravitational force} \) in Eq. (21).

Taking into consideration the expressions above described, then deriving the small fluctuations during the steady-state, and rendering the equations linear, we obtain from Eq. (21),

\[
\frac{L}{A_2} \frac{\partial \delta W_0}{\partial \bar{t}} = D_i \delta f + D_i \delta p - K_i \delta W_0, \tag{22}
\]

where

\[
\begin{align*}
\bar{L} & = L + \frac{A_2}{A_{2\text{at}}} L_{\text{at}} + L_{\text{at}} + \frac{A_2}{A_{2\text{sub}}} (L_{\text{sub}} + L_{\text{at}}), \\
& + \frac{A_2}{A_{2\text{up}}} L_{\text{up}} + A_{1\text{p}} L_{\text{p}} + A_{1\text{p}}, \\
D_i & = \left( \frac{1}{v_e} - \frac{1}{v_s^2} \right) L_e + h L_e - \delta L_{\text{at}}, \\
D_i & = \left( \frac{1}{v_s^2} \frac{\partial \nu_s}{\partial \rho} - \frac{1}{v_s^2} \frac{\partial \nu_s}{\partial \rho} \right) x(L_e + h L_e - \delta L_{\text{at}}), \\
k & = f_i \bar{f} \text{ and } k' = f_0 \bar{f}.
\end{align*}
\]

6. Determination of Void Fraction

There are many parameters that influence the void fraction, e.g., the steam quality, inlet speed, subcooling, pressure, pressure etc. The void fraction in a natural circulation boiling water reactors however is determined uniquely by only one of the above parameters, provided the feed water enthalpy is fixed. This fact may be explained as follows:

The steady-state thermal balance equation is

\[
Q' = W_i (i - i_{\text{mw}}), \tag{23}
\]

indicating that, under a specified pressure condition, the steam flow is proportional to the reactor power when the feed water enthalpy is determined.

From the steady-state momentum balance equation, the in-core fluid density, which is a function of the void fraction, can be related to the exit steam quality:

\[
\sum \rho_i \delta l = K W_s = K W_{\rho_s} x_s \tag{24}
\]

The Bankoff formula, introduced for the slip ratio, gives another relation between the void fraction and steam quality:

\[
x_s = \frac{W_{\rho_i}}{W_0} = \gamma \frac{f_0 \rho_s}{\rho_w} = k' \frac{f_0 \rho_s}{\rho_w} \tag{25}
\]

Both Eqs. (24) and (25) are monotonic curves, so that the one operating point can be exclusively determined by these two equations for a given reactor power or steam flow and feed water enthalpy.

In Fig. 3, one group of curves, \( a \) and \( a' \), represents Eq. (25), and the other group, \( b \) and \( b' \), Eq. (24).
Figure 4 shows the average void fraction vs. the exit quality calculated for various feed water or core inlet enthalpies, and various inlet flow speeds. It is seen that the exit steam quality has the largest influence upon the average void fraction, whereas the feed water enthalpy is less influential or even negligible. Consequently, $f$ can be expressed in one general form

$$f = Kx_e = K \frac{W_{st}}{W_0 + W_{lmt}}. \quad (26)$$

Figure 5 shows a block diagram of the dynamics model developed from the equations already given.

IV. JPDR TRANSIENT ANALYSIS

1. Application of the Model to JPDR Transient Analysis

In order to confirm the adequacy of the model developed, it was applied to the analysis of the results of the JPDR transient tests.

For the purpose of comparison between the experimental and calculated data, four kinds of transient tests were performed, comprising, as described below, turbine bypass regulator (BPR) and/or turbine initial pressure regulator (IPR) tests and BPR oscillation test. Figure 6 shows the BPR and IPR systems which actuate the valve concerned upon receiving a pressure change signal.
curves and the thin lines the calculated.

In Fig. 7 The control rod was moved \( \pm 30^\circ \) under BPR control at rated power.

In Fig. 8 The BPR set point was changed by about \( \pm 10 \) psi under BPR control at the same power.

In Fig. 9 The IPR set point was changed by about \( \pm 10 \) psi under IPR control also at the same power.

In Fig. 10 The BPR set point was changed under both BPR and IPR controls at the same power.

All the figures show that the calculated curves are very close to the experimental, evidencing that the model applied to the analysis of JPDR transient is quite valid and gives results consistent with experiment.

It should be noted however that:
(a) Continuous flux fluctuation in the category of what is known as void noise, with
small amplitude and short time constant, is beyond the analysis by the present model.

(b) The rather small calculated value of steam flow overshoot just after the BPR set point change, shown in Fig. 8 is thought to be due to the insufficiently accurate simulation of BPR in the frequency range above 1 cycle/sec.

(c) The type of plant operation such as represented by Fig. 10, i.e., simultaneous BPR and IPR control, is very seldom undertaken in practice. The difference between calculated and experimental, $W_{s1}$ passing through the BPR system is mainly due to the nonlinearity of the BPR valve position against the steam flow. Nevertheless, marked dips appear in the calculated curve both when the flow is increasing and when decreasing. This is typical of this kind of operation, and is due to the slower motion of the IPR system compared to the BPR.

(2) BPR Oscillation Test

In order to check the frequency dependency of the JPDR dynamic characteristics, a sinusoidal signal was applied to the BPR to oscillate the steam flow through the BPR valve.

Figures 11 (a) and (b) show the measured and calculated results, (a) representing the gain and (b) the phase. It will be seen that the calculated values are very close to the measured.
The neutron flux peak, between values around 0.7 and 1.25 cycles/sec, is thought to be due to void noise and the present analytical model cannot explain it.

3. Discussion

(1) Graphical Analysis of BPR and IPR Tests

Figure 12 (a) is a graphical analysis of the BPR set point change test. The set point reduction, equivalent to the apparent increase of reactor pressure, causes the BPR valve to open, resulting in an increase of the steam flow through the BPR valve, and also in a pressure reduction. The transient locus, therefore, is that seen as curve A-1. When the set point steps up, the locus will be A-2.

Figure 12(b) also shows a graphical analysis of the BPR set point change test under BPR+IPR operation.

Due to the faster response of the BPR system to BPR set point change, as compared to the IPR, the locus starts to describe a curve that looks as if it were heading toward P″. This is indicated by the dotted line in Fig. 12(b). In the mean time, IPR starts to close the turbine control valve upon receiving a "pressure low" signal. This is equivalent to a displacement of the locus destination along the line P″, P, P₁, ..., and finally the movement ends at point P, where the steam flow reduction, due to turbine control valve closure, balances with the steam flow increase through the BPR valve. Therefore the transient locus will be A-1 when the set point steps down, and A-2 when it steps up.

This kind of graphical analysis can be applied to other transient tests, such as those under automatic reactor control[7].

(2) Comparison of the Present Model with Others

In order to verify the validity of this model, a comparison was made with other models. Examples chosen for the comparison are the BPR and IPR tests and the BPR oscillation test.

Miida's and Hogle's models were selected. Some values they used for the constant terms were corrected when the values differed greatly from those newly determined.

Figure 13 shows the transient data of the BPR set point change test using Hogle's model. It is seen that slight discrepancies are seen between the calculated and the measured values in the overshoot, oscillation time constant and attenuation factors.

Figures 14 (a) and (b) show the frequency characteristics of the transfer function from the steam flow to the neutron flux, (a) giving the gain and (b) the phase; the four curves represent respectively the measured, and the calculated ones by three models, i.e. the author's, Miida's and Hogle's. Although the four curves are close to one another, that of Hogle shows a tendency to deviate from the measured values as the frequency becomes lower, in Miida's, the deviation increases if carry-under which was not considered in his original model is taken into account.

Figures 15 (a) and (b) represent the transfer functions from the steam flow to the pressure.

Four curves are shown as before, again found quite close to one another. Miida's model gives a curve very close to the measured in the region of 0.02 to 0.1 cycle/sec, but the curve deviates gradually in other frequency ranges.
regions. The author’s model gives a curve running roughly parallel to the measured one but with a small discrepancy of about 3 db (gain curve).

From these calculations and comparisons, it can be concluded that the present model is very effective in calculating the actual BWR transient characteristics, compared with other models.

(3) Limitations of the Present Model

Because this model employs lumped parameters and is linear, it is best suited to “macroscopic” core transient analysis; the effect of void noise cannot be treated, neither the continuous random fluctuation of the neutron flux; nor can it provide precise information on local changes of the various parameters within the reactor core.

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Fig. 13 BPR Set Point Change by Hogle’s Model

Fig. 14 BPR Oscillation $\delta \phi/\delta W_2$

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(a) Grain

(b) Phase


V. CONCLUSION

The present paper first discussed the analytical model with distributed parameters, currently used for BWR core transient analysis. From actual experience with JPDR plant operation, it was found that some of the assumptions on which the model was based were inadequate.

With the view to developing a model with assumptions more suitable for analyzing the actual plant dynamics, a model with lumped parameters rather than distributed was conceived. The new model, refined in several points, was found to be better than the current model for dynamic calculations.

The model developed was applied to the analysis of the JPDR transient tests under BPR and IPR control, and BPR oscillation test.

A comparison was made between this model and others, and it was found that this model could give a very valid analysis of the BPR transients.

The fundamental dynamics analysis of any BWR will be possible with the model, but minor modification may be necessary when applied to the forced-circulation type.

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[Nomenclature]

A: Area  B: Buckling

F: External force, arbitrary function
**PPD:** Frictional pressure drop

**f:** Void fraction  
**g:** Gravity force

**i:** Enthalpy  
**J:** Joule

**K:** Arbitrary constant  
**k:** Arbitrary constant

**L:** Diffusion length  
**l:** Neutron life

**M:** Mass  
**N:** Neutron density

**P:** Reactor power  
**p:** Pressure

**Q:** Thermal flux or power  
**q:** Apparent force upon void

**R:** Martinelli constant

**s:** Laplacian transform variable

**t:** Time  
**U:** Apparent void velocity

**u:** Internal energy  
**V:** Volume

**v:** Specific volume  
**W:** Mass flow

**w:** Velocity  
**x:** Steam quality

**z:** Height

**β:** Delayed neutron fraction

**γ:** Slip ratio  
**ϕ:** Arbitrary function

**Φ:** Arbitrary function  
**ϕ:** Neutron flux

**λ:** Decay factor  
**τ:** Time constant

**ρ:** Density

**Suffix**

0: Core inlet; zero; outer
1: Boiling boundary; one
2: Core outlet; two  
3: Water surface
4: Top of downcomer  
BPR: Bypass regulator
C: Clean-up system, core

**cb:** Chimney  
**D:** Downcomer

**e:** Exit  
**eff:** Effective

**F:** Fuel  
**FW:** Feedwater

**IPR:** Initial pressure regulator

**i:** i-th, inner  
**LD:** Lower dome

**LP:** Lower plenum

---**REFERENCES**---

(7) JPDR Division: JAERI-1067, (1964).